PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

★ INTRODUCTION

In class IX, we have read about linear equations in two variables. A linear equation is a rational and integral equating of the first degree.

For example, the equations : $3x + 2y = 7$, $2x - \sqrt{3}y = \sqrt{5}$, $y - 4x = \sqrt{3}$ are linear equations in two variables, since in each case

(i) Neither x nor y is under a radical sign i.e., x and y rational .

(ii) Neither x nor y in the denominator.

(ii) The exponent f x and y in each term is one.

In general, $ax + by + c = 0 : a, b, c \in R$; $a \neq 0$ and $b \neq 0$ is a linear equation in two variables. A linear equation in two variables has an infinite number of solutions. The graph of a linear equation in two variables is always a straight line . In this chapter, we shall study about systems of linear equations in two variables, solution of system of linear equations in two variables and graphical and algebraic methods of solving a system of linear equations in two variables. In the end of the chapter, we shall be discussing some applications of linear equations in two variables in simple problems areas.

★ HISTORICAL FACTS

Diophentus, the last genius of Alexandria and the best algebraic mathematician of the Greeko Roman Era, has made a unique contribution in the development of Algebra and history of mathematics. He was born in the 3rd century and lived for 84 years. Regarding his age it has been told in a VINODIKA of Greek collections.

"He spent one-sixth of his life in childhood, his beared grew after one twelfth more, after another one-seventh he married, five years later his son was born, the son lived to half the father's age, and Diophentus died four years after his son."

i.e. $\frac{x}{6} + \frac{x}{12} + \frac{x}{7} + 5 + \frac{x}{2} + 4 = x \Rightarrow 9x = 756 \Rightarrow x = 84$ years.

He was known as the father of Algebra. Arithimetica is his famous book.

★ RECALL

(i) **Equation :** An statement of equality of two algebraic expressions which involve one or more unknown quantities is known as an equation.

(ii) **Linear Equation :** An equation in which the maximum power of variable is one is called a linear equation.

(iii) **Linear Equation in One Variable :** An equation of the form $ax + b = 0$ where x is a variable, a, b are real number and $a \neq 0$ is called a linear equation in one variable.

(iv) **Linear Equation in Two Variables :** An equation of the form $ax + by + c = 0$, where a, b, c are real number , $a \neq 0$, $b \neq 0$ and x, y are variables is called linear equation in two variables.

Any pair values of x & y which satisfies the equation $ax + by + c = 0$ is called a root or solution it.

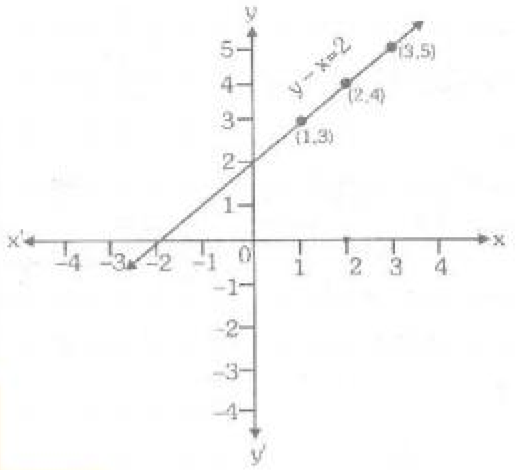
Ex. $(x = 1, y = 1)$ is a solution of $4x - y - 3 = 0$.

Remark : A linear equation in two variables have infinite number of solutions.

(v) **Graph of a Linear Equation in two Variables :** Assume y – x = 2 be a linear equation in two variables. The following table exhibits the abscissa and ordinates of points on the line represented by the equation y – x = 2

x	1	2	3
y	3	4	5

Plotting the points (1, 3), (2, 4) and (3, 5) one the graph paper and drawing the line joining them we obtain the graph of line represented by the given equation as shown in fig.



★ SIMULTANEOUS LINEAR EQUATIONS IN TWO RABBLES

A pair of linear equations in two variables is said to form a system of simultaneous linear equations.

