CONSTRUCTIONS

★ **INTRODUCTION**

In class IX, we have discussed a number of constructions with the help of ruler and compass e.g. bisecting a line segment, bisecting an angle, perpendicular bisector of line segment, some more constructions of triangles etc. with their justifications. In this chapter we will discuss more constructions by using the knowledge of the earlier construction.

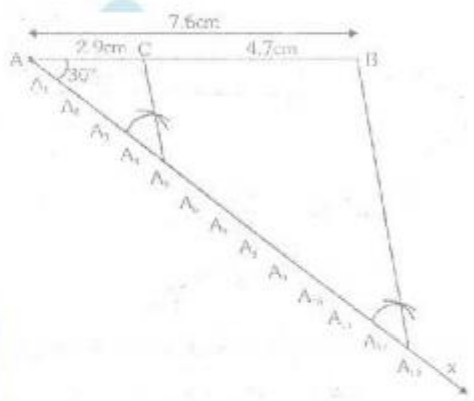
★ **DIVISION OF A LINE SEGMENT**

Let us divide the given line segment in the given ratio say 5 : 8. This can be done in the following two ways:

(i) Use of Basic Proportionality Theorem.

(ii) Constructing a triangle similar to a given triangle.

Construction – 1: Draw a segment of length 7.6 cm and divide it in the ratio 5 : 8. Measure the two parts . (NCERT)



Steps of Constructions:

Step 1 : Draw any ray AX making an angle of 30^o with AB.

Step 2 : Locate 13 points : $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}, A_{11}, A_{12}$ and A_{13} So that:

$AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = \ldots\ldots = A_{11}\,A_{12} = A_{12}\,A_{13}$

Step 3 : Join B with A_{13}.

Step 4 : Through the point A_5, draw a line $A_5C \| A_{13}B$ such that $\angle AA_5C$ = corr. $\angle AA_{13}B$ intersecting AB at a point C. Then AC : CB = 5 : 8.

Let us see how this method gives us the required division.

Since A_5C is parallel to A_{13} B.

Therefore $\dfrac{AA_5}{A_5A_{13}} = \dfrac{AC}{CB}$ (Basic Proportionality Theorem)

By construction, $\dfrac{AA_5}{A_5A_{13}} = \dfrac{5}{8}$

Therefore $\dfrac{AC}{CB} = \dfrac{5}{8}$

This given that C divides AB in the ratio 5 : 8.

By measurement, we find, AC = 2.9 cm, CB = 4.7 cm.

By Calculation: $AC = \dfrac{7.6x5}{13} = \dfrac{38}{13} = 2.9$

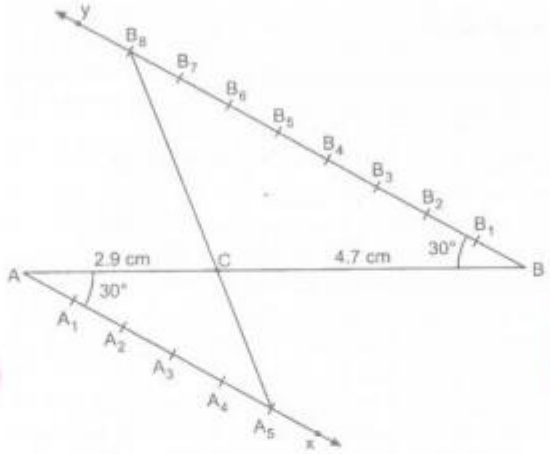
$BC = \dfrac{7.6x8}{13} = \dfrac{60.8}{13} = 4.67 = 4.7\,\text{cm}.$

Alternative Solutions

Step 1 : Draw a line segment AB = 7.6 cm and to be divided in the ratio 5 : 8.

Step 2 : Draw any ray AX making an angle of 30° with AB.

Step 3 : Draw a ray BY parallel to AX by making $\angle ABY$ equal to $\angle BAX$. i.e. $\angle ABY$ = corr. $\angle BAX$.



Step 4 : Locate the points A_1, A_2, A_3, A_4, A_5, on AX and $B_1, B_2, B_3, B_4, B_5, B_6, B_7$, and B_8 on BY such that :

$AA_1 = A_1A_2 = \ldots\ldots = A_4A_5 = BB_1 = B_1B_2 = \ldots\ldots = B_6B_7 = B_7B_8$.

