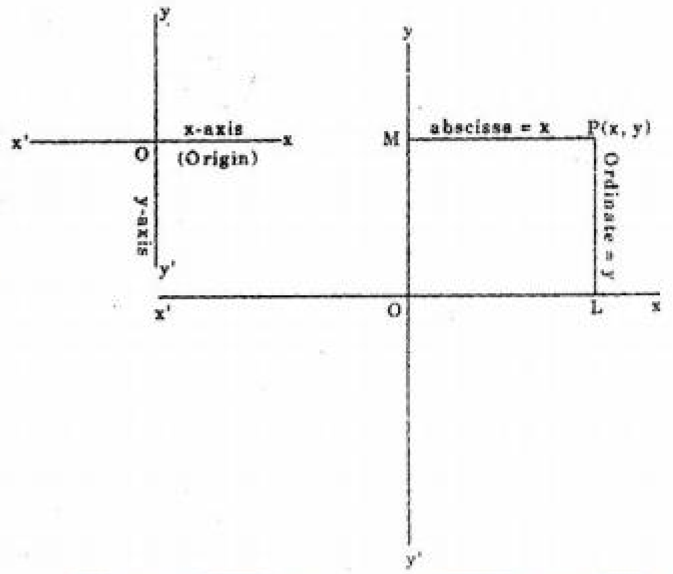
MATHEMATICS

LESSON 9

COORDINATE GEOMETRY

1. **Coordinate Geometry** is the branch of Mathematics which makes use of Algebra to prove results in Geometry. Hence it is known as Algebraic Geometry or Analytical Geometry. This was first introduced by the eminent French Mathematician Descartes. In this, the position of a point in a plane is determined by its coordinates.

2. **Coordinates of a point in a plane:** Let xOx', a horizontal line and yOy', a vertical line meet at O. xOx' and yOy' are called the axes of coordinates. The point O is called the origin of coordinates. xOx' is called the x-axis and yOy' is called the y-axis. The position of any point P in the plane is determined by its distance from the x end y axes. The perpendicular distance of the point p from the y-axis is called the x coordinate of the point or the abscissa of the point. The perpendicular distance of the point P from the x-axis is celled the y-coordinate of the point or the ordinate of the point.



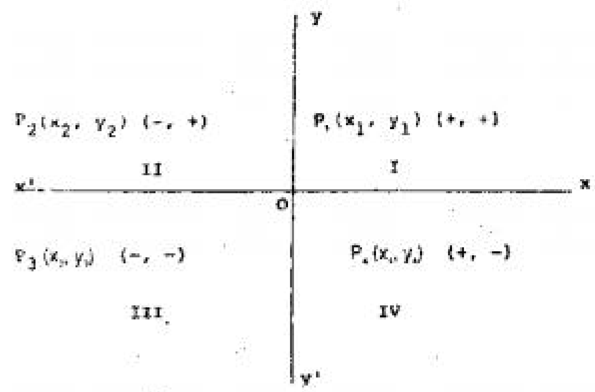
If MP = x and LP = y, the coordinates of the point P are (x, y)
OLPM is a rectangle
$\therefore$ OL = MP = x
OM = LP = y

By convention the x-coordinates (abscissa) of all points to the right of y-axis are positive and the x-coordinates of all points to the left of y-axis are negative.

Also the y-coordinates (ordinates) of all points above x-axis are positive and the y-coordinates of all points below the y-axis are negative.

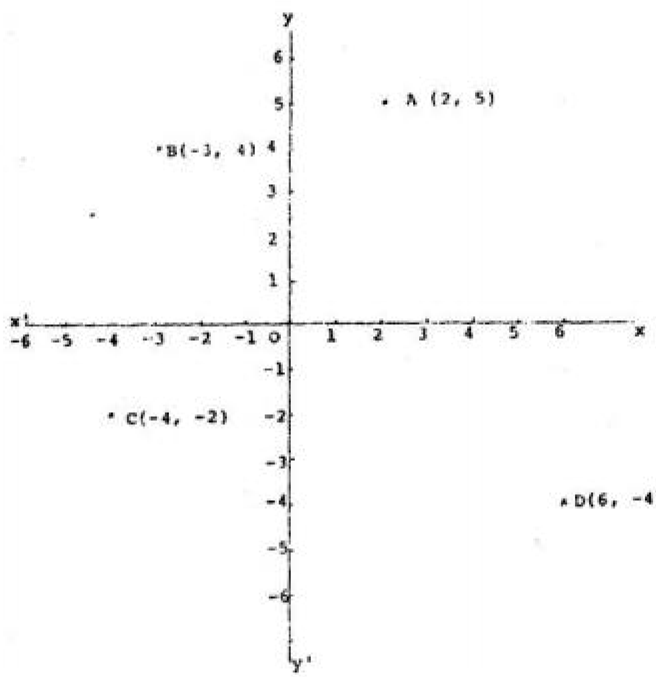
The x-axis xOx' and the y-axis yOy' divide the plane into four parts and each of these parts is called a quadrant.