General Form : $a_1x + b_1y\ c_1 = 0$

and $a_2x + b_2y + c_2 = 0$, where a_1, a_2, b_1, b_2, c_1 and c_2 are era number ; $a_1^2 + b_1^2 \neq 0$ and $a_2^2 + b_2^2 \neq 0$ and x, y are variables.

Ex. Each of the following pairs of linear equations form a system of two simultaneous linear equations in two variables.

(i) x – 2y = 3, 2x + 5y = 5 (ii) 3x+5y+7=0, 5x + 2y + 9 = 0

★ SOLUTION OF THE SYSTEM OF EQUATIONS

Consider the system of simultaneous linear equations : $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

A pair of value of the variables x and y satisfying each one of the equations in a given system of two simultaneous linear equations in x and y is called a solution of the system .

Ex. x = 2, y = 3 is a solution of the system of simultaneous linear equations.

2x + y = 7 , 3x + 2y = 12

The given equations are 2x + y = 7 ......(i)

3x + 2y = 12 ......(ii)

Put x = 2, y = 3 in LHS of equation (i), we get

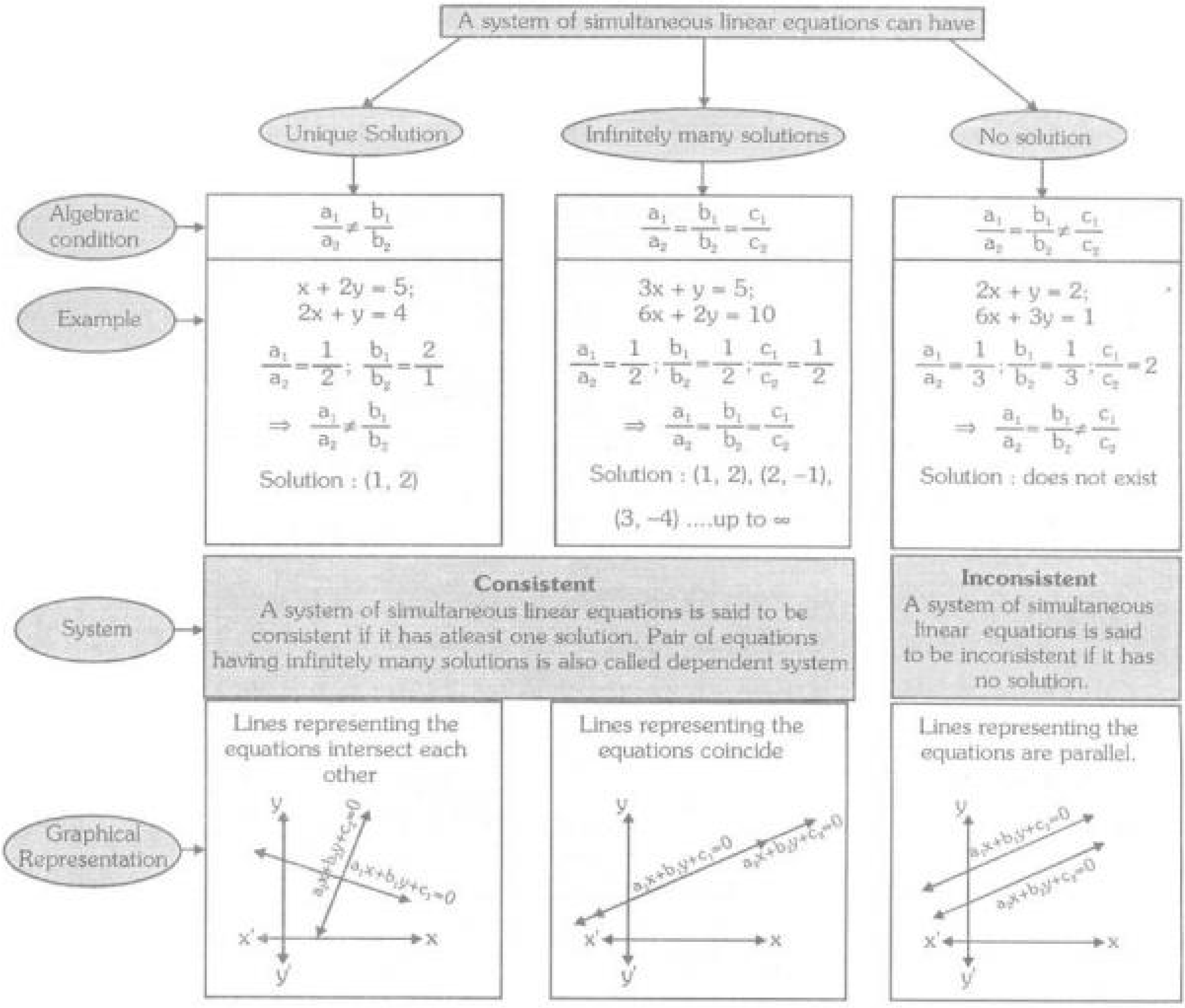
LHS = 2 × 2 + 3 = 7 = RHS

Put x = 2, y = 3 in LHS of equation (ii), we get

LHS = 3 × 2 + 2 × 3 = 12 = RHS

The value x = 2, y = 3 satisfy both equations (i) and (ii).

Hence x = 2, y = 3 is a solution of the given system.



Remark :	An equation involving two variables cannot give value of both the variables. For values of both the variables we required two equations. Similarly for three variables we require three equations and so on, i.e. to find n variables we need n equates.

★ **HOMOGENEOUS SYSTEM OF EQUATIONS**

A system of simultaneous equations is said to be homogenous, if all of the constant terms are zero.

General Form : $a_1x + b_1y = 0$ and $a_2x + b_2y = 0$

Homogeneous equation of the form ax + by = 0 is a line passing through the origin.

Therefore , the system is always consistent.

(i) When $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ the system of equation has only one solution.

