ARITHMETIC PROGRESSIONS

★ INTRODUCTION

Consider the following arrangement of numbers :

(i) 1, 3, 5, 7, …….. (ii) 3, 6, 12, 24, …….. (iii) 1, 4, 9, 16, ……..

In each of the above arrangements, we observe some patterns. In (i) we find that the succeeding terms are obtained by adding a fixed number [i.e. 2], in (ii) by multiplying with a fixed number [i.e. 2], in (iii) we find that they are squares of natural numbers.

In this chapter, we shall discuss one of these patterns in which succeeding terms are obtained by adding a fixed number to the preceding terms. We shall also see how to find their n^{th} terms and the sum of n consecutive terms, and use this knowledge in solving some daily life problems.

★ HISTORICAL FACTS

Gauss was a very talented and gifted mathematician of 19^{th} century who developed the formula :

$1 + 2 + 3 + 4 + \ldots. + (n-1) + n = \frac{n(n+1)}{2}$ for the sum of first n natural numbers at the age of 10. He did this in the following way :

$$S = 1 + 2 + 3 \ldots.. + (n-2) + (n-1) + n$$
$$S = \underline{n + (n-1) + (n-2) + \ldots.. + 3 + 2 + 1}$$
$$2S = (n+1) + (n+1) + (n+1) + \ldots. + (n+1) + (n+1) + (n+1)$$
$$= (n+1)(1 + 1 + 1 + \ldots \text{ upto n times})$$
$$2S = (n+1)\,n \Rightarrow S = \frac{n(n+1)}{2}$$

Even when he was a little child of three he could read and make mathematical calculation himself. Gauss proved the fundamental theorem of Algebra when he was 20 years old. His contribution to mathematics has been immense because his formulae were used in applied field of Astronomy, Differential Geometry and Electricity widely all over the world by scientists.

★ SEQUENCE

In our daily life, we come across the arrangement of numbers or objects in an order such as arrangement of students in a row as per their roll numbers, arrangement of books in the library, etc.

An arrangement of numbers depends on the given rule :

Given Rule	Arrangement of numbers
Write 3 and then add 4 successively	3. 7, 11, 15, 19,….
Write 3 and then multiply 4 successively	3, 12, 48, 192,…..
Write 4 and then subtract 3 successively	4, 1, - 2, -5,…..
Write alternately 5 and – 5	5, - 5, 5, -5,…

Thus, a sequence is an ordered arrangement of numbers according to a given rule.

Terms of a Sequence : The individual numbers that form a sequence are the terms of a sequence.

For example : 2, 4, 6, 8, 10,…… forming a sequence are called the first, second third, fourth and fifth,…. terms of the sequence.

The terms of a sequence in successive order is denoted by 'T_n' or 'a_n' The nth term 'T'n is called the general term of the sequence.

★ **SERIES**

The sum of terms of a sequence is called the series of the corresponding sequence. $T_1 + T_2 + T_3 + \ldots$ is an infinite series, where as $T_1 + T_2 + T_3 + \ldots + T_{n-1} + T_n$ is a finite series of n terms.

$$S_n = T_1 + T_2 + T_3 + \ldots + T_{n-2} + T_{n-1} + T_n$$
$$S_{n-1} = T_1 + T_2 + T_3 + \ldots + T_{n-2} + T_{n-1}$$
$$S_n - S_{n-1} = T_n$$

OR $\boxed{T_n = S_n - S_{n-1}}$

Ex.1 Write the first five terms of the sequence, whose nth term is $a_n = \{1 + (-1)^n\}n$.

Sol. $a_n = \{1 + (-1)^n\}n$

Substituting n = 1, 2, 3, 4 and 5, we get

$a_1 = \{1 + (-1)^1\}\ 1 = 0$; $a_2 = \{1 + (-1)^2\}\ 2 = 4$;
$a_3 = \{1 + (-1)^3\}\ 3 = 0$; $a_4 = \{1 + (-1)^4\}\ 4 = 8$;
$a_5 = \{1 + (-1)^5\}\ 5 = 0$

Thus, the required terms are : 0, 4, 0, 8 and 0.

Ex.2 Find the 20th term of the sequence whose nth term is, $a_n = \dfrac{n(n-2)}{n+3}$

Sol. $a_n = \dfrac{n(n-2)}{n+3}$. Putting n = 20, we obtain $a_{20} = \dfrac{20(20-2)}{20+3}$

Thus, $a_{20} = \dfrac{360}{23}$

Ex3. The Fibonacci sequence is defined by $a_1 = 1 = a_2$; $a_n = a_{n-1} + a_{n-2}$ for n > 2. Find $\dfrac{a_{n+1}}{a_n}$, for n = 1,2,3,4,5,

Sol. We have $a_1 = a_2 = 1$ and $a_n = a_{n-1} + a_{n-2}$

Substituting n = 3, 4, 5 and 6, we get,

$a_3 = a_2 + a_1 = 1 + 1 = 2$ $\quad$ $a_4 = a_3 + a_2 = 2 + 1 = 3$
$a_5 = a_4 + a_3 = 3 + 2 = 5$ $\quad$ and $\quad$ $a_6 = a_5 + a_4 = 5 + 3 = 8$

Now, we have to find $\dfrac{a_{n+1}}{a_n}$ for n = 1, 2, 3, 4 and 5

For, $n = 1,\ \dfrac{a_2}{a_1} = \dfrac{1}{1} = 1$

$n = 2,\ \dfrac{a_3}{a_2} = \dfrac{2}{1} = 2$

$n = 3,\ \dfrac{a_4}{a_3} = \dfrac{3}{2}$ $\quad \Rightarrow \quad$ $n = 4,\ \dfrac{a_5}{a_4} = \dfrac{5}{3}$

$n = 5,\ \dfrac{a_6}{a_5} = \dfrac{8}{5}$

Hence, the required values are 1, 2, $\dfrac{3}{2}, \dfrac{5}{3}$ and $\dfrac{8}{5}$

COMPETITION WINDOW

SERIES OF NATURAL NUMBERS

1. The sum of first n natural numbers i.e. $1 + 2 + 3 + \ldots\ldots + n$ is usually written as $\sum n$.

$$\boxed{\sum n = \frac{n(n+1)}{2}}$$

2. The sum of squares of first n natural numbers i.e. $1^2 + 2^2 + 3^2 + \ldots.. + n^2$ is usually written as $\sum n^2$.

$$\boxed{\sum n^2 = \frac{n(n+1)(2n+1)}{6}}$$

3. The sum of cubes of first n natural numbers i.e. $1^3 + 2^3 + 3^3 + \ldots. + n^3$ is usually written as $\sum n^3$.

$$\boxed{\sum n^3 = \left(\frac{n(n+1)^2}{2}\right) = \left(\sum n\right)^2}$$

★ **PROGRESSON**

It is not always possible to write each and every sequence of some rule

For example of prime numbers 2, 3, 5, 7, 11,… cannot be expressed explicitly by stating a rule and we do not have any expression for writing the general term of this sequence.

The sequence that follows a certain pattern is called a progression. Thus, the sequence 2, 3, 5, 7, 11,… is not a progression. In a progression, we can always write the nth term.

Consider the following collection of numbers : (i) 1, 3, 5, 7… (ii) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots.$

From the above collection of numbers, we observe that

(i) Each term is greater than the previous by 2.

(ii) In each term the numerator is 1 and the denominator is obtained by adding 1 to the preceding denominator.

Thus, we observe that the collection of numbers given in (i) and (ii) follow a certain pattern and as such are all progressions.

★ **ARITHMETIC PROGRESSIONS**

An arithmetic progression is that list of numbers in which the first term is given and each term, other than the first term is obtained by adding a fixed number 'd' to the preceding term.

The fixed term **'d'** is known as the **common difference** of the arithmetic progression. It's value can be positive, negative or zero. The **first term** is denoted by **'a'** or **'a_1'** and the **last term** by 'ℓ'.

Ex. Consider a sequence 6, 10, 14, 18, 22, …

Hence, $a_1 = 6, a_2 = 10, a_3 = 14, a_4 = 18, a_5 = 22$

$a_2 - a_1 = 10 - 6 = 4$

$a_3 - a_2 = 14 - 10 = 4$

$a_4 - a_3 = 18 - 14 = 4$

Therefore, the sequence is an arithmetic progression in which the first term a = 6 and the common difference d = 4.

Symbolical form : Let us denote the first term of an AP by a_1, second term by a_2,….nth term by a_n and the common difference by d. Then the AP becomes $a_1, a_2, a_3, \ldots\ldots a_n$.

So, $a_2 - a_1 = a_3 - a_2 = \ldots = a_n - a_{n-1} = d$.

General form : In general form, an arithmetic progression with first term 'a' and common difference 'd' can be represented as follows :

$$a, a + d, a + 2d, a + 3d, a + 4d, \ldots$$

Finite AP : An AP in which there are only a finite number of terms is called a finite AP. It may be noted that each such AP has a last term.

Ex. (a) The heights (in cm) of some students of school standing in a queue in the morning assembly are 147, 148, 149,…., 157.

(b) The minimum temperatures (in degree Celsius) recorded for a week in the month of January in a city arranged in ascending order are – 3.1, - 3.0, - 2.9, - 2.8, - 2.7, - 2.6, - 2.5

Infinite AP : An AP in which the number of terms is not finite is called infinite AP. It is note worthy that such APs do not have a last term.

Ex. (a) 1, 2, 3, 4,……

(b) 100, 70, 40, 10,….

Least Information Required : To know about an AP, the minimum information we need to know is to know both – the first term a and the common difference d.

For instance if the first term a is 6 and the common difference d is 3, then AP is 6, 9, 12, 15,…

Similarly, when

a = - 7, d = - 2, the AP is – 7, - 11, -13,….

a = 1.0, d = 0.1, the AP is 1.0, 1.1, 1.2, 1.3,…

So if we know what a and d are we can list the AP.

Ex.4 In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

(i) The taxi fare after each km when the fare is Rs. 15 for the first km and Rs 8 for each additional km.

(ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every metre of digging, when it costs Rs.150 for the first metre and rises by Rs. 50 for each subsequent metre.

(iv) The amount of money in the account every year, when Rs. 10000 is deposited at compound interest 8% per annum. **[NCERT]**

Sol. (i) Taxi fare for 1 km = Rs. 15 = a_1

Taxi fare for 2 kms = Rs. 15 + 8 = Rs. 23 = a_2

Taxi fare for 3 kms = Rs. 23 + 8 = Rs. 31 = a_3

Taxi fare for 4 kms = Rs. 31 + 8 = Rs. 39 = a_4 and so on.

$a_2 - a_1$ = Rs. 23 – 15 = Rs. 8

$a_3 - a_2$ = Rs. 31 – 23 = Rs. 8

$a_4 - a_3$ = Rs. 39 – 31 = Rs. 8

i.e., $a_{k+1} - a_k$ is the same everytime.

So, this list of numbers form an arithmetic progression with the first term a = Rs 15 and the common difference d = Rs. 8

(ii) Amount of air present in the cylinder = x units (say) = a_1

Amount of air present in the cylinder after one time removal of air by the vacuum pump $= x - \frac{x}{4} = \frac{3x}{4}$ units a_2

Amount of air present in the cylinder after two time removal of air by the vacuum pump $= -\frac{3x}{4} - \frac{1}{4}\left(\frac{3x}{4}\right) = \frac{3x}{4} - \frac{3x}{16} = \frac{9x}{16}$ units $= \left(\frac{3}{4}\right)^2$ x units = a_3

Amount of air present in the cylinder after three times removal of air by the vacuum pump $= \left(\frac{3}{4}\right)^2 x - \frac{1}{4}\left(\frac{3}{4}\right)^2 x \Rightarrow \left(1-\frac{1}{4}\right)\left(\frac{3}{4}\right)^2 x \Rightarrow \left(\frac{3}{4}\right)\left(\frac{3}{4}\right)^2 x = \left(\frac{3}{4}\right)^3$ x units = a_4 and so on.