The quadrant xOy is the first quadrant, x'Oy is the second quadrant, x'Oy' is the third quadrant and xOy' is the fourth quadrant.



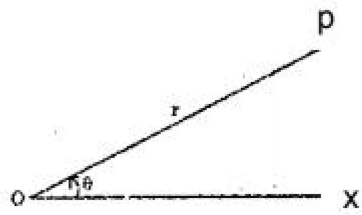
The signs of coordinates of points in the four quadrants are indicated against the coordinates of the points in the four quadrants. The coordinates of points indicated above are called the Cartesian rectangular coordinates.

Example 1. Plot the following points (2, 5), (-3, 4), (-4, -2) (6, -4)
The positive x-coordinates are to the right of y-axis. The negative x coordinates are to the left of y-axis. The positive y coordinates are above the x-axis and the negative y coordinates are below the x-axis.



3. **Polar coordinates of a point**

In polar coordinates, the position of a given point is determined by its distance from a fixed point (called the pole) and the angle which the straight line joining the point and the pole makes with a fixed line through the pole (called the initial line) is measured in the anticlockwise sense.



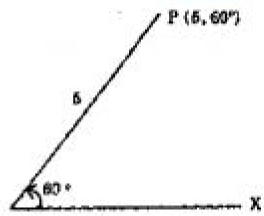
OP = distance of point P from the pole O. OX is the initial line.

OP = r is called the radius vector of the point P. $\angle XOP = \theta$ is called the vectorial angle of the point. The polar coordinates of the point P are represented by (r, θ).

Example 2: Locate the point (6, 60°)

$OP = 5$

$\angle XOP = 60°$



4. **Relations between Cartesian rectangular and polar coordinates**

Let OX and OY *be* the axes of coordinates. Let the coordinates of P be (x, y). If PL be perpendicular to OX,

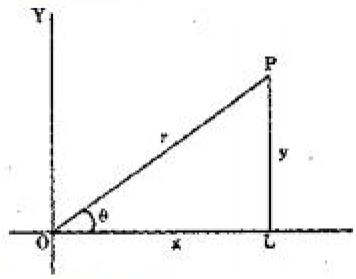
OL=x; PL= y.

Let OP = r be the radius vector. Let $\angle POX = \theta$ be the vectorial angle.

From right-angled triangle OLP

$x = r\cos\theta$ ……..(i)

$y = r\sin\theta$ ……..(ii)



From triangle OLP, $x^2 + y^2 = r^2$

(by Pythagoras theorem)

$\therefore r = \sqrt{x^2 + y^2}$

Dividing (ii) by (i),

$$\frac{r\sin\theta}{r\cos\theta} = \frac{y}{x}$$

Hence, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

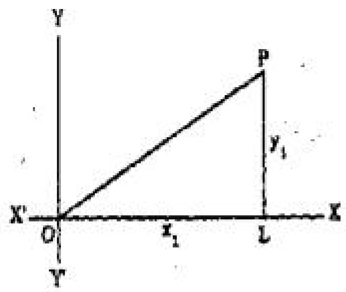
5. The distance between the origin and the point $(x_1, y_1$, is given by $\sqrt{x_1^2 + y_1^2}$

Proof: Let the coordinates of any point P with respect to axes OX and OY be (x_1, y_1).

Draw PL perpendicular to OX. Joint OP.

$OP^2 = OL^2 + LP^2$. From right-angled triangle OLP,

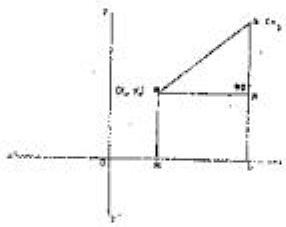
$OP^2 = x_1^2 + y_1^2$



$\therefore$ OP = $\sqrt{x_1^2 + y_1^2}$

6. **The distance between the points** (x_1, y_1) and (x_2, y_2) and is given by $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Proof: Let A be $(x_1, y,)$ and B be (x_2, y_2). Be Two points and let Ox and Oy be the x and y axes.