(ii) When $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ the system of equation has infinitely many solutions.

Ex.1 On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following points of linear equations are consistent of inconsistent.

(i) 3x + 2y = 5, 2x – 3y = 7 **(ii) 2x – 3y = 8, 4x – 6y = 9**

Sol. **(i)** We have, $3x + 2y = 5 \Rightarrow 3x + 2y - 5 = 0$ and $2x - 3y = 7 \Rightarrow 2x - 3y - 7 = 0$

$$\frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{2}{-3} \text{ and } \frac{c_1}{c_2} = \frac{5}{7} \qquad \therefore \qquad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Therefore, the given pair of linear equations is consistent.

(ii) We have, $2x - 3y = 8 \Rightarrow 2x - 8 = 0$ and $4x - 6y = 9 \Rightarrow 4x - 6y - 9 = 0$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{-8}{-9} = \frac{8}{9}$$

$$\therefore \qquad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, the given pair of linear equations is inconsistent .

Ex.2 **For what value of k, the system of equations $x + 2y = 5, 3x + ky + 15 = 0$ has**

(i) a unique solution

(ii) No solution ?

Sol. We have , $x + 2y = 5 \Rightarrow x + 2y - 5 = 0$ and $3x + kt + 15 = 0$.

(i) The required condition for unique solution is : $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$$\therefore \qquad \frac{1}{3} \neq \frac{2}{k} \Rightarrow k \neq 6$$

Hence, for all real values of k except 6, the given system of equations will have a unique solution.

(ii) The required condition for no solution is : $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\therefore \qquad \frac{1}{3} = \frac{2}{k} \neq \frac{-5}{15} \Rightarrow \frac{1}{3} = \frac{2}{k} \text{ and } \frac{2}{k} \neq \frac{-5}{15}$$

$$\Rightarrow \qquad k = 6 \text{ and } \frac{2}{k} \neq \frac{-1}{3} \Rightarrow k = 6 \text{ and } k \neq -6$$

Hence the given system of equations will have no solution when k = 6.