$a_2 - a_1 = \frac{3x}{4} - x = \frac{x}{4}$ units $\Rightarrow$ $a_3 - a_2 = \left(\frac{3}{4}\right)^2 x - \frac{3}{4}x = -\frac{3}{16}$ x units

As $a_2 - a_1 \neq a_3 - a_2$, this list of numbers does not form an AP.

(iii) Cost of digging the well after 1 metre of digging = Rs. 150 = a_1

Cost of digging the well after 2 metres of digging = Rs. 150 + 50 = Rs 200 = a_2

Cost of digging the well after 3 metres of digging = Rs. 200 + 50 = Rs 250 = a_3

Cost of digging the well after 4 metres of digging = Rs. 250 + 50 = Rs 300 = a_4

and so on.

$a_2 - a_1$ = Rs 200 – 150 = 50 $a_3 - a_2$ = Rs 250 – 200 = 50

$a_4 - a_3$ = Rs 300 – 250 = 50

i.e., $a_{k-1} - a_k$ is the same everytime. So this list of numbers forms an AP with the first term a = Rs. 150 and the common difference d = Rs.50

(iv) Amount of money after 1 year = Rs. 10000 $\left(1+\frac{8}{100}\right) = a_1$

Amount of money after 2 year = Rs. 10000$\left(1+\frac{8}{100}\right)^2 = a_2$

Amount of money after 3 year = Rs. 10000 $\left(1+\frac{8}{100}\right)^3 = a_3$

Amount of money after 4 years = Rs. 10000 $\left(1+\frac{8}{100}\right)^4 = a_4$

$a_2 - a_1$ = Rs. 10000 $\left(1+\frac{8}{100}\right)^2$ - Rs. 10000 $\left(1+\frac{8}{100}\right)$

= Rs. 10000 $\left(1+\frac{8}{100}\right)$ $\left(1+\frac{8}{100}-1\right) \Rightarrow$ Rs. 10000 $\left(1+\frac{8}{100}\right)\left(\frac{8}{100}\right)$

$a_3 - a_2$ = Rs. 10000 $\left(1+\frac{8}{100}\right)^3$ - Rs. 10000 $\left(1+\frac{8}{100}\right)^2 \Rightarrow$ Rs. 10000 $\left(1+\frac{8}{100}\right)^2$ $\left(1+\frac{8}{100}-1\right)$ =

Rs. 10000 $\left(1+\frac{8}{100}\right)^2$ $\left(\frac{8}{100}\right)$

As $a_2 - a_1 \neq a_3 - a_2$, this list of numbers does not form an AP.

Ex.5 Write first four terms of the AP, when the first term a and the common difference d are given as follows
(i) $a = 4, d = 5$ (ii) $a = -1.25, d = -0.25$

Sol. (i) $a = 4, d = 5$

First term, $a = 4$

Second term $= 4 + d = 4 + 5 = 9$

Third term $= 9 + d = 9 + 5 = 14$

Fourth term $= 14 + d = 14 + 15 = 19$

Hence, first four terms of the given AP are 4, 9, 14, 19.

(ii) $a = -1.25, d = -0.25$

First term = a = -1.25

Second term = $-1.25 + d = -1.25 + (-0.25) = -1.50$

Third term = $-1.50 + d = -1.50 + (-0.25) = -1.75$

Fourth term = $-1.75 + d = -1.75 + (-0.25) = -2.00$

Hence, first four terms of the given AP are $-1.25, -1.50, -1.75, -2.00$

Ex.6 For the AP $\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}, \ldots\ldots$ write the first term a and the common difference d. Also write the next two terms after the given last term $-\frac{3}{2}$.

Sol. We have $a_1 = \frac{3}{2}, a_2 = \frac{1}{2}, a_3 = -\frac{1}{2}, a_4 = -\frac{3}{2}$ and so on.

Thus, $a = \frac{3}{2}$

$a_2 = a_1 = \left(\frac{1}{2}\right) - \left(\frac{3}{2}\right) = -1,$

$a_3 - a_2 = \left(-\frac{1}{2}\right) - \left(\frac{1}{2}\right) = -1,$

$a_4 - a_3 = \left(-\frac{3}{2}\right) - \left(-\frac{1}{2}\right) = -1$, and so on.

$\Rightarrow$ $d = -1$

Now, we find the successor of $-\frac{3}{2}$.

$a_5 = \left(-\frac{3}{2}\right) + d = \left(-\frac{3}{2}\right) + (-1) = -\frac{5}{2}$

Then $a_6 = a_5 + d = \left(-\frac{5}{2}\right) + (-1) = -\frac{7}{2}$

Hence, the next two terms after the given term $-\frac{3}{2}$ *are* $-\frac{5}{2}, -\frac{7}{2}$.

COMPETITION WINDOW

GEOMETRIC PROGRESSION

1. A sequence of non-zero numbers $a_1, a_2, a_3,\ldots,a_n$ is said to be a geometric sequence or G.P.

iff $\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \ldots\ldots$

i.e. iff $\frac{a_{n+1}}{a_n}$ = a constant for all n.

This constant is called the common ratio of the G.P. and is usually denoted by 'r'. e.g., 3, 9, 27, 81,….
A general G.P. is a, ar, ar^2,…
When the terms of a geometric sequence are added, we get a geometric series.

HARMONIC PROGRESSION

A sequence of non-zero numbers $a_1, a_2,\ldots a_n$ is said to be a harmonic sequence or H.P.

iff $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots. \frac{1}{a_n}$ are in A.P.

e.g., (i) 12, 6, 4, 3….. (ii) 10, 30, - 30, - 10, - 6,…..

A general H.P. is $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d} + \ldots.$

Where is the first term and d is the common difference of the A.P.

★ **GENERAL TERM OF AN ARITHEMETIC PROGRESSION**

The formula for writing general term or the nth term of an arithmetic progression is

$a_n = a + (n - 1)d$

Where, a is the first term of arithmetic progression,
and d is the common difference of arithmetic progression.

★ **r^{th} TERM OF FINITE ARITHMETIC PROGRESSION FROM THE END**

Let there be an arithmetic progression with first term a and common difference d. If there are n terms in the arithmetic progression, then

r^{th} term from the end = a + (n – r)d

Also, if ℓ is the last term of the arithmetic progression then r^{th} term from the end is the r^{th} term of an arithmetic progression whose first term is ℓ and common difference is – d.

r^{th} term form the end = ℓ + (r – 1) (- d)

Ex.7 Find the 30th term of the AP : 10, 7, 4…. **[NCERT]**

Sol. The given A.P. is 10, 7, 4,…..
Here, a = 10, d = 7 – 10 = - 3 and n = 30
we have $a_n = a + (n - 1)d$
So, $a_{30} = 10 + (30 - 1)(-3) \Rightarrow a_{30} = 10 - 87 \Rightarrow a_{30} = -77$
∴ The 30^{th} term of the given AP is – 77.

Ex.8 The 6^{th} term of an arithmetic progression is – 10 and the 10^{th} term is – 26. Determine the 15^{th} term of the AP.

Sol. Let first term and the common difference of the AP be a and d respectively.

6^{th} term = - 10 (Given)

$\Rightarrow$ a + 5 (6 – 1)d = - 10 [$\because a_n = a + (n - 1)d$]

$\Rightarrow$ a + 5d = - 10 ....(i)

10^{th} term = - 26 (Given)

$\Rightarrow$ a + (10 – 1)d = - 26

$\Rightarrow$ a + 9d = - 26 ....(ii)

Solving (i) and (ii) we get

a = 10, d = - 4

Therefore, 15^{th} term of the AP

= a + (15 – 1)d [$\because a_n = a + (n - 1)d$]

= a + 14d

= 10 + 14 (-4)

= 10 – 56 = - 46

Hence, the 15^{th} term of AP is – 46.

Ex.9 Find the 6th term from the end of the AP 17, 14, 11,...., - 40.

Sol. The given AP 17, 14, 11,...., - 40

Here, a = 17, d = 14 – 17 = - 3, ℓ = - 40

Let there be n terms in the given AP.

Then, nth term = - 40

$\Rightarrow$ a + (n – 1)d = - 40 [$\because a_n = a + (n - 1)d$]

$\Rightarrow$ 17 + (n – 1) (-3) = - 40

$\Rightarrow$ (n – 1) (-3) = - 40 – 17

$\Rightarrow$ (n – 1) (-3) = - 57

$\Rightarrow$ $n - 1 = \frac{-57}{-3}$

$\Rightarrow$ n – 1 = 19

$\Rightarrow$ n = 19 + 1

$\Rightarrow$ n = 20

Hence, there are 20 terms in the given AP, Now, 6th term from the end

= a + (20 – 6)d [$\because$ rth term from the end = a + (n – r)d]

= a + 14d

= 17 + 14(-3)

= 17 – 42 = - 25

Hence, the 6th term from the end of the given AP is – 25.

Ex.10 is 200 any term of the sequence 3, 7, 11, 15,…?

Sol. The given sequence is 3, 7, 11, 15,…

$a_2 - a_1 = 7 - 3 = 4 \quad \Rightarrow \quad a_3 - a_2 = 11 - 7 = 4 \quad \Rightarrow \quad a_4 - a_3 = 15 - 11 = 4$

As $a_{k+1} - a_k$ is the same for k = 1, 2, 3, etc., the given sequence form an AP.

Here, a = 3, d = 4

Let 200 be the nth term of the given sequence. Then,

$a_n = 200$

$\Rightarrow a + (n - 1)d = 200 \qquad \Rightarrow 3 + (n - 1)\,4 = 200$

$\Rightarrow (n - 1) = \frac{197}{4} \qquad \Rightarrow n = \frac{197}{4} + 1 \Rightarrow n = \frac{201}{4}.$

But n should be a positive integer. So, 200 is not term of the given sequence.

COMPETITION WINDOW

GENERAL TERM OF A.G.P.

The nth terms of a G.P. is a, ar^2,….ar^{n-1} is $\boxed{T_n = ar^{n-1}}$

rth TERM FROM THE END OF FINITE G.P.

Let a be the first term and r be the common ratio of a finite G.P. consisting of n terms, then

$\boxed{\text{rth term from the end} = ar^{n-r}}$

Also, if ℓ is the last term of the G.P.then

$$\boxed{\text{rth term from the end} = \ell\left[\frac{1}{r}\right]^{n-1}}$$

GENERAL TERM OF A H.P.

To find the nth term of an H.P., find the nth term of the corresponding A.P. obtained by the reciprocals of the terms of the given H.P. Now the reciprocal of the nth term of an A.P., will be the nth term of the H.P.

Try out the Following:

1. Find the 9th term and the general term of the progression $\frac{1}{4}, \frac{-1}{2}, 1, -2....$

2. Which term of the G.P. 5, 10, 20, 40, ….is 5120?

3. The fourth, seventh and the last term of a G.P. are 10, 80 and 2560 respectively Find the first term and the number of terms in the G.P.

4. Find the 9th term of progression $\frac{1}{7} + \frac{1}{14} + \frac{1}{21} + \frac{1}{28} + ...$

5. If the pth term of a H.P. is qr and it's qth term is pr, then find it's rth term.

6. Find the 6th term of the series $2 + 1\frac{3}{4} + 1\frac{5}{9}, ...$

ANSWER KEY

1. 64; $(-1)^{n-1}\, 2^{n-3}$ **2.** 11^{th} term **3**. $\frac{10}{8}$;12 terms **4**. $\frac{1}{63}$ **5**. pq **6.** $\frac{7}{6}$

★ SELECTION OF TERMS IN AN AP

Sometimes we require certain number of terms in AP. The following ways of selecting terms are generally very convenient.

Number of terms	Terms	Common difference
3	a-d, a, a + d	d
4	a – 3d, a – d, a + d, a + 3d	2d
5	a – 2d, a – d, a, a + d, a + 2d	d
6	a – 5d, a – 3d, a – d, a + 3d, a + 5d	2d

It should be noted that in case of an odd number of terms, the middle term is a and the common difference is d while in case of an even number of terms the middle terms are a – d, a + d and the common difference is 2d.