Step 5 : Join A_5B_8. Let it intersect AB at a point C. then AC : CB = 5 : 8.

Here ΔAA_5C is similar to ΔBB_8C

Then $$\frac{AA_5}{BB_8} = \frac{AC}{BC}$$

Since by construction, $$\frac{AA_5}{BB_8} = \frac{5}{8}$$ Therefore $$\frac{AC}{CB} = \frac{5}{8}$$

By measurement : AC = 2.9 cm, BC = 4.7 cm.

Constructions – 2 : Construct a triangle with sides 5 cm, 6 cm and 7 cm and then another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle. (NCERT)

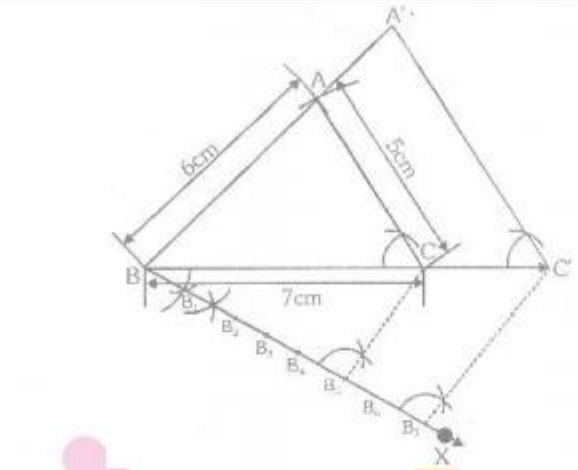
Sol. First all we are to construct a triangle ABC with given sides, AB = 6 cm, BC = 7 cm, CA = 5 cm. Given a triangle ABC, we are required to construct a triangle whose sides are $\frac{7}{5}$ of the corresponding sides of ΔABC.

Steps of Construction :

Step 1 : Draw any ray BX making an angle of 30° with the base BC of ΔABC on the opposite side of the vertex A.

Step 2 : Locate seven points $B_1, B_2, B_3, B_4, B_5, B_6$ and B_7 on BX so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6 = B_6B_7$.

[Note that the number of points should be greater of m and n in the scale factor $\frac{m}{n}$.]



Step 3 : Join B_5 (the fifth point) to C and draw a line through B_7 parallel to B_5C, intersecting the extended line segment BC at C'.

Step 4 : Draw a line through C' parallel to CA intersecting the extended line segment BA at A'. Then, A'B'C is the required triangle.

For justification of the construction.

$$\Delta ABC \approx \Delta A'BC'$$

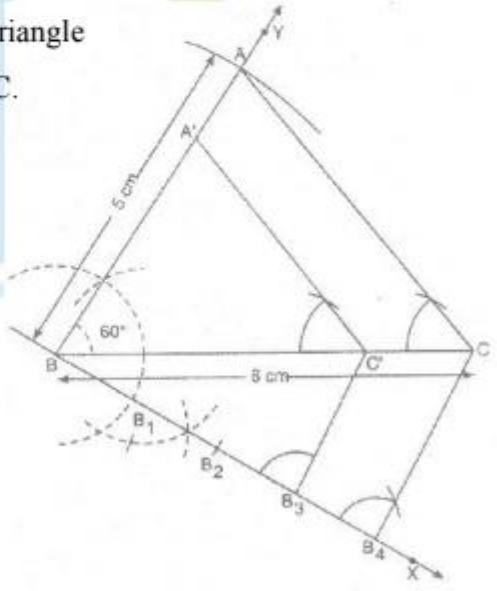
Therefore, $$\frac{AB}{A'B} = \frac{AC}{A'C} = \frac{BC}{BC'}$$

But $$\frac{BC}{BC'} = \frac{BB_5}{BB_7} = \frac{5}{7}$$

Therefore $$\frac{A'B}{AB} = \frac{A'C}{AC} = \frac{BC'}{BC} = \frac{7}{5}$$

Construction – 3 : Draw a triangle ABC with side BC = 6 cm, AB = 5 cm and $\angle ABC = 60^\circ$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC. (NCERT)

Sol. Given a triangle ABC, we are required to construct another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC.