Draw AL, BM perpendicular to x-axis.
Draw BN perpendicular to AL.
From right-angled triangle ANB

$AB^2 = BN^2 + AN^2$ … (1)

$BN = ML = OL - OM = x_1 - x_2$ … (2)

$AN = AL - NL = AL - BM = y_1 - y_2$ … (3)

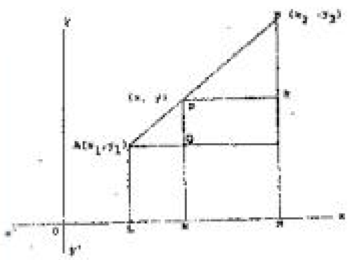
From (1), (2) and (3)

$AB^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$

$\therefore$ AB = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

7. **The coordinates of the point of division the straight line segment joining the points (x_1, y_1) and (x_2, y_2) in the ratio m1: m2 are given by**

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} \text{ and } y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$



Proof: Let A be (x_1, y_1) and B (x_2, y_2). Let the point P(x, y) divides segment AB in the ratio m_1: m_2

ie. $\frac{AP}{PB} = \frac{m_1}{m_2}$

Draw AL, BM and PN perpendicular to X-axis.
Draw AQ perpendicular to PN and PR perpendicular to BM at Q.
In triangles AQP and PRB.

$\angle AQP = \angle PRB = 90°$

$\angle PAQ = \angle BPR$ (corresponding angles)

$\therefore$ The 2 triangles are equiangular and similar.

$\therefore \frac{AQ}{PR} \frac{AP}{PB} = \frac{m_1}{m_2}$ … (1)

$AQ = LN = ON - OL = x - x_1$ … (3)

$PR = NM = OM - ON = x_2 - x$ … (3)

From (1), (2), (3)

$$\frac{x - x_1}{x_2 - x} = \frac{m_1}{m_2}$$

Cross multiplying

$m_1 (x_2 – x) = m_2 (x – x1)$

ie. $x (m_1 + m_2) = m_1 x_2 + m_2 x_1$

$$\therefore\ x = \frac{m_1 x_2 + m_2 x_1}{m_1 - m_2}$$

Again $\frac{AQ}{BR} = \frac{AP}{PB} = \frac{m_1}{m_2}$ … (4)

$PQ = PN – QN = PN - AL = y – y_1$ … (5)

$BR = BM – RM = BM – PN = y_2 – y$ … (6)

From (4), (5) and (6)

$$\frac{y - y_1}{y_2 - y} = \frac{m_1}{m_2}$$

$\therefore\ m_1 (y_2 – y) = m_2 (y – y_1)$

i.e., $y (m_1 + m_2) = m_1 y_2 + m_2 y_1$

$$\therefore\ y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

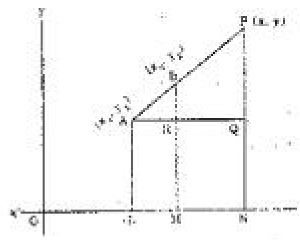
The coordinates the point of division are given by

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

8. The coordinates of the point which divides the straight line segment joining the points A (x_1, y_1) and B (x2, y_2) externally in the ratio m: n is given by

$$x = \frac{\mathbf{mx_2 - nx_1}}{\mathbf{m - n}} ; \mathbf{y} = \frac{\mathbf{my_2 - ny_1}}{\mathbf{m - n}}$$



Proof: Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two Points Let (Px, y) be the point which divides the straight line segment AB externally in the ratio m: n

i.e., $\frac{AP}{PB} = \frac{m}{n}$ or AP: PB = m: n

Draw AL, BM and PN perpendicular to X-axis.

Draw ARQ parallel to x-axis to meet BM at R and PN at Q.

In triangles AQP and ARB.

$\angle AQP = \angle ARB = 90°$

$\angle APQ$ = corresponding $\angle ABR$

Hence, the two triangles are equiangular and hence similar.

$\therefore \frac{AP}{AB} = \frac{AQ}{AR}$

$\therefore \frac{AP}{AP - AB} = \frac{AQ}{AQ - AR}$

i.e., $\frac{AP}{BP} = \frac{AQ}{RQ}$

i.e., $\frac{m}{n} = \frac{AQ}{RQ}$

AQ = LN = ON – OL = $x - x_1$

RQ = MN = ON – OM = $x - x_2$

$$\therefore \frac{x - x_1}{x - x_2} = \frac{m}{n}$$

ie. $m(x - x_2) = n(x - x_1)$

$\therefore x(m - n) = mx2 - nx1$

$\therefore x = \frac{xm_2 - nx_1}{m - n}$

Similarly $y = \frac{my_2 - ny_1}{m - n}$

9. Prove that the coordinates of the mid-point of (x_1, y_1) and (x_2, y_2) are given by

$$x = \frac{x_1 + x_2}{2}; y = \frac{y_1 + y_2}{2}$$

Proof: It P divides the straight line segment joining the points (x_1, y_1) and $(x_2\ y_2)$ internally in the ratio m: n, then the coordinates of the point of division are given by

$$x = \frac{xm_2 + nx_1}{m + n}$$

$$y = \frac{my_2 + ny_1}{m + n} \qquad \text{where } \frac{AP}{PB} = \frac{m}{n}$$

When m = n the point P becomes the mid point of AB.