Remark–1 : If the sum of terms is not given, then select terms as a, a + d, a + 2d,….

Remark–2 : If three numbers a, b, c in order are in AP. Then

$$b - a = \text{Common difference} = c - b$$

$$\Rightarrow \quad b - a = c - b \quad \Rightarrow \quad 2b = a + c$$

Thus, a, b, c are in AP if and only if $\boxed{2b = a + c}$

Remark–3 : If a, b, c are in AP, then b is known as the arithmetic mean (AM) between a and c.

Remark–4 : If a, x, b are in AP Then,

$$2x = a + b \Rightarrow x = \frac{a+b}{2}$$

Thus, AM between a and b is $\frac{a+b}{2}$.

Ex.11 The sum of three numbers in AP is -3, and their product is 8. Find the numbers.

Sol. Let the numbers be (a – d), a, (a + d). Then,

Sum = - 3 $\Rightarrow$ (a – d) + a + (a + d) = - 3 $\Rightarrow$ 3a = - 3 $\Rightarrow$ a = - 1

Now, product = 8

$\Rightarrow$ (a – d) (a) (a +d) = 8 $\Rightarrow$ $a(a^2 - d^2) = 8$

$\Rightarrow$ $(-1)(1 - d^2) = 8$ [$\because$ a = - 1]

$\Rightarrow$ $d^2 = 9 \Rightarrow d = \pm 3$

If d = 3, the numbers are – 4, -1, 2. If d = - 3, the numbers are 2, - 1, - 4

Thus, the numbers are – 4, - 1, 2 or 2, - 1, - 4

Ex.12 Find four numbers in AP, whose sum is 20 and the sum of whose squares is 120.

Sol. Let the numbers be (a – 3d), (a – d), (a + d), (a + 3d). Then,

Sum = 20

$\Rightarrow$ (a – 3d) + (a –d) + (a + d) + (a + 3d) = 20 $\Rightarrow$ 4a = 20 $\Rightarrow$ a = 5

Now sum of the squares = 120

$\Rightarrow$ $(a - 3d)^2 + (a - d)^2 + (a + d)^2 + (a + 3d)^2 = 120$

$\Rightarrow$ $4a^2 + 20d^2 = 120$

$\Rightarrow$ $a^2 + 5d^2 = 30 \Rightarrow 25 + 5d^2 = 30$

$\Rightarrow$ $25 + 5d^2 = 30 \Rightarrow 5d^2 = 5 \Rightarrow d = \pm 1$

If d = 1, then the numbers are 2, 4, 6, 8. If d = - 1, then the numbers are 8, 6, 4, 2.

Thus, the numbers are 2, 4, 6, 8 or 8, 6, 4, 2.

Ex.13 If 2x, x + 10, 3x + 2 are in AP. Find the value of x.

Sol. Since, 2x, x + 10, 3x + 2 are in AP.

$\therefore$ 2 (x + 10) = 2x + (3x + 2)

$\Rightarrow$ 2x + 20 = 5x + 2

$\Rightarrow$ 3x = 18

$\Rightarrow$ x = 6.

COMPETITION WINDOW

SELECTION OF TERMS IN G.P.

Sometimes it is required to select a finite number of terms in G.P. It is always convenient if we select the terms in the following manner :

No. of Terms	Terms	Common Ratio
3	$\frac{a}{r}$, a, ar	R
4	$\frac{a}{r^3}, \frac{a}{r}$, ar, ar^3	r^2
5	$\frac{a}{r^2}, \frac{a}{r}$, a, ar, ar^2	r

If the product of the numbers is not given, then the numbers are taken as a, ar, ar^2, ar^3,…

TRY OUT THE FOLLOWING

1. If the sum of three numbers in G.P. is 38 and their product is 1728, find them.

2. Find the three numbers in G.P. is whose sum is 13 and the sum of whose squares is 91.

3. Find four numbers in G.P. whose sum is 85 and product is 4096.

4. Three numbers are in G.P. whose sum is 70. If the extremes be each multiplied by 4 and the means by 5, they will be in A.P. Find the numbers.

5. Find four numbers in G.P. in which the third term is greater than the first by 9 and the second term is greater than the fourth by 18.

6. The product of first three terms of a G.P. is 1000. If 6 is added to it's second term and 7 added to its third term, the terms become in A.P. Find the G.P.

ANSWERS

1. 8, 12, 18, or 18, 12, 8 **2.** 1, 3, 9 or 9, 3, 1 **3.** 1, 4, 16, 64 or 64, 16, 4, 1

4. 10, 20, 40, or 40, 20, 10 **5.** 3, - 6, 12, -24 **6.** 5, 10, 20,… or 20, 10, 5…

★ **SUM TO N TERMS OF AN ARITHMETIC PROGRESSION**

The sum S_n of n terms of an arithmetic progression with first term 'a' and common difference 'd' is

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

OR $$S_n = \frac{n}{2}[a + \ell]$$

Where ℓ = last term.

Remark–1 : In the formula $S_n = \frac{n}{2}[2a + (n-1)d]$, there are four quantities viz. S_n, a, n and d. If any three of these are known, the fourth can be determined. Sometimes, two of these quantities are given.
In such a case, remaining two quantities are provided by some other relation.

Remark–2 : If the sum S_n of n terms of a sequence is given, then n^{th} term a^n of the sequence can be determined by using the following formula :

$\mathbf{a_n = S_n - S_{n-1}}$

i.e., the nth term of an AP is the difference of the sum to first n terms and the sum to first (n – 1) terms of it.

Ex.14 Find the sum of the AP: $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots.,$ to 11 terms. **[NCERT]**

Sol. Here, $a = \frac{1}{15}$

$d = \frac{1}{12} - \frac{1}{15} = \frac{1}{60}$

n = 11

We know that

$\Rightarrow \quad S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad S_{11} = \frac{11}{2}\left[2\left(\frac{1}{15}\right) + (11-1)\left(\frac{1}{60}\right)\right] \Rightarrow S_{11} = \frac{11}{2}\left[\frac{2}{15} + \frac{1}{6}\right]$

$\Rightarrow \quad S_{11} = \frac{11}{2}\left[\frac{3}{10}\right] \Rightarrow S_{11} = \frac{33}{20}$

So, the sum of the first 11 terms of the given AP is $\frac{33}{20}$.

Ex.15 Find the sum : 34 + 32 + 30 + …. + 10

Sol. 34 + 32 + 30 + … + 10

This is an AP

Here, a = 34

d = 32 – 34 = - 2

ℓ = 10

Let the number of terms of the AP be n.

We know that

$a_n = a + (n-1)d$

$\Rightarrow \quad 10 = 34 + (n - 1)(-2) \Rightarrow (n - 1)(-2) = -24$

$\Rightarrow \quad n - 1 = \dfrac{-24}{-2} = 12 \Rightarrow n = 13$

Again, we know that

$$S_n = \frac{n}{2}(a + \ell) \Rightarrow S_{13} = \frac{13}{2}(34 + 10)$$

$\Rightarrow \quad S_{13} = 286$

Hence, the required sum is 286.

Ex.16 Find the sum of all natural numbers between 100 and 200 which are divisible by 4.

Sol. All natural numbers between 100 and 200 which are divisible by 4 are

104, 108, 112, 116,….196

Here, $a_1 = 104$

$a_2 = 108$

$a_3 = 112$

$a_4 = 116$

$\vdots \quad \vdots$

$\therefore \quad a_2 - a_1 = 108 - 104 = 4$

$a_3 - a_2 = 112 - 108 = 4$

$a_4 - a_3 = 116 - 112 = 4$

$\vdots \quad \vdots \quad \vdots$

$\because \quad a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots\ldots(= 4 \text{ each})$

$\therefore$ This sequence is an arithmetic progression whose common difference is 4.

Here, $a = 104$

$d = 4$

$\ell = 196$

Let the number of terms be n. Then

$\ell = a + (n - 1)d$

$\Rightarrow \quad 196 = 104 + (n - 1)4$

$\Rightarrow \quad 196 - 104 = (n - 1)4$

$\Rightarrow \quad 92 = (n - 1)4 \quad \Rightarrow \quad (n - 1)4 = 92$

$\Rightarrow \quad n - 1 = \dfrac{92}{4} \quad \Rightarrow \quad n - 1 = 23$

$\Rightarrow \quad n = 23 + 1 \Rightarrow n = 24$

Again, we know that

$$S_n = \frac{n}{2}(a + \ell)$$

$\Rightarrow \quad S_{24} = \left(\frac{24}{2}\right)(104 + 196)$

$= \quad (12)(300) = 3600$

Hence, the required sum is 3600.

Ex.17 Find the number of terms of the AP 54, 51, 48,…so that their sum is 513.

Sol. The given AP is 54, 51, 48,….

Here, a = 54, d = 51 – 54 = - 3

Let the sum of n terms of this AP be 513.

We know that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\Rightarrow \quad 513 = \frac{n}{2}[2(54) + (n-1)(-3)] \qquad \Rightarrow 513 = \frac{n}{2}[108 - 3n + 3]$

$\Rightarrow \quad 513 = \frac{n}{2}[111 - 3n] \qquad \Rightarrow 1026 = n[111 - 3n]$

$\Rightarrow \quad 1026 = 111n - 3n^2 \qquad \Rightarrow 3n^2 - 111n + 1026 = 0$

$\Rightarrow \quad n^2 - 37n + 342 = 0$ [Dividing throughout by 3]

$\Rightarrow \quad n^2 - 18n - 19n + 342 = 0 \qquad \Rightarrow \quad n(n-18) - 19(n-18) = 0$

$\Rightarrow \quad (n-18)(n-19) = 0 \qquad \Rightarrow \quad n - 18 = 0 \text{ or } n - 19 - 0$

$\Rightarrow \quad n = 18, 19$

Hence, the sum of 18 terms or 19 terms of the given AP is 513.

Note : Actually 19^{th} term

$= \quad a_{19}$

$= \quad a + (19-1)d \qquad [\because a_n = a + (n-1)d]$

$= \quad a + 18d$

$= \quad 54 + 18(-3)$

$= \quad 54 - 54 = 0$

Ex.18 Find the AP whose sum to n terms is $2n^2 + n$.

Sol. Here, $S_n = 2n^2 + n$ (Given)

Put n = 1, 2, 3, 4,…., in succession, we get

$S_1 = 2(1)^2 + 1 = 2 + 1 = 3 \qquad S_2 = 2(2)^2 + 2 = 8 + 2 = 10$

$S_3 = 2(3)^2 + 3 = 18 + 3 = 21 \qquad S_4 = 2(4)^2 + 4 = 32 + 4 = 36$

and so on.

$\therefore \quad a_1 = S_1 = 3$

$a_2 = S_2 - S_1 = 10 - 3 = 7$

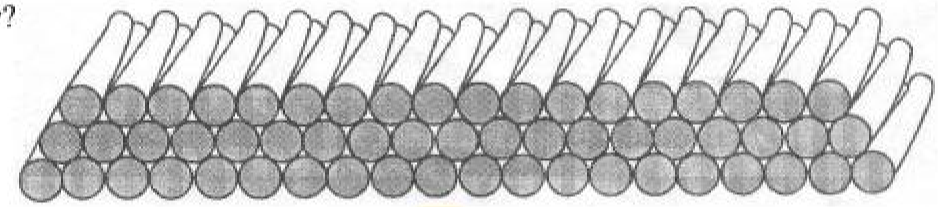
$a_3 = S_3 - S_2 = 21 - 10 = 11$

$a_4 = S_4 - S_3 = 36 - 21 = 15$

and so on.

Hence, the required AP is 3, 7, 11, 15,…

Ex.19 200 logs are stacked in the following manner : 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see figure). In how many rows are the 200 logs placed and how many logs are in the top row?