Step of Constructions :

Step 1 : Draw a line segment BC = 6 cm.

Step 2 : At B construct $\angle CBY = 60^\circ$ and cut off AB = 5 cm, join AB and AC. ABC is the required Δ.

Step 3 : Draw any ray BX making an acute angle say 30° with BC on the opposite side of the vertex A, $\angle CBX = 30^\circ$ downwards.

Step 4 : Locate four (the greater of 3 and 4 in $\frac{3}{4}$) points B_1, B_2, B_3 and B_4 on BX, so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.

Step 5 : Join B_4C and draw a line through B_3 (the 3rd point) parallel to B_4C to intersect BC at C'.

Step 6 : Draw a line through C' parallel to the line CA to intersect BA at A'.

Then A'BC' is the required triangle whose each side is $\frac{3}{4}$ times the corresponding sides of them ΔABC,

Let us now see how this construction gives the required triangle.

For justification of the construction.

$$\frac{BC'}{C'C} = \frac{3}{1}$$

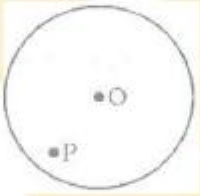
Therefore $$\frac{BC}{BC'} = \frac{BC' + C'C}{BC'} = \frac{BC'}{BC'} + \frac{C'C}{BC'} = 1 + \frac{1}{3} = \frac{4}{3}$$

$\Rightarrow$ $BC' = \frac{3}{4}$ BC, Also C'A' is parallel to CA.

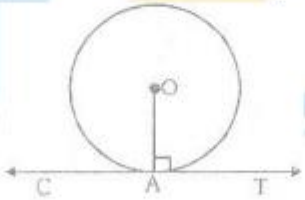
Therefore $\Delta A'BC' \approx \Delta ABC \Rightarrow \frac{A'B}{AB} = \frac{A'C'}{AC} = \frac{BC'}{BC} = \frac{3}{4}$

★ **CONTRUCTION OF TANGENTS TO A CIRCLE**

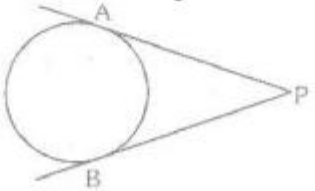
(a) If a point lies inside a circle, we can not draw any tangent to the circle i.e., No tangent is possible in this case



(b) If a point lies on the circle, then there is only one tangent to the circle at this point. The tangent to a circle at any point is perpendicular to the radius passing through the point of contact.

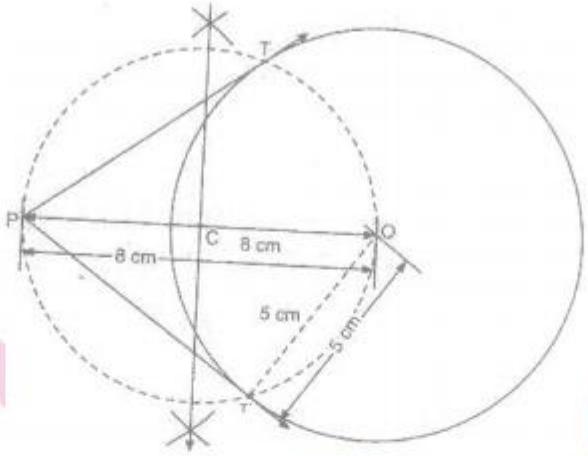


(c) Two tangents are drawn from an external point to circle, they are equal in length.



Construction 4 : Draw a circle of radius 5 cm. From a point 8 cm away from its centre, construct pair of tangents to the circle measure their lengths.

Sol.



Steps of Construction :

Step-1 : Draw a circle with radius 5 cm whose centre is O.

Step-2 : Take a point P at a distance 8 cm from the centre O such that OP = 8cm.

Step-3 : Bisect the line segment OP at the point C such that OC = CP = 4 cm.

Step-4 : Taking C as centre and OC as arc, draw a dotted circle to intersect the given circle at the points T and T'.

Step-5 : Join PT and PT'

PT and PT' are the required pair of tangents to the circle.