$\therefore$ Coordinates of mid-point of AB are given by

$$x = \frac{mx_2 + mx_1}{m + m} = \frac{m(x_1 + x_2)}{2m} = \frac{x_1 + x_2}{2}$$

$$y = \frac{my_2 + my_1}{m + m} = \frac{m(y_1 + y_2)}{2m} = \frac{y_1 + y_2}{2}$$

Example 3: Write the polar coordinates of the (2, 2)

Solution: $\frac{y}{x} = \frac{2}{2} = 1 = \tan\theta$ ie. $\theta = 45°$

$r^2 = x^2 + y^2 = 2^2 + 2^2 = 8$

$\therefore r = \sqrt{8} = 2\sqrt{2}$

Polar coordinates of a point are given by (r,θ)

∴ Polar coordinates of (2, 2) are $r2\sqrt{2}$, 45°)

Example 4: Write the Cartesian rectangular coordinates of the point (4, 30)

Solution: Relations connecting rectangular coordinates and polar coordinates are given by x = r $\cos\theta$; y = r $\sin\theta$

For the point (4, 30°)

r = 4; θ = 30°

$\therefore\ x = 4\cos 30° = 4\times\frac{\sqrt{3}}{2}=2\sqrt{3}$

$y = 4\sin 30° = 4\times\frac{1}{2}=2$

Hence the point is $(2\sqrt{3},2)$

Example 5: Find the distance between the points (3, 4) and (-5, -2)

Solution: The distance between two points (x_1, y_1) and (x_2, y_2) is given by

$$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

∴ The distance between the given points is

$$= \sqrt{[3-(-5)]^2+[4-(-2)]^2}$$

$$= \sqrt{[3+5)^2+(4+2)^2}=\sqrt{8^2+6^2}$$

$$= \sqrt{64+36}=\sqrt{100}=10$$

Example 6: Show that the points A(-9. 7); B(3, 3); C(6, 2) are collinear.

Solution: AB = $\sqrt{(-9-3)^2+(7-3)^2}=\sqrt{144+16}=\sqrt{160}$

ie. AB = $\sqrt{16\times10}=4\sqrt{10}$ ... (i)

AC = $\sqrt{(-9-6)^2+(7-2)^2}=\sqrt{225+25}=\sqrt{250}$

(ie) AC = $\sqrt{25\times10}=5\sqrt{10}$ ... (ii)

BC = $\sqrt{(3-6)^2+(3-2)^2}=\sqrt{(9+1)}=\sqrt{10}$ ... (iii)

From (i), (ii), (iii) it is seen that

AC = AB + BC sin $5\sqrt{10}=4\sqrt{10}+\sqrt{10}$

∴ A, B, C are collinear.

Example 7: Show that the triangle formed by the points A(-2, 0), B(2, 0), $C(0,2\sqrt{3})$ is equilateral.

Solution: $AB^2 = (-2-2)^2 + (0-0)^2 = (-4)^2 = 16$

$AC^2=(-2-0)^2+(0-2\sqrt{3})^2=(-2)^2+(-2\sqrt{3})^2=4+12=16$

$BC^2=(2-0)^2+(0-2\sqrt{3})^2=4+12=16$

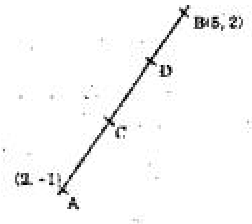
$\therefore\ AB^2 = AC^2 = BC^2$

$\therefore \quad AB = AC = BC$

Hence the triangle formed by the three points is equilateral.