[NCERT]

Sol. The number of logs in the bottom row, next row, row next to it and so on form the sequence 20, 19, 18, 17, …….

$a_2 - a_1 = 19 - 20 = -1$

$a_3 - a_2 = 18 - 19 = -1$

$a_4 - a_3 = 17 - 18 = -1$

i.e., $a_{k+1} - a_k$ is the same everytime.

So, the above sequence forms an AP.

Here, $a = 20$

$d = -1$

$S_n = 200$

We know that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\Rightarrow \quad 200 = \frac{n}{2}[2(20) + (n-1)(-1)] \quad \Rightarrow \quad 200 = \frac{n}{2}[40 - n + 1]$

$\Rightarrow \quad 200 = \frac{n}{2}[41 - n] \quad \Rightarrow \quad 400 = n[41 - n]$

$\Rightarrow \quad n[41 - n] = 400 \quad \Rightarrow \quad 41n - n^2 = 400$

$\Rightarrow \quad n^2 - 41n + 400 = 0 \quad \Rightarrow \quad n^2 - 25n - 16n + 400 = 0$

$\Rightarrow \quad n(n - 25n) - 16(n - 25) = 0 \quad \Rightarrow \quad (n - 25)(n - 16) = 0$

$\Rightarrow \quad n - 25 = 0 \text{ or } n - 16 = 0 \quad \Rightarrow \quad n = 25 \text{ or } n = 16$

$\Rightarrow \quad n = 25, 16$

Hence, the number of rows is either 25 or 16.

Now, number of logs in row

= Number of logs in 25th row

= a_{25}

= $a + (25 - 1)d \qquad [\because a_n = a + (n-1)d]$

= $a + 24d$

= $20 + 24(-1) \quad \Rightarrow 20 - 24 = -4$

Which is not possible.

Therefore, n = 16 and

Number of log in top row

= Number of logs in 16th row

= a_{16}

= $a + (16 - 1)d \qquad [\because a_n = a + (n-1)d]$

= a + 15 d

= 20 + 15 (-1)

= 20 – 15 = 5

Hence, the 200 logs are placed in 16 rows and there are 5 logs in the top row.

COMPETITION WINDOW

SUM OF n TERMS OF A.G.P.

If S_n is the sum of first n terms of the G.P. a, ar, ar^2,….,

i.e., $S_n = a + ar + ar^2 + ... + ar^{n-1}$, then $\boxed{S^n = \frac{a(1-r^n)}{1-r} or \frac{a(r^n-1)}{r-1}}$,$r \neq 1$

Also, $\boxed{S^n = \frac{a-r\ell}{1-r} or \frac{r\ell - a}{r-1}}$, $r \neq 1$, where ℓ is the last term i.e. the nth term .

For r = 1, Sn = na

SUM OF AN INFINITE G.P.

The sum of an infinite G.P. with first term a and common ratio r, where $-1 < r < 1$, is

$$\boxed{S_\infty = \frac{a}{1-r}}$$

If $r \geq 1$, then the sum of an infinite G.P. tends to infinity.

SUM OF n TERMS OF A H. P.

There is no specific formula to find the sum of n terms of H.P. To solve the questions of this progression, first of all convert it in A.P. then use the properties of A.P.

TRY OUT THE FOLLOWING

1. Find the sum of seven terms of the G.P. 3, 6, 12, ….
2. Find the sum to 7 terms of the sequence $\left[\frac{1}{5}+\frac{2}{5^2}+\frac{3}{5^3}\right], \left[\frac{1}{5^4}+\frac{2}{5^5}+\frac{3}{5^6}\right], \left[\frac{1}{5^7}+\frac{2}{5^8}+\frac{3}{5^9}\right].$
3. Find the sum of the series 2 + 6 +18 +… + 4374.
4. How many terms of the sequence 1, 4, 16, 64…. will make the sum 5461?
5. Find the sum to infinite of the G.P. $\frac{-5}{4}, \frac{5}{16}, \frac{-5}{64}$,….
6. The first term of a G.P. is 2 and the sum of infinity is 6. Find the common ratio
7. If each term of an infinite G.P. is twice the sum of the terms following it, then find the common ratio of the G.P.

ANSWERS

1. 381 **2**. $\frac{19}{62}\left[1-\frac{1}{5^{21}}\right]$ **3**. 6560 **4**. – 1 **5**. $\frac{2}{3}$ **6**. $\frac{1}{3}$

★ **PROPERTIES OF ARITHMETICAL PROGRESSIONS**

1. If a constant is added to or subtracted from each term of an A.P., then the resulting sequence is also an A.P. with the same common difference.
2. If each term of a given A.P. is multiplied or divided by a non-zero constant K, then the resulting sequence is also an A.P. with common difference Kd or d/K, where d is the common difference of the given A.P.
3. In a finite A.P., the sum of the terms equidistant from the beginning and end is always same and is equal to the sum of first and last term.

4. Three numbers a, b, c, are in A.P. iff 2b = a + c.
5. A sequence is an A.P. iff it's nth term is a linear expression in n i.e., $a_n = A_n + B$ are constants. In such a case, the coefficient of n is the common difference of the A.P.
6. A sequence is an A.P. iff the sum of it's first n terms is of the form $An^2 + Bn$, where A, B are constants. independent of n. In such a case, the common difference is 2 A.
7. If the terms of an A.P. are chosen at regular intervals, then they form an A.P.

COMPETITION WINDOW

ARITHMETIC MEANS

1. If there numbers a, b, c are in A.P. then b is called the arithmetic mean (A.M.) between a and c.
2. The arithmetic mean between two numbers a and b is $\frac{a+b}{2}$
3. $A_1, A_2 \ldots, A_n$ are said to be n A.M.s between two numbers a and b. iff a, $A_1, A_2, \ldots, A_n$, b are in A.p. Let d be the common difference of the A.P.

Clearly, b = (n + 2)th term of the A.P.

$\Rightarrow$ b = a + (n + 1) d

$\Rightarrow$ $d = \frac{b-a}{n+1}$

Hence, $A_1 = a + d = a + \frac{b-a}{n+1}$, $A_2 = a + 2d = a + \frac{2(b-a)}{n+1}$

……………………………………………………

……………………………………………………

$A_n = a + nd = a + \frac{n(b-a)}{n+1}$

4. The sum of n A.M.'s between two numbers a and b is n times the single A.M. between then i.e., $n\left[\frac{a+b}{2}\right]$

GEOMETRIC MEANS

1. If three non-zero numbers a, b, c are in G.P. then b is called the geometric mean (G.M.) between a and b.
2. The geometric mean between two positive numbers a and b is $\sqrt{ab}$

HARMONIC MEAN

1. If three non-zero numbers a, b, c are in H.P., then is called the harmonic mean (H.M.) between a and b,
2. The harmonic mean between numbers a and b is $\frac{2ab}{a+b}$

Remark : If A, G, H denote respectively, the A.M., the G.M. and the H.M. between two distinct positive numbers, then

(i) A, G, H are in G.P. (ii) A > G > H

★ **SYNOPSIS**

1. **Sequence :** A sequence is an ordered arrangement of numbers according to a given rule.
2. **Terms :** The numbers in a sequence are called its terms.
3. **Series :** The sum of terms of a sequence is called the series of the corresponding sequence.
4. **Progression :** A progression is a sequence whose terms obey a certain pattern.
5. **Arithmetic Progression :** Arithmetic progression is a sequence if the difference of a term a and its predecessor is always constant.

6. **Common Difference :** The difference between two successive terms of an A.P. is called common difference.

7. **General Term :** General term or nth term or last term of an A.P. is $T_n = \ell = a_n = a + (n - 1)d$, where 'a' is the first term and 'd' the common difference.

8. **Sum of n terms of an A.P. :** $S_n = \frac{n}{2}\{2a + (n-1)d\} = \frac{n}{2}\{a + \ell\}$

Where ℓ = last term = $a + (n - 1)d$

nth term, $a_n = S_n - S_{n-1}$

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOOSE THE CORRECT ONE

1. If the sum of first n terms of an AP be $3n^2 - n$ and it's common difference is 6, then its first term is :
(A) 2 (B) 3 (C) 1 (D) 4

2. If 7^{th} and 13^{th} terms of an A.P. be 34 and 64, respectively, then it's 18^{th} term is :
(A) 87 (B) 88 (C) 89 (D) 90

3. The sum of all 2-digit odd numbers is :
(A) 2475 (B) 2530 (C) 4905 (D) 5049

4. The fourth term of an A.P. is 4. Then the sum of the first 7 terms is :
(A) 4 (B) 28 (C) 16 (D) 40

5. In an A.P. $s_1 = 6$, $s_7 = 105$, then $s_n : s_{n-3}$ is same as :
(A) $(n + 3) : (n - 3)$ (B) $(n + 3) : n$ (C) $n : (n - 3)$ (D) None of these

6. In an A.P. $s_3 = 6$, $s_6 = 3$, then it's common difference is equal to :
(A) 3 (B) – 1 (C) 1 (D) None of these

7. The number of terms common to the two A.P. s
$2 + 5 + 8 + 11 + \ldots + 98$ and $3 + 8 + 13 + 18 + \ldots + 198$
(A) 33 (B) 40 (C) 7 (D) None of these

8. (p + q)th and (p – q)th terms of an A.P. are respectively m and n, The P^{th} term is :
(A) $\frac{1}{2}(m + n)$ (B) $\sqrt{mn}$ (C) $m + n$ (D) mn

9. The first, second and last terms of an A.P. are a, b and 2a. The number of terms in the A.P. is:
(a) $\frac{b}{b-a}$ (B) $\frac{b}{b+a}$ (C) $\frac{a}{b-a}$ (D) $\frac{a}{a+b}$

10. Let s_1, s_2, s_3 be the sums of n terms of three series in A.P., the first term of each being 1 and the common differences 1, 2, 3 respectively. If $s_1 + s_3 = \lambda s_2$, then the value of λ is :
(A) 1 (B) 2 (C) 3 (D) None of these

11. Sum of first 5 terms of an A.P. is one fourth of the sum of next five terms. If the first term = 2, then the common difference of the A.P. is :
(A) 6 (B) – 6 (C) 3 (D) None of these

12. If x, y, z are in A.P., then the value of $(x + y - z)(y + z - x)$ is equal to :
(A) $8yz - 3y^2 - 4z^2$ (B) $8yz - 3z^2 - 4y^2$ (C) $8yz + 3y^2 - 4z^2$ (D) $8yz - 3y^2 + 4z^2$

13. The number of numbers between 105 and 1000 which are divisible by 7 is :
(A) 142 (B) 128 (C) 127 (D) None of these

14. If the numbers a, b, c, d, e form an A.P. then the value of $a - 4b + 6c - 4d + e$ is equal to :
(A) 1 (B) 2 (C) 0 (D) None of these

15. If s_n denotes the sum of first n terms of an A.P., whose common difference is d, then $s_n - 2s_{n-1} + s_{n-2}$ $(n > 2)$ is equal to :
(A) 2d (B) – d (C) d (D) None of these

16. The sum of all 2-digited numbers which leave remainder 1 when divided by 3 is:
(A) 1616 (B) 1602 (C) 1605 (D) None of these

17. The first term of an A.P. of consecutive integers is $p^2 + 1$. The sum of 2p + 1 terms of this series can be expressed as :
(A) $(p + 1)^2$ (B) $(2p + 1)(p + 1)^2$ (C) $(p + 1)^3$ (D) $p3 + (p + 1)^3$

18. If the sum of n terms of an AP is $2n^2$ 5n, then its nth term is –
(A) 4n – 3 (B) 3n – 4 (C) 4n + 3 (D) 3n + 4

19. If the last term of an AP is 119 and the 8th term from the end is 91 then the common difference of the AP is –
(A) 2 (B) 4 (C) 3 (D) – 3

20. If $(a_n) = \{2.5, 2.51, 2.52, \ldots\}$ and $\{b_n\} = \{3.72, 3.73, 3.74, \ldots\}$ be two AP's then $a_{100005} - b_{100005} =$
(A) – 1.22 (B) 1.22 (C) 1.2 (D) – 1.02