Ex.3 **Find the value of k for which the system of equations $4x + 5y = 0, kx + 10y = 0$ has infinitely many solution .**

Sol. The given system is of the form $a_1x + b_1y = 0, a_2x + b_2y = 0$

$a_1 = 4, a_2 = k, b_1 = 5, b_2 = 10$

If $\frac{a_1}{a_2} = \frac{b_1}{b_2}$, the system has infinitely many solutions.

$$\frac{4}{k} = \frac{5}{10} \Rightarrow k = 8$$

Ex.4 **Find the value of a and b for which the given system of equations has an infinite number of solutions :**
$2x + 3y = 7$; $(a + b + 1)x + (a + 2b + 2)y = 4(a + b) + 1$

Sol. We have $2x + 3y = 7 \Rightarrow 2x + 3y - 7 = 0$ and $(a + b + 1)x + (a + 2b + 2)y = 4(a + b) + 1$

$$\Rightarrow \qquad (a + b + 1)x + (a + 2b + 2)y - \{4(a + b) + 1\} = 0$$

The required condition for an infinite number of solutions is $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\therefore \qquad \frac{2}{a+b+1} = \frac{3}{a+2b+2} = \frac{-7}{-\{4(a+b)+1\}}$

$\Rightarrow \qquad \frac{2}{a+b+1} = \frac{3}{a+2b+2}$ and $\frac{3}{a+2b+2} = \frac{7}{4(a+b)+1}$

$\Rightarrow \qquad$ 2a + 4b + 4 = 3a + 3b + 3 and 12a + 12b + 3 = 7a + 14b + 14 $\qquad \Rightarrow \qquad$ a – b – 1 = 0 and 5a – 2b = 11

$\Rightarrow \qquad$ a – b = 1 …(i)

and 5a – 2b = 11 …(ii)

Multiplying (i) by 2 we get 2a – 2b = 2 …(iii)

Subtracting (iii) from (ii) we get 3a = $\Rightarrow$ a = $\frac{9}{3}$ = 3

Put a = 3 in (i), we get 3 – b = $\Rightarrow$ b = 2

Hence, the given system of equations will have infinite number of solutions when a = 3 and b = 2.

★ GRAPHICAL METHOD OF SOLVING A SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS

To solve a system of two linear equations graphically,

(i) Draw graph of he first equation.

(ii) On the same pair of axes, draw graph of the second equation.

(iii)(a) If the two lines intersect at a point, read the coordinates of the point of intersection of obtain the solution and verify your answer.

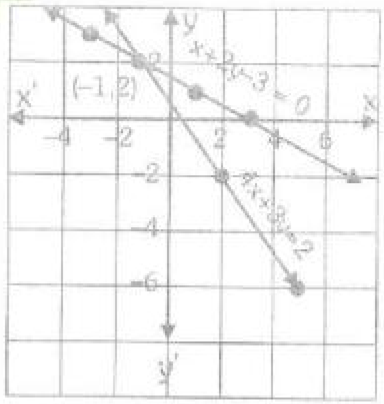
(b) If the two lines are parallel, there is no point of intersection, write the system as inconsistent. Hence , no solution

(c) If the two lines have the same graph, then write the system as consistent with infinite number of solutions

Ex.5 Which of the following pairs of linear equations are consistent / inconsistent ? If consistent, obtain the solution graphically.

(i) x + 2y – 3 = 0, 4x + 3y = 2 (ii) 3x + y = 1, 2y = 2 – 6x

(iii) 2x – y = 2, 2y – 4x = 2



Sol. (i) x + 2y – 3 = 0 $\Rightarrow$ y = $\frac{3-x}{2}$

x	1	2	-3
y	1	0	3

Points are (1, 1), (3, 0), (-3, 3)

4x + 3y $\Rightarrow$ y = $\frac{2-4x}{3}$

x	2	-1	5
y	-2	2	-6

points are (2, -2), (-1, 2), (5, -6)

From the graph, we see that the two lines intersect at a point (-1, 2)

So the solution of the pair of linear equations is x = - 1, y = 2

i.e., the given pair of equations is consistent.