Example 8: Find the points of trisection of the line segment joining the points (2, - 1) and (5, 2)

Solution: Let A bc (2, 1); B be (5, 2)



Let C and D be points which divide the Line segment AB into three equal parts Ie. AC = CD = DB

C and D are the points of trisection of AB

$AC \cdot CB = 1:2$

$AD \cdot DB = 2:1$

Let C be (x_1, y_1) and be (x_2, y_2)

Then $x_1 = \frac{1(5)+2(2)}{1+2} = \frac{9}{3} = 3$

$y_1 = \frac{1(2)+2(-1)}{1+2} = \frac{0}{3} = 0$

$\therefore C = (x_1, y_1) = (3, 0)$

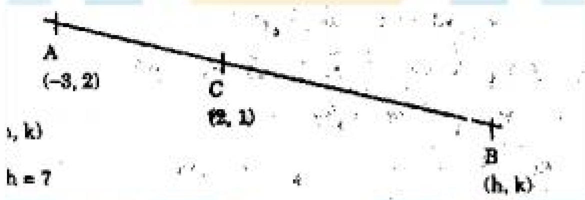
$x_2 = \frac{2(5)+1(2)}{2+1} = \frac{12}{3} = 4$

$y_2 = \frac{2(2)+1(-1)}{2+1} = \frac{3}{3} = 1$

$\therefore D = (x_2, y_2) = (4, 1)$

Example 9: The mid-point of a line segment is the point (2, 1) and one end point of the segment is (-3, 2). Find the other en point.

Solution



Let the other end point be (h, k)

$\therefore \frac{h-3}{2} = 2 \text{ie.} h-3=4 \text{or} h=7$

$\frac{k+2}{2} = 1 \text{ie.} k+2=2 \therefore k=0$

$\therefore$ the other end-point = (h, k) = (7, 0)

Example 10: A segment AB is divided in the ratio 3: 2 the point of division being (0, - 3). If A is the point (- 6, - 9) find B.

Solution:

Let B be (h, k)

$$\frac{3h-12}{3+2}=0 \text{ ie. } 3h-12=0 \text{ ie. } h=4$$

$$\frac{3k-18}{3+2}=-3 \therefore 3k-18=-15 \text{ ie. } 3k=3 \text{ or } k=1$$

$\therefore$ the point B is (h, k) = (4, 1)

10. Area of a triangle

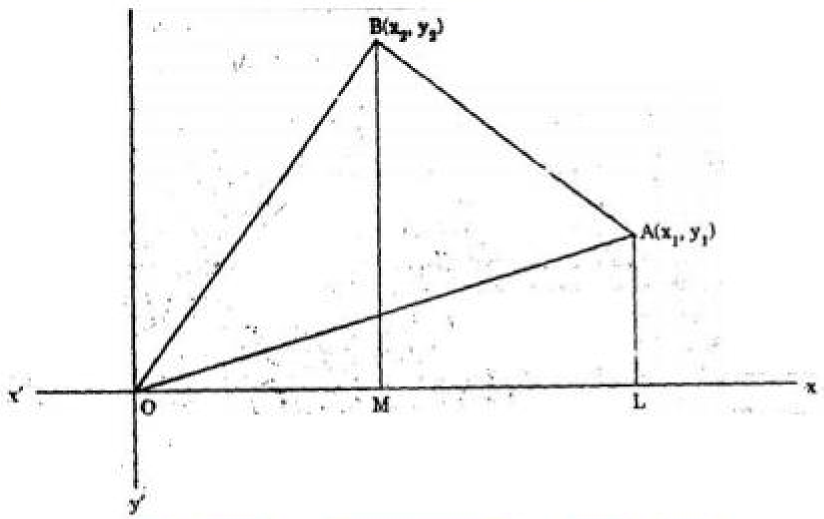
(1) Find the area of the triangle formed by joining the origin to the points (x_1, y_1) and (x_2, y_2)

Proof: Let A be (x_1, y_1) and B be (x_2, y_2)

Join AB, OA and OB. Draw AL, BM, perpendicular to x-axis

The area of $\Delta OAB = \frac{1}{2} OM \cdot MB + \frac{1}{2} (AL + BM) ML - \frac{1}{2} OL \cdot LA$

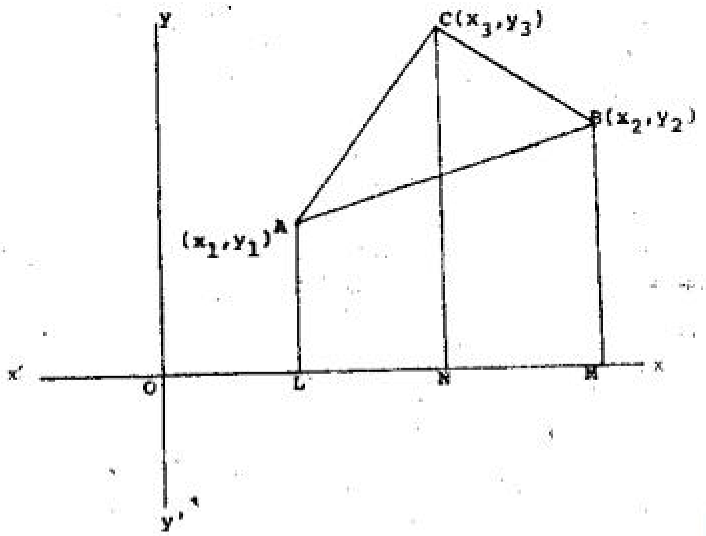
$$\Delta OAB \ \frac{1}{2}\times x_1y_2+\frac{1}{2}(y_1+y_2)(x_1-x_2)-\frac{1}{2}\times x_1y_1$$