OBJECTIVE						ANSWER KEY									EXERCISE – 1
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	A	C	A	B	A	B	C	A	A	B	B	A	C	C	C
Que.	**16**	**17**	**18**	**19**	**20**										
Ans.	C	D	C	B	A										

EXERCISE – 2 (FOR SCHOOL/BOARD EXAMS

SUBJECTIVE TYPE QUESTIONS

VERY SHORT ANSWER TYPE

1. Write the first five terms of each of the sequences, whose nth terms are:
   (i) $a_n = \frac{(-1)^n(2n+1)}{6}$ (ii) $a_n = \frac{1}{n} + (-1)^n$ (iii) $a_n = n\left[\frac{n^2+5}{4}\right]$ (iv) $a_n = (-1)^{n-1}\, 5^{n+1}$
2. Find the indicated terms in each of the following sequences whose nth terms are:
   (i) $a_n = 2^n + n^3$; a^3 (ii) $a_n = \frac{n^2 - n + 1}{n}$, a_{10} (iii) $a_n = (-1)^{n-1}n^3$; a_9 (iv) $a_n = (n-1)(2-n)(3+n)$; a_{20}
3. Write the first five terms of each of the following sequences and obtain the corresponding series.
   (i) $a_1 = 1$, $a_n = a_{n-1} + 2$, $n \geq 2$ (ii) $a_1 = 4$, $a_{n+1} = 2na_n$
4. Write the first term a and the common difference d of the AP : -5, -1, 3, 7….
5. Write the first term a and the common difference d of the AP : -1.1, -3.1, - 5.1, - 7.1……
6. Write the arithmetic progression when first term a = - 1 and common difference d = $\frac{1}{2}$.
7. Write the arithmetic progression when first term a = - 1.5 and common difference d = - 0.5.
8. Find the common difference and write the next four terms of the AP : $1, \frac{1}{4}, \frac{3}{2}, \ldots$..
9. Write the sequence with nth term, $a_n = 3 + 4n$.
10. Find out whether the sequence 3, 3, 3, 3, … is an AP. If it is, find out the common difference.
11. Find the common difference and write the next two terms of the AP : 1.8, 2.0, 2.2, 2.4…
12. Find out whether the sequence $1^2, 3^2, 5^2\, 7^2, \ldots$ is an AP. If it is, find out the common difference.
13. Find the common difference and write the next two terms of the AP $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \ldots$.
14. Find the 18^{th} term of the AP. $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2} \ldots$.
15. Which term of the AP : 84, 80, 76,… is 0?
16. Is 302 a term of the AP : 3, 8, 13,…?
17. How many terms are there in the AP : 7, 13, 19,…..205 ?
18. Find the sum of the arithmetic progression : -26, -24, -22,…. to 36 terms.
19. Show that the sequence defined by $a_n = 2n^2 + 3$ is not an A.P.
20. Find the 14th term of the A.P. 9, 5, 1, - 3…
21. Find the nth term of the sequence m – 1, m – 3, m – 5,…
22. Is – 150 a term of the A.P. 11, 8, 5, 2….

SHORT ANSWER TYPE QUESTIONS

1. Show that the sequence defined by $a_n = 5n - 7$ is an AP. Find its common difference.
2. Prove that no matter what the real numbers a and b are, the sequence with nth term a + nb is always an AP. What is the common difference?
3. The first term of an AP is 5, the common difference is 3 and the last term is 80, find the number of terms.
4. If the nth term of the AP. 9, 7, 5… is same as the nth term of the AP. 15, 12, 9,…., find n.
5. Find the 12th term from the end of the arithmetic progression : 3, 5, 7, 9,…..201?
6. Find n if the given value of x is the nth term of the AP : $5\frac{1}{2}, 11, 16\frac{1}{2}, 22, \ldots$; x = 550.
7. Which term of the AP : 3, 10, 17,…. will be 84 more than its 13^{th} term?

8. Find the 8th term from the end of the AP : 7, 10, 13,….184
9. Find the value of x for which (8x + 4), (6x – 2) and (2x + 7) are in AP.
10. If the sum of a certain number of terms starting from first of an AP 25, 22, 19,…, is 116. Find the last term.
11. How many terms of he sequence 18, 16, 14,…. Should be taken so that their sum is zero?
12. Find the sum of first n odd natural numbers.
13. Find the sum of all even integers between 101 and 999.
14. Find the sum : $7 + 10\frac{1}{2} + 14 + \ldots + 84$.
15. Find the sum of the first 15 terms of the sequence having nth term as : $a_n = 3 + 4n$.
16. The 6th and 17th terms of an AP. are 19 and 41 respectively, find the 40th term.
17. In a certain AP, the 24th term is twice the 10th term. Prove that the 72nd term is twice the 34th term.
18. Find the second term and nth term of an AP whose 6th term is 12 and the 8th term is 22.
19. An AP consists of 60 terms. If the first and the last terms be 7 and 125 respectively, find 32nd term.
20. Find $a_{30} - a_{20}$ for the AP : a, a + d, a + 2d, a + 3d,….
21. Two arithmetic progressions have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?
22. Find the term of the arithmetic progression 9, 12, 15, 18,…. Which is 39 more than its 36th term.
23. The sum of three terms of AP. Is 21 and the product of the first and the third terms exceeds the second term by 6, find three terms.
24. The sum of three numbers in AP. is 12 and the sum of their cubes is 288. Find the numbers.
25. Show that $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ are in AP.
26. How many terms are there in the AP whose first and fifth terms are – 14 and 2 respectively and the sum of the terms is 40?
27. The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?
28. Show that the sum of all odd integers between 1 and 1000 which are divisible by 3 is 83667.
29. In an AP if the 5th and 12th terms are 30 and 65 respectively, what is the sum of first 20 terms?
30. Find the sum of the first 25 terms of an AP whose nth terms is given by $a_n = 2 - 3n$.
31. If x, x + 10 and 3x + 2 are in A.P., find the value of x.
32. If x + 1, 3x and 4x + 2 are in A.P., find the fifth term of A.P.
33. If $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P., then prove that $2b^2 = a^2 + c^2$.
34. If a, $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ and b are in A.P. and $a \neq b$, then find the value of n.
35. Find the nth term and 100th term of the sequence 7 + 3 – 1 – 5….
36. Which term of the A.P., 3, 8, 13, 18,…., is 78?
37. Which term of the A.P., 21, 18, 15,… is – 81?
38. Which term of the A.P., 121, 117, 113,…., is it’s first negative term?
39. Which term of the sequence 114, 109, 104,… is the first negative term?
40. How many terms are there in the A.P. $-1, \frac{-5}{6}, -\frac{2}{3}, \frac{-1}{2}, \ldots, \frac{10}{3}$?
41. How many two digit numbers are divisible by 3?
42. How many three digit numbers are divisible by 5?
43. If the 5th and 21st terms of an A.P. are 14 and – 14 respectively, then which term of the A.P. is zero?
44. In an A.P., the fourth term exceeds four times the 12th term by one and the third term exceeds twice the tenth term by five, find the A.P.
45. Determine the A.P. whose fourth term is 15 and the difference of 6th term from 10th term is 16.
46. The 4th term of an A.P. is zero, prove that its 25th term is triple its 11th term.

47. If the pth term of an A.P. is $\frac{1}{q}$, and qth term of an A.P. is $\frac{1}{q}$, then show that its (pq)th term is 1.

48 . The sum of three numbers in A.P. is 21 and their product is 231. Find the numbers.

49. The sum of three numbers in A.P. is 3 and their product is – 35. Find the numbers.

50. Divided 20 into four parts which are in A.P. such that the product of the first and fourth and the product of the second and third is in the ratio 2 : 3.

51. If the sum of first n terms of an A.P. is given by $s_n = 5n^2 + 3n$, find the nth term of the A.P.

52 The sum of the first 9 terms of an A.P. is 81 and the sum of it's first 20 terms is 400. Find the first term, the common difference and the sum upto 15^{th} term

53. The sum of n terms of two arithmetic progressions are in the ratio (3n + 8) : (7n + 15).Find the ratio of their
(i) 12th terms (ii) 15th terms.

54. If $s_n = n^2p$ and $s_m = m^2p$, $m \neq n$ is an A.P. prove that $s_p = p^3$.

55. The first, second and the last terms of an A.P. are m, n and 2m respectively. Show that it's sum is $\frac{3mn}{2(n-m)}$

56. If the mth term of an A.P. is 20 and nth term is 10, then show that sum of it's first (m + n) terms is $\frac{m+n}{2}\left[30+\frac{10}{m-n}\right]$.

57. If s_1, s_2, s_3 are the sum of n terms of three arithmetic progressions, the first term of each being unity and the respective common difference being 1, 2, 3; prove that $s_1 + s_3 = 2s_2$.

58. Two A.P.'s have the same common difference. If the first term of the two A.P's are 3 and 8 respectively, find the difference between their sum to first 30 terms.

59. If in an A.P., the sum of 12 terms is equal to 18 and the sum of 18 terms. is equal to 12, then prove that the sum of 30 terms is – 30.

60. Find the sum of all two digit natural numbers which are divisible by 4.

61. Find the sum of all 3-digit natural numbers which are divisible by 13.

62. Find the sum of all 2-digit numbers which when divided by 5 leave remainder 1.

63. Find the sum of all multiples of 9 lying between 300 and 700.

LONG ANSWER TYPE QUESTION

1. If pth, qth and rth terms of an AP are a, b, c respectively, then show that :
(i) a(q – r) + b (r – p) + c (p – q) = 0, (ii) (a – b)r + (b – c)p + (c – a)q = 0

2. In a garden bed, there are 23 rose plants in the first row, twenty one in the second row, nineteen in the third row and so on. There are five plants in the last row. How many rows are there of rose plants?

3. If the mth term of an AP is $\frac{1}{n}$ and the nth term is $\frac{1}{m}$, show that the sum of mn terms is $\frac{1}{2}$(mn + 1).

4. If the sum of m terms of an AP is the same as the sum of its n terms, show that the sum of its (m + n) terms is zero.

5. The sum of the first p,q,r terms of an AP. are a,b,c respectively. Show that:
$$\frac{a}{q}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$$

6. A manufacturer of radio sets produced 600 units in the third year and 700 units in the seventh year. Assuming that the product increases uniformly by a fixed number every year, find (i) the production in the first year
(ii) the total product in 7 years and (ii) the product in the 10th year.

7. A man is employed to count Rs. 10710. He counts at the rate of Rs. 180 per minute for half an hour. After this he counts at the rate of Rs 3 less every minute thatn the preceding minute. Find the time taken by him to count the entire amount.

8. A man arranges to pay off a debt of Rs. 3600 by 40 annual installments which form an arithmetic series. When 30 of the installments are paid, he dies leaving one-third of the debt unpaid, find the value of the first installment.

9. A ladder has rungs 25 cm apart. The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and bottom rungs are 2.5 metre apart, what is the length of the wood required for the rungs?

10. The digits of a positive integer, having three digits, are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number.

11. Two cars start together in the same direction from the same place. The first goes with uniform speed of 10 km/h. The second goes at a speed of 8 km/h in the first hour and increases the speed by $\frac{1}{2}$ km each succeeding hour. After how many hours will the second car overtake the first car if both cars go non – stop?

12. A man repays a loan of Rs. 3250 by paying Rs. 20 in the first month and then increases the payment by Rs. 15 every month. How long will it take him to clear the loan?

13. 150 workers were engaged to finish a piece of work in a certain number of days. Four workers dropped the second day, four more workers dropped the third day and so on. It takes 8 more days to finish the work now. Find the number of days in which the work was completed.

14. Along a road lie an odd number of stones place at intervals of 10 metres. These stones have to be assembled around the middle stone. A person can carry only one stone at a time. A man carried the job with one of the end stones by carrying them in succession. In carrying all the stones he covered a distance of 3 km. Find the number of stones.