(ii) 3x + y = 1 $\Rightarrow$ y = 1 – 3x

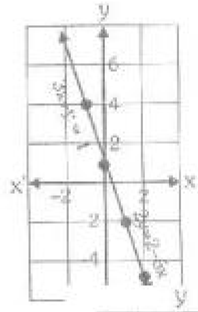
x	0	1	2
y	1	-2	-5

Points are (0, 1), (1, -2), (2, -5)

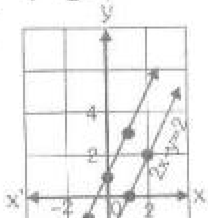
2y = 2 – 6x $\Rightarrow$ y = $\frac{2-6x}{2}$

X	-1	1	-2
y	4	-2	7

Points are (-1, 4), (1, -2), (-2, 7)



The two equations have the same graph. Thus system is consistent with infinite



number of solutions, i.e., the system is dependent.

(iii) $2x - y = 2 \Rightarrow y = 2x - 2$

x	0	1	2
y	-2	0	2

Points are (0, -2), (1, 0), (2, 2)

$2y - 4x = 2 \Rightarrow y = \dfrac{4x+2}{2}$

x	0	1	-1
y	1	3	-1

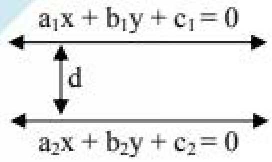
Points are (0, 1), (1, 3), (-1, -1)

The graph of the system consists of two parallel lines. Thus, the system is inconsistent. It has no solution.

COMPETITION WINDOW

DISTANCE BETWEEN TWO PARALLEL LINES

Consider pair of parallel lines



$a_1x + b_1y + c_1 = 0$ …(i) $a_2x + b_2y + c_2 = 0$ …(ii)

$\because$ The lines are parallel .

$\therefore$ $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = k$ (say) $\Rightarrow a_1 = a_2 k$ & $b_1 = b_2 k$

Putting these values in (i), we get : $a_2kx + b_2ky + c_1 = 0$ or $a_2x + b_2y + \dfrac{c_1}{k} = 0$

or $a_2x + b_2y + c_3 = 0$ …(iii) $\left[c_3 = \dfrac{c_1}{k}\right]$

Clearly in equation (ii) and (iii), coefficients of x and y are same but the constant term is different in both the equations. The perpendicular distance (d) between the two lines can be calculated by using the following formula :

$$d = \frac{|c_2 - c_3|}{\sqrt{a_2^2 + b_2^2}}$$

e.g, The distance between the parallel liens 3x – 4y 9 = 0 and 6x – 8y – 15 = 0 can be calculated as follows :

$3x - 4y + 9 = 0$ …(i), $6x - 8y - 15 = 0$ or $3x - 4y - \dfrac{15}{2} = 0$ …(ii)

Required perpendicular distance, $d = \dfrac{\left|9 - \left(-\dfrac{15}{2}\right)\right|}{\sqrt{(3)^2 + (4)^2}} = \dfrac{\left|9 + \dfrac{15}{2}\right|}{\sqrt{25}} = \dfrac{33}{10}$

★ ALGEBRAIC METHOD OF SOLVING A PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Some times, graphical number does not given an accurate answer. While reading the co-ordinate of a point on a graph paper we are likely to make an error. So we require some precise method to obtain accurate result. The algebraic methods are given below :

(i) Method of elimination by substitution.

(ii) Method of elimination by equating the coefficients.

(iii) Method of cross multiplication.