$$\Delta OAB = \frac{1}{2}[x_1y_2+(x_1y_1+x_1y_1-x_2y_1-x_2y_2)-x_1y_1]$$

ie. $\Delta OAB = \frac{1}{2}(x_1y_2-x_2y_1)$ square units

(2) Find the area of the triangle formed by joining the points (x_1, y_1); $(x_2\ y_2)$; (x_3, y_3)



Proof: Area of ΔABC

= Area of trapezium ALNC + Area o trapezium CNMB. Area of trapezium ALMB

$= \frac{1}{2}(AL+CN)+\frac{1}{2}(CN+BM)AM-\frac{1}{2}(AL+BM)LM$

$= \frac{1}{2}(y_1+y_3)(x_3-x_1)+\frac{1}{2}(y_3+y_2)(x_2-x_3)-\frac{1}{2}(y_1+y_2)(x_2-x_1)$

Area of ΔABC = $\frac{1}{2}x_1[(y_1+y_2)-(y_1+y_3)\ +x_2[(y_3+y_2)-(y_1+y_2)]$

$+x_3(y_1+y_3)-(y_3+y_2)]$

ie. Area of ΔABC $=\frac{1}{2}x_1[(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

In order to ensure that the value of the expression on the right-hand side is positive it is necessary that the points A(x_1, y_1) B(x_2, y_2) C(x_3, y_3) are arranged in the order such that the ΔABC is described in the anti-clockwise direction.

Example 11: Find the area of the triangle joining the points (1, - 3), (3, 1), (1, 4)

Solution: Let (x_1, y_1) = (1, - 3); (x_2, y_2) = (x_2, 1); (x_3, y_3) = 1.4)

Area of Δ $= \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)x_3(y_1-y_2)]$

$= \frac{1}{2}[1_1(1-4)+3(4+3)+1(3-1)]$

$= \frac{1}{2}(-3+21-4)=\frac{1}{2}(14)=7$ square units

Example 12: Find the area of the quadrilateral formed by joining the points (5, 0), (3, 4), (- 4, 1), (2, - 3)

The area of the quadrilateral ABCD

= area of ΔABC + area of ΔACD

Area of ΔABC

$= \frac{1}{2}[5(4-1)+3(1-0)-4(0-4)]$

$= \frac{1}{2}[15+3+16)=\frac{34}{2}=17$

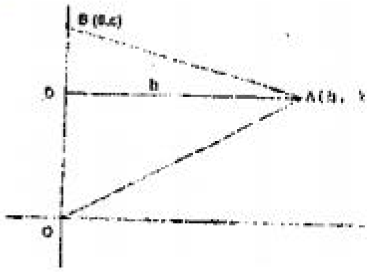
Area of ΔACD

$= \frac{1}{2}[5(1+3)-4(-3-0)+2(0-1)]$

$= \frac{1}{2}[20+12-2)=\frac{30}{2}=15$

∴ Area of quadrilateral ABCD = area of Δ BC + area of ΔACD = 17 + 15 = 32 square units.

Example 13: Find the area of of the triangle having for its vertices the points (0, 0), (0, c), h.k)



Solution: Let A be (b, k) and B be (0, c)

Join OA.AB

Draw AD perpendicular to OB

In ΔOAB, if OB is considered as the base and AD as the height

The area of ΔOAB $-\frac{1}{2}OB\times AD=\frac{1}{2}c\times h=ch$

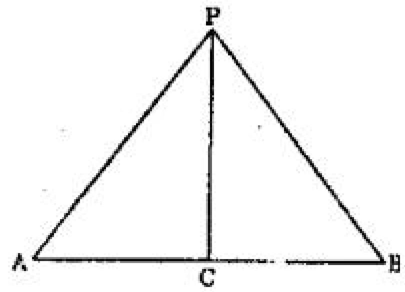
11. **Locus:** The path traced out by a point moving under certain geometrical conditions is called the locus of the point.

The following are well known loci (plural of locus is loci)

(i) The locus of point which moves so that its distance from a fixed point is constant is the circumference of a circle. The fixed point is called the centre of the circle and the fixed distance is called the radius of the circle.