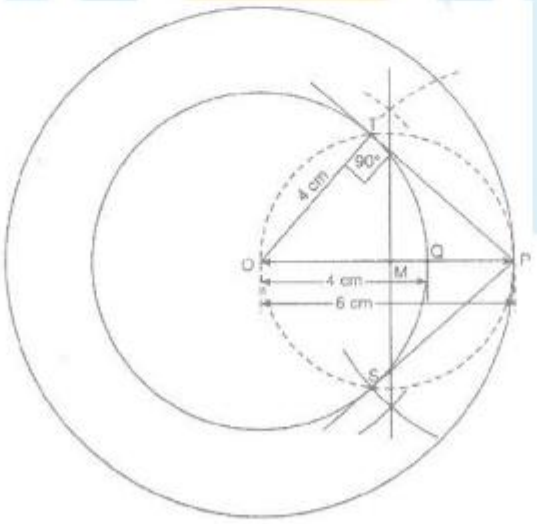
By measurement we obtain PT = PT' = 6.2 cm (Answer)

Verification: PT = PT' = $\sqrt{8^2 - 5^2} = \sqrt{64 - 25} = \sqrt{39} = 6.2$ cm (Answer)

Construction 5. Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also verify the measurement by actual calculation.

(NCERT)

Sol.



Steps of Construction:

Step 1 : Draw two concentric circles with centre O and radii 4 cm and 6 cm such that OP = 6 cm, OQ = 4 cm.

Step 2 : Join OP and bisect it at M. i.e. M is the mid-point of OP i.e. OM = PM = 3 cm.

Step 3 : Taking M as centre with OM as radius draw a circle intersecting the smaller circle in two points namely T and S.

Step 4 : Join PT and PS.

PT and PS are the required tangents from a point P to the smaller circle, whose radius is 4 cm. By measurement: PT = 4.5 cm.

Verification. OTP is right Δ at T

$OP^2 = OT^2 + PT^2$

$6^2 = 4^2 + PT^2 \Rightarrow PT^2 = 36 - 16 = 20$

$PT = \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5} = 2 \times 2.24 = 4.48$ cm

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

SUBJECTIVE TYPE QUESTIONS

1. Draw a line segment of length 7.5 cm and divide it internally in the ratio 3 : 2. Measure the two parts.
2. Divide a line segment 8.8 cm long internally in the ratio 4 : 7 and measure the two parts.
3. Draw a line segment of length 13.5 cm and divide it internally in the ratio 2 : 3 : 4. Measure each part.
4. Construct a triangle with sides AB = 4 cm. BC = 5 cm and AC = 6 cm and then another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC.
5. Construct a triangle ABC whose sides are 4 cm, 5 cm, 7 cm. Construct another triangle similar to ΔABC and with sides $\frac{2}{3}$rd of the corresponding sides of triangle ABC.
6. Draw a right triangle in which the sides (other than hypotenuse) are of length 5 cm and 12 cm. Then construct another triangle whose sides are $\frac{7}{5}$ times the corresponding sides of the given triangle.
7. Construct an isosceles triangle whose base is 6 cm and altitude 3 cm and then another triangle whose sides are $\frac{4}{5}$ times the corresponding sides of the isosceles triangle.
8. Draw a triangle ABC with sides BC = 8 cm, $\angle B = 30^\circ$, $\angle A = 45^\circ$. Then construct a triangle whose sides are $\frac{5}{4}$ times the corresponding sides of ΔABC.
9. Construct a ΔABC, whose perimeter is 10.5 cm and base angles are 60° and 45°. Construct another Δ whose sides are $\frac{4}{3}$ of the corresponding sides of the ΔABC.
10. Draw two tangents to a circle of radius 4 cm from a point P at a distance 7 cm from its centre. Also measure the length of the two tangents. Are they equal? Give reasons for your answer.
11. Construct a circle with radius equal to 3 cm. Draw two tangents to it inclined at an angle of 60° at their point of intersection. Measure their lengths and verify the results by calculation.
12. Draw two tangents to a circle of radius 4 cm inclined at an angle of 45° to each other.
13. Construct a tangent to a circle of radius 3 cm from a point on the concentric circle of radius 5 cm and measure its length. Also verify the measurement by actual calculation.
14. Draw a circle of radius 2.5 cm. Take two points P and Q on one of its extended diameter each at a distance of 7.5 cm from its centre. Draw tangents to the circle from these two points P and Q.
15. Draw a line segment AB of length 10 cm. Taking A as centre, draw a circle of radius 5 cm and taking B as centre, draw a circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle.