★ ALGEBRAIC SOLUTION BY SUBSTITUTION METHOD

To solve a pair linear equations in two variables x and y by substitution method, we follow the following steps :

Step – I : Write the given equations

$$a_1x + b_1y + c_1 = 0 \quad ..(i)$$

and $$a_2x + b_2y + c_2 = 0 \quad ...(ii)$$

Step –II : Choose one of the two equations and express y in terms of x (or x in terms of y), i.e. express, one variable in terms of the other.

Step –III : Substitute this value of y obtained in step-II, in the other equation to get a linear equation in x.

Step-IV : Solve the linear equation obtained in step-III and get the value of x.

Step-V : Substitute this value of x in the relation obtained in step-II and find the value of y.

Ex.6 Solve for x and y : 4x + 3y = 24, 3y – 2x = 6.

Sol. $4x + 3y = 24 \quad ...(i)$

$3y - 2x = 6 \quad ...(ii)$

From equation (i), we get

$$y = \frac{24 - 4x}{3} \quad ...(iii)$$

Substituting in equation (ii), we get

$$3\left(\frac{24 - 4x}{3}\right) - 2x = 6$$

$\Rightarrow 24 - 4x - 2x = 6 \quad \Rightarrow \quad -6x = -24 + 6$

$\Rightarrow 6x = 18 \quad \Rightarrow \quad x = 3$

Substituting x = 3 in (iii), we get

$$\Rightarrow y = \frac{24 - 12}{3}$$

$$\Rightarrow \frac{12}{3} = 4$$

Hence , x + 3, y = 4

Ex.7 **Solve the following pair of linear equations by the substitution method.**

$\sqrt{2}x + \sqrt{3}y = 0$ **and** $\sqrt{3}x - \sqrt{8}y = 0$

Sol. We have,

$\sqrt{2}x + \sqrt{3}y = 0$ ...(i)

and $\sqrt{3}x - \sqrt{8}y = 0$ ...(ii)

From (i), we get $y = \frac{-\sqrt{2}x}{\sqrt{3}}$ ...(iii)

Substituting $y = \frac{-\sqrt{2}x}{\sqrt{3}}$ in (ii), we get $\sqrt{3}x - \sqrt{8}\left(\frac{-\sqrt{2}x}{\sqrt{3}}\right) = 0$

$\Rightarrow \quad \sqrt{3}x + \frac{4x}{\sqrt{3}} = 0 \Rightarrow 3x + 4y = 0 \Rightarrow 7x = 0 \Rightarrow x = 0$

Substituting x = 0 in (iii), we get $y = \frac{-\sqrt{2} \times 0}{\sqrt{3}} = 0$

Hence, the solution is x = 0 and y = 0.

★ ALGEBRAIC SOLUTION BY ELIMINATION METHOD

To solve a pair of linear equations x and y by elimination method, we follow the following steps :

Step-I : Write the given equation

$a_1x + b_1y + c_1 = 0$ ...(i)

and $a_2x + b_2y + c_2 = 0$ ...(ii)

Step-II : Multiply the given equations by suitable numbers so that the coefficient of one of the variables are numerically equal .

Stpe-III : If the numerically equal coefficients are opposite in sign , then add the new equations otherwise subtract

Step-IV : Solve the linear equations in one variable obtained in step–III and get the value of one variable .

Step-V : Substitute this value of the variable obtained in step-IV in any of the two equations and find the value of the other variable.

Ex.8 **Solve the following pair of linear equations by elimination method : 3x + 4y = 10 and 2x – 2y = 2.**

Sol. We have, 3x + 4y = 10 ...(i)

and 2x – 2y = 2 ...(ii)

Multiplying (ii) by 2, we get 4x – 4y = 4 ...(iii)

Adding (i) and (iii), we get 7x = 14 ⇒ x = 2

Putting x = 2 in equation (ii), we get 2 × 2 – 2y = 2 ⇒ y = 1

Hence, the solution is x = 2 and y = 1.