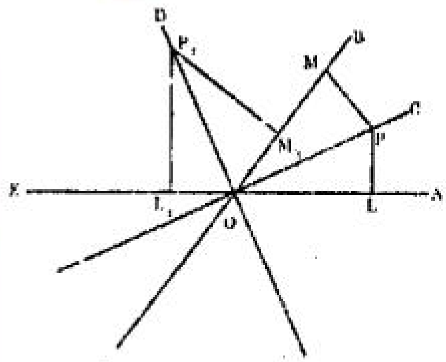
(ii) The locus of point, which moves so that its distances from two fixed points are equal, is the perpendicular bisector of the line segment joining the two points.

CP is the perpendicular bisector of the line segment AB. It will be found that AP = PB [$\because$ Δs PCA and PCB are congruent]. Any point on CP will be at equal distances from A and B.



(iii) The locus of points equi-distant from two given lines will be the pair of bisectors of the angles between the lines.

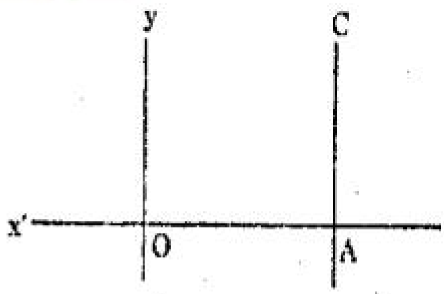
Any point P on OC is equi-distant from OA and OB. ie., PL = PM, where PL and PM are lengths of perpendiculars from P to OA and OB. any point p_1 on Odis equi-distant OA and OB i.e., $P_1L_1 = P_1M_1$, where P_1L_1 and P_1M_1 are lengths of perpendiculars drawn from P to CE (or AO produced) and OB.



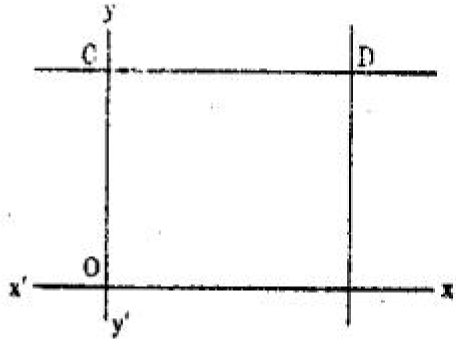
12. **Equation to locus**

(i) Let us take a straight line parallel to y-axis and at a distance of 5 units from it.

The distances of the straight line AC from y-axis is 5 units. Since AC is parallel to y-axis, the distances of all points on AC from y-axis, the distances of all points on AC from y-axis are equal and equal to 5 units. Ie., the x coordinates of all points on AC = 5. $\therefore x = 5$ is the equation of AC or x = 5 is the equation of the locus.



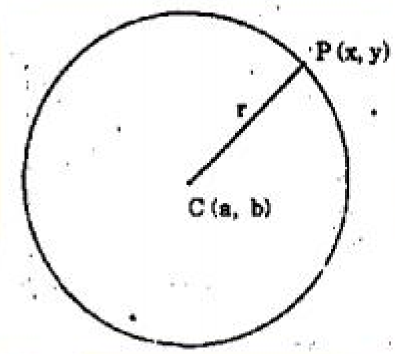
(ii) Let us consider a straight line which is parallel to x-axis and at a distance of 4 units from it.
Since the line CD is parallel to x-axis, all points on CD are at equal distances from Ox, equal to 4 units. Hence, the y coordinates of all points on CD = 4.



$\therefore$ y = 4 equation of CD or y = 4 is the equation of the locus.

(iii) Let us consider the locus of a point which moves such that its distance from a fixed point is constant.

The locus of the point is a circle. Let (x, y) be any point on the circle and C (a, b) the fixed point and r be the constant distance.



CP = r
$CP^2 = r^2$
$(x - a)^2 + (y - b)^2 = r^2$ is the equation of the locus.

Example 14: Find the equation of the point, which moves such that its distances from the two points (0, a) and (0, - a) are equal.

Solution: Let A be the point (0, a) and B be the point (0, - a). Let p (x, y) be any point on te locus. $\therefore$ PA = PB.

$\sqrt{(x-0)^2+(y-a)^2} = \sqrt{(x-0)^2+(y+a)^2}$

Squaring, $x^2 + (y - a)^2 = x^2 + (y + a)^2$

$\therefore (y + a)^2 - (y - a)^2 = 0$ i.e., 4ay = 0

y = 0 which is the x-axis.

$\therefore$ the required locus is the x-axis.