EXERCISE – 2 (FOR SCHOOL/BOARD EXAMS)

SUBJECTIVE TYPE QUESTIONS

PREVIOUS YEARS BOARD (CBSE) QUESTIONS

1. Draw a line segment AB = 7 cm. Divide it internally in the ratio of (i) 3 : 5, (ii) 5 : 3. **[2000 C]**

2. From a point P on the circle of radius 4 cm, draw a tangent to the circle with using the centre. Also write the steps of construction. **[2000]**

3. Draw circle of radius 4.5 cm. Take a point P on it. Construct a tangent at the point P without using the centre of the circle. Write the steps of construction. **[2001]**

4. Divide a line segment of length 5.6 cm internally in the ratio (i) 3 : 2 (ii) 2 : 3. **[2001]**

5. Construct a ΔABC in which base AB = 6 cm, $\angle C = 60^\circ$ and the median CD = 5 cm. Construct a $\Delta AB'C'$ similar to ΔABC with base AB' = 8 cm. **[2002]**

6. Draw a circle of radius 3.5 cm. From a point P on the circle draw a tangent to the circle without using its centre.. **[2003]**

7. Draw a circle of radius 5 cm. Take a point P on it, without using the centre of the circle, construct a tangent at the point P. Write the steps of construction also. **[2003]||**

8. Draw a circle of diameter 12 cm. From a point P, 10 cm away from its centre, construct a pair of tangent to the circle. Measure the lengths of the tangent segments. **[2004 C]**

9. Draw a circle of radius 3.5 cm. Form a point P, outside the circle at a distance of 6 cm from the centre of circle, draw two tangent to the circle. **[2005]**

10. Construct a ΔABC in which AB = 6.5 cm, $\angle B = 60^\circ$ and BC = 5.5 cm. Also construct a triangle AB'C' similar to ΔABC, whose each side is $\frac{3}{2}$ of the corresponding side of the ΔABC. **[Delhi-2008]**

11. Draw a ΔABC with side BC = 6 cm, AB = 5 cm and $\angle ABC = 60^\circ$. Construct a $\Delta AB'C'$ similar to ΔABC such that sides of $\Delta AB'C'$ are $\frac{3}{4}$ of the corresponding sides of ΔABC. **[AI-2008]**

12. Draw a right triangle in which the sides containing the right angle are 5 cm and 4 cm. Construct a similar triangle whose sides are $\frac{5}{3}$ times the sides of the above triangle. **[Foreign-2008]**

13. Construct a ΔABC in which BC = 6.5 cm, AB = 4.5 cm and $\angle ABC = 60^\circ$. Construct a triangle similar to this triangle whose sides are $\frac{3}{4}$ of the corresponding sides of triangle ABC. **[Delhi-2008]**

14. Draw a right triangle in which sides (other than hypotenuse) are of lengths 8 cm and 6 cm. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the first triangle. **[AI-2009]**

15. Draw a circle of radius 3 cm. From a point P, 6 cm away from it's centre, construct a pair of tangents to the circle. Measure the lengths of the tangents. **[Foreign-2009]**

16. Construct a triangle ABC in which AB = 8 cm, BC = 10 cm and AC = 6 cm. Then construct another triangle whose sides are $\frac{4}{5}$ of the corresponding sides of Δ ABC. **[AI-2010]**

17. Construct a triangle ABC in which BC = 9 cm, $\angle B = 60^{\circ}$ and AB = 6 cm. Then construct another triangle whose sides are $\frac{2}{3}$ of the corresponding sides of Δ ABC. **[AI-2010]**

18. Construct a triangle ABC in which BC = 8 cm, $\angle B = 60^{\circ}$ and $\angle C = 45^{\circ}$. Then construct another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of Δ ABC. **[AI-2010]**