Ex.9 Solve : ax + by = c, bx + ay = 1 + c

Sol. $ax + by = c$ ...(i)

$bx + ay = 1 + c$ ...(ii)

Adding (i) and (ii), we get

$(a + b)\, x + (a + b)\, y = 2c + 1$

$\Rightarrow \quad x + y = \dfrac{2c+1}{a+b}$ ...(iii)

Subtracting (ii) and (i), we get

$(a - b)\, x - (a - b)\, y = -1$

$\Rightarrow \quad x - y = \dfrac{-1}{a-b}$ ...(iv)

Adding (iii) and (iv), we get

$$2x = \frac{2x+1}{a+b0} - \frac{-1}{a-b} = \frac{2ac - 2bc + a - b - a - b}{a^2 - b^2}$$

$$\Rightarrow \quad 2x = \frac{2ac - 2bc - 2b}{a^2 - b^2}$$

$$\Rightarrow \quad x = \frac{ac - bc - b}{a^2 - b^2}$$

Subtracting (iv) from (iii) we get

$$2y = \frac{2c+1}{a+b} + \frac{-1}{a-b} = \frac{2ac - 2bc + a - b + a + b}{a^2 - b^2}$$

$$\Rightarrow \quad 2y = \frac{2ac - 2bc + 2a}{a^2 - b^2}$$

$$\Rightarrow \quad y = \frac{ac - bc + a}{a^2 - b^2}$$

Hence , $x = \dfrac{ac - bc - b}{a^2 - b^2}, y = \dfrac{ac - bc + a}{a^2 - b^2}$

★ ALGEBRAIC SOLUTIONS BY CROSS-MULTIPLICATION METHOD

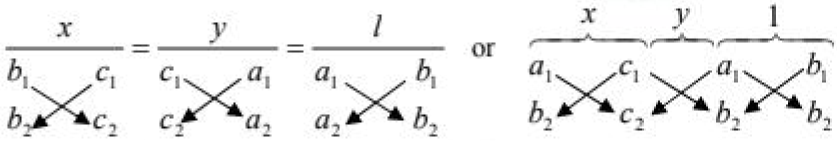
Consider the system of linear equations

$a_1x + b_1y + c_1 = 0$ ...(i)

$a_2x + b_2y + c_2 = 0$ ...(ii)

To solve it by cross multiplication method , we follow the following steps :

Step-I : Write the coefficients as follows :



The arrows between the two numbers indicate that they are to be multiplied. The products with upward arrows are to be subtracted from the products with downward arrows.

To apply above formula, all the terms must be in left to the equal sign in he system of equations –

New, by above mentioned rule, equation (i) reduces to

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$\Rightarrow \quad x = \dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

Case-I : If $a_1b_2 - a_2b_1 \neq 0 \Rightarrow$ x and y have some finite value, with unique solution for the system of equations.

Case-II : If $a_1b_2 - a_2b_1 = 0 \Rightarrow \dfrac{a_1}{a_2} = \dfrac{b_1}{b_2}$

Here two cases arise :

(a) If $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2} = \lambda(\lambda \neq 0)$

Then $a_1 = a_2\lambda$, $b_1 = b_2\lambda$, $c_1 = c_2\lambda$

Put these values in equation $a_1x + b_1y + c_1 = 0$ …(i)

$\Rightarrow \quad a_2\lambda x + b_2\lambda + c_2\lambda = 0$

$\Rightarrow \quad \lambda(a_2x + b_2y + c_2) = 0$ but $\lambda \neq 0$

$\Rightarrow \quad a_2x + b_2y + c_2 = 0$ …(ii)

So (i) and (ii) are dependent, so there are infinite number of solutions.

(b) If $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2} \Rightarrow a_1b_2 - b_1a_2 = 0$

But $x = \dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$\Rightarrow \quad x = \dfrac{\text{Finite value}}{0}$ = does not exist

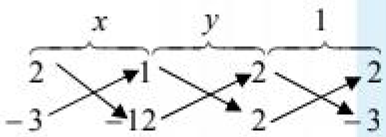
and $\quad y = \dfrac{\text{Finite value}}{0}$ = does not exist

So system of equations is inconsistent.