15. The interior angles of a polygon are in arithmetic progression. The smallest angle is 120°, and the common difference is 5°. Find the number of sides of the polygon.

16. A man saved Rs. 16500 in ten years after the first he saved Rs. 100 more than he did in the preceding year. How much did he save in the first year?

17. A man arranges to pay off a debt of Rs. 3600 by 40 annual installments which form an arithmetic series. When 30 of the installments are paid, he dies leaving one-third of the debt unpaid, find the value of the first installment.

18. A piece of equipment cost a certain factory Rs. 600.000. If it depreciates in value, 15% the first, 13.5% the next year, 12% the third year and so on. What will be it's value at the end of 10 years, all percentages applying to the original cost?

- **VERY SHORT ANSWER TYPE QUESTIONS**

1. (i) $\frac{-1}{2}, \frac{5}{6}, \frac{-7}{6}, \frac{3}{2}$ *and* $\frac{-11}{6}$ (ii) $0, \frac{3}{2}, \frac{-2}{3}, \frac{5}{4}$ *and* $\frac{-4}{5}$ (iii) $\frac{3}{2}, \frac{9}{2}, \frac{21}{2}, 21$ *and* $\frac{75}{2}$ (iv) 25, - 125, 625, - 3125 and 15625

2. (i) 35 (ii) $\frac{91}{10}$ (iii) 729 (iv) – 7866 **3.** (i) 1, 3, 5, 7, 9 ; 1 + 3 + 5 + 7 +9 (ii) 4, 8, 32, 192, 1536 : 4 + 8 32 + 192 + 1536

4. a = - 5, d = 4 **5.** a = -1.1, d = -2 **6.** $-1, -\frac{1}{2}, 0, \frac{1}{2}, 1, \ldots\ldots$ **7.** –1.5, -2, -2.5, -3,….

8. $\frac{5}{4}; a_4 = \frac{11}{4}, a_5 = \frac{16}{4}, a_6 = \frac{21}{4}, a_7 = \frac{26}{4}$ **9.** 7, 11, 15, 19,…. **10.** Yes, d = 0 **11.** d = 0.2, a_5 = 2.6, a_6 = 2.8

12. No **13.** $d = \frac{1}{4}; a_5 = 1, a_6 = \frac{5}{4}, a_7 = \frac{26}{4}$ **14.** $35\sqrt{2}$ **15.** 22^{nd} term **16.** No **17.** 34

18. 324 **20.** – 43 **21.** m – 2n + 1 **22.** No

- **SHORT ANSWER TYPE QUESTIONS**

1. 5 **2.** b **3.** 26 **4.** 7 **5.** 179 **6.** 100 **7.** 25th **8.** 163 **9.** $\frac{15}{2}$ **10.** 4 **11.** 19 **12.** n^2 **13.** 246950

14. $\frac{2093}{2}$ **15.** 525 **16.** 87 **18.** a_2 = - 8, a_n = 5n – 18 **19.** 69 **20.** 10d **21.** 100 **22.** 49th **23.** 1, 7, 13

24. 2, 4, 6 or 6, 4, 2 **26.** 10. **27.** 38,6973 **29.** 1150 **30.** – 925 **31.** x = 9 **32.** 24 **34.** 1
35. 11 – 4n, - 389 **36.** 16th **37.** 15th, 8^{th} **38.** 32nd **39.** 24th **40.** 27 **41.** 30 **42.** 180
43. 3rd **44.** 27, 25, 23, 21 **45.** 3, 7, 11, 15, 19,…. **48.** 3, 7, 11 or 11, 7, 3 **49.** – 5, 1, 7 or 7, 1, - 5
50. 2, 4, 6 and 8 **51.** 10 n – 2 **52.** a = 1, d = 2, s_{15} = 225 **53.** (i) 7 : 16 (ii) 95 : 218 **58.** 150 **60.** 1188
61. 37674 **62.** 963 **63.** 21978

- **LONG ANSWER TYPE QUESTIONS**

2. 10 rows **6.** **(i)** 550 **(ii)** 4375 **(iii)** 775 **7.** 89 minutes **8.** Rs. 51 **9.** 3.5 minters **10.** 852
11. 9 hours **12.** 20 months **13.** 25 days **14.** 25 stones **15.** 9 sides **16.** Rs. 1200 **17.** Rs. 51 **18.** 10500

PERVIOUS YEARS BOARD (CBSE) QUESTIONS

1. The nth term (t_n) of an Arithmetic progression is given by $t_n = 7n + 1$. Find the sum of the first 30 terms of Arithmetic progression. **[Foreign – 2004]**
2. The 10th term of an Arithmetic progression (A.P.) is 57 and its 15th term is 87. Find the Arithmetic Progression. **[Foreign – 2004]**
3. If the sum of first n terms of an A.P. is given by $S_n = 3n^2 + 2n$, find the nth term of the A.P.

OR

If m times the mth terms of an A.P. is equal to n times its nth term, find its (m + n)th term. **[Delhi-2004C]**

4. How many terms of the A.P.3,5,7,… must be taken so that the sum is 120? **[Delhi-2004C]**
5. If the sum of first n terms of an A.P. is given by $S_n = 4n^2 – 3n$, find the nth term of the A.P. **[Delhi-2004C]**
6. If the sum of first of an A.P. is given by $S_n = 2n^2 + 5n$, find the nth term of the A.P. **[Delhi-2004C]**
7. Find the sum of first 15 terms of an A.P. whose nth term is 9 – 5n.

OR

If the sum to first n terms of an A.P. is given by $S_n = 5n^2 + 3n$, find the nth term of the A.P. **[AI-2004C]**

8. Find 10^{th} term from end of an A.P. 4, 9, 14,…, 254 **[Delhi-2005]**
9. Find the number of terms of the A.P. 54, 51, 48,….so that their sum is 513.

OR

If the nth term of an A.P. is (2n + 1), find the sum of first n terms of the A.P. **[Delhi-2005]**

10. Find the sum of all two digits odd positive numbers. **[AI-2005]**
11. The 8th term of an Arithmetic Progression is zero. Prove that its 38th term is triple of it 18th term. **[AI-2005]**
12. Find the sum of all two digit positive numbers divisible by 3. **[Foreign-2005]**
13. If fifth term of the A.P. is zero, show that its 33rd term is four times its 12th term **[Foreign-2005]**
14. Which term of the A.P. 5, 9, 13,... is 81? Also find the sum 5 + 9 + 13 +… + 81 **[Delhi-2005C]**
15. The sum of first n terms of an A.P. is given by $(n^2 + 3n)$. Find the 20th term of the progression. **[Delhi-2005C]**
16. Find the sum of the first 51 terms of the A.P. whose 2nd term is 2 and 4th term is 8. **[AI-2005C]**
17. The sum of the first n terms of an A.P. is given by $S_n = 3n^2 – n$. Determine the A.P. and its 25^{th} term.

OR

The sum of three numbers in A.P. is 27 and their product is 405. Find the numbers. **[AI-2005C]**

18. The 6^{th} term of an Arithmetic progression (A.P.) is -10 and the 10th term is – 26 Determine the 15th term of the A.P. **[Delhi-2006]**

19. Find the sum of all the natural numbers less than 100 which are divisible by 6. **[AI-2006]**

20. How many terms are there in A.P. whose first term and 6th term are – 12 and 8 respectively and sum of all its terms is 120?

21 Using A.P., find the sum of all 3-digit natural numbers which are multiples of 7. **[Delhi-2006C]**

22. In an A.P. the sum of first n terms is $\frac{5n^2}{2}+\frac{3n}{2}$ Find its 20th term. **[AI-2006C]**

23. Find the sum of first 25 terms of an A.P. whose nth term is $1 - 4n$. **[Delhi-2007]**

24. Which term of the A.P. 3, 15, 27, 39, …will be 132 more than its 54th term? **[Delhi-2007]**

25. In an A.P., the sum of its first n terms is $n^2 + 2n$. Find its 18th term. **[AI-2007]**

26. The first term, common difference and last term of an A.P. are 12, 6 and 252 respectively. Find the sum of all terms of this A.P. **[AI-2007]**

27. The nth term of an A.P. is $7 - 4n$. Find its common difference. **[Delhi-2008]**

28. The sum of n terms of an A.P. is $5n^2 - 3n$. Find the A.P. Hence, find its 10th term. **[Delhi-2008]**

29. The nth term of an A.P. is $6n + 2$. Find it's common difference. **[Delhi-2008]**

30. Find the 10th term from the end of the A.P. 8, 10, 12,…., 126 **[Delhi-2008]**

31. Write the next term of the A.P. $\sqrt{8}, \sqrt{18}, \sqrt{32}, ….$ **[AI-2008]**

32. The sum of the 4th and 8th terms of an A.P. is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the A.P. **[AI-2008]**

33. The first term of an A.P. is p and it's common difference is q. Find it's 10th terms is. **[AI-2008]**

34. For what value of n are the terms of two A.P.'s 63, 65, 67,….., and 3, 10, 17 ,…. equal?

OR

If m times the mth tem of an A.P. is equal to n times it's nth term, find the $(m + n)$th term of the A.P. **[Foreign-2008]**

35. In an A.P. the first term is 8, nth term is 33 and sum to first n terms is 123. Find n and d, the common difference. **[Foregin-2008]**

36. In an A.P., the term is 22, nth term is – 11, and sum to first n terms is 66. Find n and d, the common difference. **[Foreign-2008]**

37. In an A.P., the first term is 22, nth term is – 11, and sum to first n terms is 66. Find n and d, the common difference. **[Foreign-2008]**

38. For what value of p, are $2p - 1$, 7 and $3p$ three consecutive terms of an A.P.? **[Delhi-2009]**

39. If S_n, the sum of first n terms of an A.P. is given by $S_n = 3n^2 - 4n$, then find its nth term. **[Delhi-2009]**

40. The sum of 4th and 8th terms of an A.P. is 24 and sum of 6th and 10th terms is 44. Find A.P. **[Delhi-2009]**

41. If S_n the sum of first n terms an A.P. is given by $S_n = 5n^2 + 3n$, then find its nth term. **[Delhi-2009]**

42. The sum of 5th 9th terms of an A.P., is 72 and the sum of 7th and 12th terms is 97. Find the A.P. **[Delhi-2009]**

43. If $\frac{4}{5}$, a, 2 are three consecutive terms of an A.P., then find the value of a. **[AI-2009]**

44. Which term of the A.P. 3, 15, 27, 39,… will be 120 more than it's 21str term? **[AI-2009]**

45. The sum of first six terms of an arithmetic progression is 42. The ratio of its 10th term to its 30th term is 1 : 3. Calculate the first and the thirteenth term of the A.P. **[AI-2009]**

46. Which term of the A.P. 4, 12, 20, 28,…. will be 120 more than it's 21st term? **[AI-2009]**

47. For what value of k, are the numbers x, 2x + k and 3x + 6 three consecutive terms of an A.P.? **[Foreign-2009]**

48. The 17th term of an A.P. exceeds its 10th term by 7. Find the common difference. **[Foreign-2009]**

49. If 9th term of an A.P. is zero, prove that its 29th term is double of it's 19th term. **[Foreign-2009]**

50. If 5th term of an A.P. is zero, prove that it's 23rd term is three times it's 11th term. **[Foreign-2009]**

51. If the 7th term of an A.P. is zero, prove that it's 27th term is five times it's 11th term. **[Foreign-2009]**

SUBJECTIVE **ANSWER KEY** EXERCISE-4 (X)-CBSE

1. 3285 **2.** 3, 9, 15, 21… **3.** $6n - 1$ or 0 **4.** 10 terms **5.** $8n - 7$ **6.** $4n + 3$

7. -465 or $10n - 2$ **8.** 209 **9.** 18 or 19 OR $n(n + 2)$ **10.** 2,475 **12.** 1,665 **14.** 860

15. 42 **16.** 3,774 **17.** 146 OR (3, 9,15) or (15, 9,3) **18.** -46 **19.** 816 **20.** 12