Example 15: A point moves so that its distance from the point (2, 4) is three times its distance from (-3, 0). Find the locus of the point.

Solution: Let A be the point (2, 4)
Let B be the point (-3, 0)
Let P be the point (x, y)

PA = 3 PB

$\therefore$ PA2 = 9 PB2

i.e., $(x - 2)^2 + (y - 4)^2 = 9 [(x + 3)^2 + (y - 0)^2]$

i.e., $(x^2 + y^2 - 4x - 8y + 20 = 9 [x^2 + y^2 + 6x + 9]$

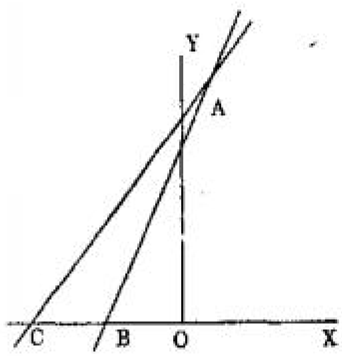
i.e., $8x^2 + 8y^2 + 58x + 8y + 61 = 0$ is the equation of the locus.

13. To find the angle between two given straight lines

Let the two straight lines be AB and AC meeting the x-axis at B and C.
Let their equation be

$y - m_1x + c_1$ and $y = m_2x + c_2$

$\left.\begin{array}{l}\tan ABX = m_1 \\ \tan ACX = m_2\end{array}\right\}$ are the slopes of $y = m_1x + c_1$ and $y = m_2x + c_2$ respectively.



Now $\angle BAC = \angle ABX - \angle ACX$

$\tan BAC = \tan [ABX - ACX]$

$$= \frac{\tan ABX - \tan ACX}{1 + \tan ABX \tan ACX}$$

$$= \frac{m_1 - m_2}{1 + m_1 m_2}$$

Therefore the required angle BAC = $\tan^{-1}\left(\frac{m_1 - m_2}{1 + m_1 m_2}\right)$

14. To find the condition that two straight lines may be parallel

Two straight lines are parallel when the angle between them is zero. Therefore the tangent of this angle is zero.

$\therefore$ $m_1 = m_2$, i.e., their slopes are equal.

Two straight lines are parallel when their "slopes (m)" are the same or in other words if their equations differ only in the constant term.

Example 16

Find the equation to the straight line which passes through the point (-5, 4) and which is parallel to the straight line

$4x + 3y + 5 = 0$

Any straight line parallel to $4x + 3y + 5 = 0$ has its equation of the form

$4x + 3y + c = 0$ … (1)

This straight line passes through the point (-5, 4) and which is parallel to the straight line

This straight line passes through the point (- 5, 4)

So, $4 (- 5) + 3 (4) + c = 0$

$- 20 + 12 + c = 0$

$c = 8$

$\therefore$ The required equation is $4x + 3y + 8 = 0$

15. To find the condition that two straight lines whose equations are given may be perpendicular.

Let the straight lines be $y = m_1x + c_1$ and $y = m_2x + c_2$

If θ be angle between them, then

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1m_2} \quad \dots (1)$$

If the lines are perpendicular, then θ and therefore $\tan\theta = \propto$

$$\text{i.e., } \frac{m_1 - m_2}{1 + m_1m_2} = \propto$$

This is possible only when the denominator is zero

$\therefore$ 1 + m1 m2 = 0 i.e., m1 m2 = - 1 (i.e., The product of the slopes of the lines = - 1)

$$\Rightarrow m_2 = \frac{-1}{m_1}$$

Note: (1) Any line perpendicular to $ax + by + c = 0$ can be taken as $bx – ay + k = 0$.

(2) Any line parallel to $ax + by + c = 0$ can be taken as $ax + by + \lambda = 0$

Example 17

Find the equation to the straight line which passing through the point (- 5, 4) and is perpendicular to $4x + 3y + 5 = 0$.

Any straight line passing through the point (- 5, 4) is $y – 4 = m(x + 5)$

This straight line is perpendicular to $4x + 3y + 5 = 0$

$$\therefore m \times \left(\frac{-4}{3}\right) = -1$$

$$m = \frac{3}{4}$$

$\therefore$ the required equation is $(y-4) = \frac{3}{4}(x+5)$

$$4y – 16 = 3x + 15$$

$$3x – 4y + 31 = 0$$

Another method

Any straight line perpendicular to $4x + 3y + 5 = 0$ is of the form

$3x – 4y + c = 0$

Now it passes through (- 5, 4)

$\therefore$ $- 15 – 16 + c = 0$

$C = 31$

$\therefore$ the required equation is $3x – 4y + 31 = 0$