21. 70, 336 **22.** 99 **23.** -1275 **24.** 65th term **25.** 38 **26.** 5412 **27.** -4

28. 2, 12, 22..; $a_{10} = 92$ **29.** 6 **30.** 108 **31.** $5\sqrt{2}$ **32.** $-13, -8, -3$

33. $p + 9q$ **34.** $n = 13$ or $a_{m+n} = 0$ **35.** $n = 6, d = 5$ **36.** $n = 15, d = -3$

37. $n = 12, d = -3$ **38.** $p = 3$ **39.** $6n - 7$ **40.** $-13, -8, -3$ **41.** $10n - 2$

42. 6, 11, 16, 21,… **43.** $a = \frac{7}{5}$ **44.** 31st term **45.** $a = 2$, $a_{13} = 26$ **46.** 36th term

47. $k = 3$ **48.** $d = 1$

EXERCISE – 4 (FOR OLYMPIADS)

CHOSSE THE CORRECT ONE

1. For a series whose nth term is $\frac{n}{x}+y$, the sum of r terms is :
   (A) $\frac{r(r+1)}{2x}+ry$ (B) $\frac{r(r-1)}{2x}$ (C) $\frac{r(r-1)}{2x}-ry$ (D) $\frac{r(r+1)}{2y}-rx$

2. If $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in A.P., then $\left[\frac{1}{a}+\frac{1}{b}-\frac{1}{c}\right]\left[\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right]$ is equal to :
   (A) $\frac{4}{ac}-\frac{3}{b^2}$ (B) $\frac{b^2-ac}{a^2b^2c^2}$ (C) $\frac{4}{ac}-\frac{1}{b^2}$ (D) None of these

3. The sum of first 24 terms of an A.P. $a_1, a_2, a_3, \ldots$; if it is known that $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$, is equal to :
   (A) 90 (B) 180 (C) 900 (D) 1800

4. A student read common difference of an AP is – 2 instead of 2 and got the sum of first five terms as – 5. The actual sum of first five terms is :
   (A) 25 (B) – 25 (C) – 35 (D) 35

5. The sum of n terms of two A.P's are in the ratio of (7n + 1) : (4n + 27). The ratio of their 11th terms is –
   (A) 2 : 3 (B) 4 : 3 (C) 5 : 4 (D) 5 : 6

6. If $1^2 + 2^2 + 3^2 + \ldots n^2 = 1015$, then the value of n is :
   (A) 13 (B) 14 (C) 15 (D) None of these

7. The sum of the series $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}+\ldots$ upto 9 terms is:
   (A) $-\frac{5}{6}$ (B) $-\frac{1}{2}$ (C) 1 (D) $-\frac{3}{2}$

8. The sum of first n odd natural numbers is:
   (A) n^2 (B) 2n (C) $\frac{n(n-1)}{2}$ (D) $\frac{n(n+1)}{2}$

9. If the roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$ are in A.P., then their common difference will be:
   (A) ± 1 (B) ± 2 (C) ± 3 (D) ± 4

10. If A.M between two numbers is 5 and their G.M. is 4, then their H.M. is:
   (A) $\frac{16}{5}$ (B) $\frac{14}{5}$ (C) $\frac{11}{5}$ (D) None of these

11. If A is the single A.M. between two numbers a and b and S is the sum of n A.M.'s between them, then $\frac{S}{A}$ depends upon:
   (A) n, a, b (B) n, a (C) n, b (D) n

12. If the A.M. between the roots of a quadratic equation is 8 and the G.M. is 5, then the equation is:
   (A) $x^2 + 10x - 25 = 0$ (B) $x^2 - 8x + 5 = 0$
   (C) $x^2 - 16x + 25 = 0$ (D) $x^2 - 16x - 25 = 0$

13. If c is the harmonic mean between a and b, then $\frac{c}{a}+\frac{c}{b}$ is equal to:
   (A) 2 (B) $\frac{a+b}{ab}$ (C) $\frac{ab}{a+b}$ (D) None of these

14. If a,b,c,d,e,f are in A.P. then e-c is equal to :
   (A) 2(c – a) (B) 2(f – d) (C) 2(d – c) (D) d – c

15. 20th term of the series : $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\ldots$ is :

(A) $\frac{441}{4}$ (B) $\frac{443}{2}$ (C) $\frac{445}{2}$ (D) $\frac{439}{2}$

16. If the value of $1^3 + 2^3 + 3^3 + \ldots + n^3 = 2025$, then the value of $1 + 2 + 3 + \ldots + n$ is:
(A) 675 (B) 81 (C) 45 (D) 285

17. If the value of $1 + 2 + 3 + \ldots + n$ is 55, then the value of $1^3 + 2^3 + 3^3 + \ldots + n^3$ is:
(A) 165 (B) 385 (C) 3025 (D) 555

18. The nth term of the series $1 + \frac{1+2}{2} + \frac{1+2+3}{3} + \ldots$ is :
(A) $\frac{n-1}{2}$ (B) $\frac{n^2+1}{2}$ (C) $\frac{n+1}{2}$ (D) $\frac{n^2-1}{2}$

19. $1^2 + 1 + 2^2 + 2 + 3^2 + 3 + \ldots + n^2 + n$ is equal to :
(A) $\frac{n(n+1)}{2}$ (B) $\left[\frac{n(n+1)}{2}\right]^2$ (C) $\frac{n(n+1)(n+2)}{3}$ (D) $\frac{n(n+)(n+2)(n+3)}{4}$

20. The next term of the sequence $\frac{1}{4}, \frac{1}{36}, \frac{1}{144} \ldots$ is :
(A) $\frac{1}{576}$ (B) $\frac{1}{400}$ (C) $\frac{1}{1296}$ (D) None of these

21. If the sum of first n natural numbers is one-fifth of the sum of their squares, then n equals:
(A) 5 (B) 6 (C) 7 (D) 8

22. The nth term of the series $1 + 3 + 6 + 10 + 15 + \ldots$ is :
(A) $\frac{n(n+1)}{2}$ (B) $n^2 - n + 1$ (C) $n(n+1)$ (D) None of these

23. The sum of the series $1^2 + 1 + 2^2 + 2 + 3^2 + 3 + \ldots +$ up to n terms is :
(A) $\frac{n(n+1)}{2}$ (B) $\frac{n(n+1)(n+2)}{3}$ (C) $\left[\frac{n(n+1)}{2}\right]^2$ (D) None of these

24. The nth term of the sequence $1, \sqrt{2}, 3^{\frac{1}{3}}, 2^{\frac{1}{2}}, \ldots$ is :
(A) $n^{\frac{1}{n}}$ (B) n^n (C) $\left(\frac{1}{n}\right)^n$ (D) None of these

25. The sum of n terms of the series $(1^2 - 2^2) + (3^2 - 4^2) + (5^2 - 6^2) +$,,, is :
(A) $\frac{n(n+1)}{2}$ (B) $\frac{-n(n+1)}{2}$ (C) $-n(2n+1)$ (D) None of these

26. If A_1 and A_2 be the two A.M.s between two numbers p and q, then $(2A_1 - A_2)(2A_2 - A_1)$ is equal to :
(A) p + q (B) p − q (C) pq (D) None of these

27. If $\frac{1}{a}, \frac{a^n + b^n}{a^{n+1} + b^{n+1}}, \frac{1}{b}$ are in A.P., then n is equal to :
(A) 0 (B) − 1 (C) $\frac{1}{2}$ (D) None of these

28. If $S_n = nP + \frac{1}{2}n(n-1)Q$ where S_n denotes the sum of the first n terms of an A.P., then the common difference of the A.P. is
(A) P + Q (B) 2P + 3Q (C) 2Q (D) Q

29. If a,b,c are positive reals, then least value of $(a + b + c)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$ is :

(A) 1 (B) 6 (C) 9 (D) None of these

30. The sum of first four terms of an A.P. is 56 and sum of last four terms is 112. If the first term is 11, then the number of terms is :

(A) 10 (B) 12 (C) 11 (D) None of these

31. For an A.P., $S_{2n} = 3S_n$. The value of $\frac{S_{3n}}{S_n}$ is equal to :

(A) 4 (B) 6 (C) 8 (D) 10

32. The ratio of the 7th to the (n – 1)th mean between 1 and 31, when n arithmetic means are inserted between then, is 5 : 9. The value of n is :

(A) 12 (B) 13 (C) 14 (D) 15

33. The first, second and last terms of an A.P. are a,b, and 2a respectively, the sum of the series is :

(A) $\frac{3ab}{2(b+a)}$ (B) $\frac{3ab}{2(b-a)}$ (C) $\frac{3ab}{2(a-b)}$ (D) None of these

34. Sum of first m terms of an A.P. is 0. If a be the first term of the A.P., then the sum of next n terms is :

(A) $\frac{-a(m+n)m}{m-1}$ (B) $\frac{-a(m+n)n}{m-1}$ (C) $\frac{-a(m+n)n}{n-1}$ (D) $\frac{-a(m+n)m}{n-1}$

35. If A_1 and A_2 be the two A.M.s between two numbers a and b, then $A_2 - A_1$ is equal to

(A) a + b (B) b – a (C) $\frac{b-a}{3}$ (D) None of these

36. The sum of terms equidistant from the beginning and end in an A.P. is equal to :

(A) Last term (B) First term
(C) Sum of the first and the last term (D) None of these

37. If the sum of the roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals then bc^2, ca^2, ab^2 are in:

(A) A.P. (B) G.P. (C) H.P. (D) None of these

38. If $a_1, a_2, a_3, \ldots a_n$ are in A.P. and $a_1 > 0$ for all I, then : $\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \ldots + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}} =$

(A) $\frac{n}{\sqrt{a_1}+\sqrt{a_n}}$ (B) $\frac{n}{\sqrt{a_n}-\sqrt{a_1}}$ (C) $\frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$ (D) None of these

39. If a,b,c are in A.P. and also $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P., then:

(A) $a = b \neq c$ (B) $a \neq b = c$ (C) $a = b = c$ (D) $a \neq b \neq c$

40. If a,b,c are in H.P., then $\frac{a-b}{b-c}$ equals :

(A) $\frac{b}{a}$ (B) $\frac{a}{b}$ (C) $\frac{a}{c}$ (D) None of these

41. An, A.P. consists pf n (odd) terms and it's middle term is m. Then the sum of the A.P. is :

(A) 2mn (B) $\frac{1}{2}mn$ (C) mn (D) mn^2

42. If A,G and H denote respectively the A.M., G.M. and H.M. between two positive numbers a and b, then A-G is equal to :

(A) a – b (B) $\frac{2ab}{a+b}$ (C) $\frac{1}{2}(\sqrt{a}-\sqrt{b})^2$ (D) None of these

43. If the roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$ are in A.P., then their common difference is :

(A) ± 1 (B) ± 2 (C) ± 3 (D) ± 4

OBJECTIVE							ANSWER KEY							EXERCISE – 4
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	A	A	C	D	B	B	D	A	C	A	D	C	A	C	A
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	C	C	C	C	B	C	A	D	A	C	C	B	D	C	C
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**		
Ans.	B	C	B	B	C	C	A	C	C	C	C	C	C		

EXERCISE – 5 (FOR IIT – JEE/AIEEE)

CHOOSE THE CORRECT ONE

1. The sum of the n terms of the series $\frac{4}{3}+\frac{10}{9}+\frac{28}{27}+\dots$ is : **[Kerala Engineering-2003]**

(A) $\frac{3^n(2n+1)+1}{2(3^n)}$ (B) $\frac{3^n(2n+1)-1}{2(3^n)}$ (C) $\frac{n3^n-1}{2(3^n)}$ (D) $\frac{3^n-1}{2}$

2. If the third term of a G.P. is p, then the product of it's first 5 terms is : **[Kerala Engineering-2003]**

(A) p^3 (B) p^2 (C) p^{10} (D) p^5

3. If $a_1, a_2, \dots\dots a_n$ are n A.M's between a and b, then $2\sum_{i=1}^{n} a_i =$ **[Kerala Engineering-2003]**

(A) ab (B) n (a + b) (C) $\frac{n(a+b)}{ab}$ (D) $\frac{a+b}{n}$

4. $4^{\frac{1}{2}} x 4^{\frac{1}{4}} x 4^{\frac{1}{8}}$ x ..... to ∞ is a root of the equation : **[Kerala Engineering-2003]**

(A) $x^2 - 4 = 0$ (B) $x^2 - 4x + 6 = 0$ (C) $x^2 - 5x + 4 = 0$ (D) $x^2 - 3x + 2 = 0$

5. If a,b,c are in A.P., then which one of the following is not true? **[Kerala Engineering-2003]**

(A) a + k, b + k, c + k are in A.P. (B) ka, kb, kc are in A.P.
(C) a^2, b^2, c^2 are in A.P. (D) a + b, c + a, b + c are in A.P.

6. The sum of the series: $\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\dots+\frac{1}{\sqrt{n^2-1}+\sqrt{n^2}}$ is equal to **[AMU–2002]**

(A) $\frac{2n+1}{\sqrt{n}}$ (B) $\frac{\sqrt{n}+1}{\sqrt{n}+\sqrt{n-1}}$ (C) $\frac{n+\sqrt{n^2-1}}{2\sqrt{n}}$ (D) n – 1

7. Sum of infinite number of terms of a G.P. is 20 and sum of their squares is 100. The common ratio of the G.P. is : **[AIEEE–2002]**

(A) 5 (B) $\frac{3}{5}$ (C) $\frac{8}{5}$ (D) $\frac{1}{5}$

8. $1^3 - 2^3 + 3^3 - 4^3 + \dots + 9^3 =$ **[AIEEE–2002]**

(A) 425 (B) – 425 (C) 475 (D) – 475

9. If $y = x - x^2 + x^3 - x^4 + \dots$ to ∞, then the value of x will be $(-1 < x < 1)$:

(A) $y + \frac{1}{y}$ (B) $\frac{y}{1+y}$ (C) $y - \frac{1}{y}$ (D) $\frac{y}{1-y}$

10. The two geometric means between 1 and 64 are: **[Kerala Engineering-2002]**

(A) 1 and 64 (B) 8 and 16 (C) 4 and 16 (D) 3 and 16

11. The sum of infinite terms of the geometric progression $\frac{\sqrt{2+1}}{\sqrt{2-1}}, \frac{1}{2-\sqrt{2}}, \frac{1}{2}, \dots$ is :

[Kerala Engineering-2002]

(A) $\sqrt{2}\left(\sqrt{2}+1\right)^2$ (B) $\left(\sqrt{2}+1\right)^2$ (C) $5\sqrt{2}$ (D) $3\sqrt{2}+\sqrt{5}$

12. If the nth term of the geometric progression, $5, -\frac{5}{2}, \frac{5}{4}, -\frac{5}{8}, \dots is \frac{5}{1024}$, then the value of n is

[Kerala Engineering-2002]

(A) 11 (B) 10 (C) 9 (D) 4

13. In a harmonic progression, pth term is q and qth term is p, then the (pq)th term is:

(A) $\frac{p+q}{pq}$ (B) 0 (C) $\frac{pq}{p+q}$ (D) 1

14. Suppose a,b,c are A.P. and a^2,b^2,c^2 are in G.P. If a < b < c and a + b + c = $\frac{3}{2}$; then the value of a is: **[IIT Screening-2002]**

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{2\sqrt{3}}$ (C) $\frac{1}{2}-\frac{1}{\sqrt{3}}$ (D) $\frac{1}{2}-\frac{1}{\sqrt{2}}$

15. Let the positive numbers a,b,c,d be in A.P., then abc, abd, acd, bcd are: **[IITScreening-2001]**

(A) Not in A.P./G.P./H.P. (B) In A.P.
(C) In G.P. (D) In H.P.

16. If the sum of the first 2n terms of the A.P. 2, 5, 8,… is equal to the sum of first n term of the A.P.57, 59, 61,… then n equals : **[IIT Scereening-2001]**

(A) 10 (B)12 (C)11 (D) 13

17. If $\frac{3+5+7+...upto\, n\, terms}{5+8+11+...upto\, 10\, terms} = 7$, then the value of n is:

(A) 35 (B) 36 (C) 37 (D) 40

18. If a, b, c are in G.P., then the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root if $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in: **[DCE-2000]**

(A) G.P. (B) A.P. (C) H.P. (D) None of these

19. Consider an infinite geometric series with first term a and common ratio r. If it's sum is 4 and the second term is $\frac{3}{4}$, then : **[IIT-Screening-2000]**

(A) $a=\frac{7}{4}, r=\frac{3}{7}$ (B) $a=2, r=\frac{3}{8}$ (C) $a=\frac{3}{2}, r=\frac{1}{2}$ (D) $a=3, r=\frac{1}{4}$

20. If 4th term of an H.P. is 5 and 5th term is 4, then it's 20th term is:

(A) Zero (B) $\frac{4}{5}$ (C) 1 (D) $\frac{5}{4}$

21. H.M. between two numbers is 4. The A.M. 'A' and the G.M. 'G' between them satisfy the relation $2A + G^2 = 27$. The numbers are:

(A) 6, 3 (B) 4, 2 (C) 6, 9 (D) 3, 5

22. The sum of an infinite G.P. is 3. The sum of the series formed by squaring its terms is also 3. The series is:

(A) $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+...$ (B) $\frac{3}{2}+\frac{3}{4}+\frac{3}{8}+\frac{3}{16}+...$

(C) $\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+...$ (D) $1-\frac{1}{3}+\frac{1}{3^2}+-\frac{1}{3^3}+...$

23. If first three terms of a sequence $\frac{1}{16}, a, b, \frac{1}{6}$ are in G.P. and last three are in H.P., then values of a and b are respectively :

(A) $-\frac{1}{4}, 1$ (B) $\frac{1}{12}, \frac{1}{9}$ (C) Both (A) and (B) are true (D) $\frac{1}{9}, \frac{1}{12}$

24. The sum of few terms of a ratio series is 728, if common ratio is 3 and last term is 486, then first term of the series is :

(A) 1 (B) 2 (C) 3 (D) 4

25. The 6th term of a G.P. is 32 and it's 8th term is 128; the common ratio of the G.P. is :

(A) – 1 (B) 2 (C) 4 (D) – 4

26. Let $a_1, a_2, \ldots\ldots, a_{10}$ be A.P., $h_1, h_2, \ldots, h_{10}$ be in H.P. If $a_1 = h_1 = 2$, $a_{10} = h_{10} = 3$, then $a_4\, h_7 =$ **[IIT]**

27. In a G.P., the first term is a, second term is b and the last term is c, then sum of the series is: **[AMU]**

28. If H is the harmonic mean between a and b, then $\frac{H+a}{H-a} + \frac{H+b}{H-b}$ is equal to: **[AMU]**

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) 2 (D) None of these

29. Let A_1, A_2 be two AMs and G_1, G_2 be two GMs between a and b, then $\frac{A_1 + A_2}{G_1\, G_2} =$

(A) $\frac{a+b}{2ab}$ (B) $\frac{2ab}{a+b}$ (C) $\frac{a+b}{ab}$ (D) $\frac{a+b}{\sqrt{ab}}$

30. a,b,c are three unequal numbers such that a,b,c are in A.P. ; b – a, c – b, a are in G.P. then a : b :c : : **[Karnataka-CET]**

(A) 1 : 2 : 4 (B) 2 : 3: 5 (C) 1 : 2: 3 (D) 1 :3 : 5

31. The first two terms of an infinite G.P. are together equal to 5 and every term is 3 times the sum of all the terms that follow it ; the common ratio of the G.P. is : **[AMU]**

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) 3 (D) 4

32. The eighth term of a G.P. is 128 and common ratio is 2. The product of it's first five terms is:

(A) 4^6 (B) 4^3 (C) 4^5 (D) 4^4

33. The sum of infinity of $\frac{1}{7} + \frac{2}{7^2} + \frac{1}{7^3} + \frac{2}{7^4} + \ldots$ is : **[AMU]**

(A) $\frac{3}{16}$ (B) $\frac{1}{5}$ (C) $\frac{1}{24}$ (D) $\frac{1}{16}$

34. p,q,r are in A.P. and each is numerically less than 1. Let : **[Karnataka-CET]**

$x = 1 + p + p^2 + \ldots\ldots$ to ∞

$y = 1 + q + q^2 + \ldots\ldots$ to ∞

$z = 1 + r + r^2 + \ldots\ldots$ to ∞, then x,y,z are in

(A) A.P. (B) G.P. (C) H.P. (D) None of these

35. If the numbers p,q,r are in A.P., then m^{7p}, m^{7q}, m^{7r} $(m > 0)$ are in :

(A) A.P. (B) G.P. (C) H.P. (D) None of these

36. If the pth, qth and rth terms of a G.P. are ℓ, m and n respectively, then $\ell^{q-r}\, m^{r-p}\, n^{p-q}$ is :

(A) 1 (B) 0 (C) pqr (D) ℓmn

37. $\frac{1}{1x2} + \frac{1}{2x3} + \frac{1}{3x4} + \ldots + \frac{1}{n(n+1)}$ equals : **[AMU]**

(A) $\frac{n+1}{n}$ (B) $\frac{n(n+1)}{6}$ (C) $\frac{n}{n+1}$ (D) $\frac{n^2}{n+1}$

38. Sum of n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+....$ is equal to :

(A) $2^n - n - 1$ (B) $1 - 2^{-n}$ (C) $2^n - 1$ (D) $n + 2^{-n} - 1$

39. If $a^x = b^y = c^z$ and a,b,c are in G.P. then x, y,z are in :

(A) A.P. (B) G.P. (C) H.P. (D) None of these

40. If a_n be the nth term of a G.P. of positive numbers and $\sum_{n=1}^{100} a_{2n} = \alpha, \sum_{n=1}^{100} a_{2n-1} = \beta$, such that $a \neq \beta$, then the common ratio of the G.P. is : **[IIT]**

(A) $\frac{\alpha}{\beta}$ (B) $\frac{\beta}{\alpha}$ (C) $\sqrt{\frac{\alpha}{\beta}}$ $\sqrt{\frac{\beta}{\alpha}}$

41. If p,q,r are in A.P. and x,y,z are in G.P, then $x^{q-r} y^{r-p} z^{p-q}$ is equal to :

(A) p + q + r (B) xyz (C) 1 (D) px + qy +rz

42. If a,b,c are in A.P., then $10^{ax+10}, 10^{bx+10}, 10^{cx+10}, x \neq 0$ are in :

(A) A.P. (B) G.P. only when $x > 0$

(C) G.P. for all x (D) G.P. only when $x < 0$

43. If the sum of roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}$ *and* $\frac{c}{b}$ are in :

(A) G.P. (B) H.P. (C) A.P. (D) None of these

44. Let α, β be the roots of $x^2 - x + p = 0$ and γ, δ be the roots of $x^2 - 4x + q = 0$. If $\alpha, \beta\ \gamma, \delta$ are in G.P., then integral values of p and q are respectively.

(A) – 2, – 32 (B) – 2, 3 (C) – 6, 3 (D) – 6,– 32

45. If a,b,c,d and x are all real and $(a^2 + b^2 + c^2)\ x^2 - 2\ (ab + bc + cd)\ x + (b^2 + c^2 + d^2) \leq 0$ then :

(A) a,b,c,d are in G.P. (B) a,b,c,d are in A.P. (C) a,b,c,d are in H.P. (D) None of these

OBJECTIVE								ANSWER KEY							EXERCISE – 4
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	B	D	B	C	C	D	B	A	D	C	A	A	D	D	D
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	C	A	B	D	C	A	B	C	B	B	D	B	C	C	C
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**
Ans.	B	C	A	C	B	A	C	D	C	A	C	C	B	A	A