Ex10 Solve by cross-multiplication method : x + 2y + 1 = 0 and 2x – 3y – 12 = 0

Sol. We have, x + 2y + 1 = 0 and 2x – 3y – 12 = 0

By cross-multiplication method , we have



$\therefore \quad \dfrac{x}{2\times(-12)-(-3)\times 1} = \dfrac{y}{1\times 2-(-12)\times 1} = \dfrac{1}{1\times(-3)-2\times 2}$

$\Rightarrow \quad \dfrac{x}{-24+3} = \dfrac{y}{2+12} = \dfrac{1}{-3-4} \Rightarrow \dfrac{x}{-21} = \dfrac{y}{14} = \dfrac{1}{-7}$

$\Rightarrow \quad x = \dfrac{-21}{-7} = 3$ and $y = \dfrac{14}{-7} = -2$

Hence the solution is x = 3 and y = −2 .

Ex.11 Solve by cross-multiplicaiton method : (a – b) x + (a + b) y = 2 ($a^2 - b^2$), (a + b) x – (a – b) y = 4ab.

Sol. Writing the equations in the standard form, we get .

$(a - b)x + (a + b)y - (a^2 - b^2) = 0$

$(a + b)x - (a - b)y - 4ab = 0$

Applying the cross-multiplication method, we get

$$\frac{x}{(a+b)-2(a^2-b^2)} = \frac{y}{-2(a^2-b^2)(a-b)} = \frac{1}{(a-b)(a+b)}$$

Simplification of the expression under x :

$-4ab(a + b) - 2(a - b)(a^2 - b^2)$

$= -2(a + b)[2ab + (a - b)^2]$

$= -2(a + b)(2ab + a^2 + b^2 - 2ab)$

$= -2(a + b)(a^2 + b^2)$

Simplification of the expression under y :

$-2(a^2 - b^2)(a + b) + 4ab(a - b)$

$= -2(a - b)[(a + b)(a + b) - 2ab]$

$= -2(a - b)(a^2 + b^2 + 2ab - 2ab)$

$= -2(a - b)(a^2 + b^2)$

Simplification of the expression under 1 :

$-(a - b)^2 - (a + b)^2$

$= -(a^2 + b^2 - 2ab) - (a^2 + b^2 + 2ab)$

$= -2(a^2 + b^2)$

Hence, $\dfrac{x}{-2(a+b)(a^2-b^2)} = \dfrac{y}{-2(a-b)(a^2-b^2)} = \dfrac{1}{-2(a^2+b^2)}$

$\Rightarrow \quad \dfrac{x}{a+b} = \dfrac{y}{a-b} = \dfrac{1}{1}$

$\Rightarrow \quad x = (a + b)$ and $y = (a - b)$

★ **EQUATIONS OF THE FORM ax + by = c AND bx + ay = d, WHERE a ≠ b.**

To solve the equations of the form. :

$ax + by = c$ ...(i)

and $bx + ay = d$ ...(ii)

where $a \neq b$, we follow the following steps :

Step-I : Add (i) and (ii) and obtain $(a+b)x + (b+a)y = c + d$, i.e., $x + y = \dfrac{c+d}{a+b}$ ...(iii)

Step-II : Subtract (ii) from (i) and obtain $(a-b)x - (a-b)y = c - d$, i.e., $x - y = \dfrac{c-d}{a-b}$ ...(iv)

Step-III: Solve (iii) and (iv) to get x and y.

Ex.12 Solve for x and y : 47x + 31y = 63, 31x + 47y = 15.

Sol. We have,

47x + 31y = 63…(i) and 31x + 47y = 15…(ii)

Adding (i) and (ii), we get : 78x + 78y = 78 $\Rightarrow$ x + y = 1 …(iii)

Subtracting (ii) from (i), we get : 16x – 16y = 48 $\Rightarrow$ x – y = 3 …(iv)

Now , adding (iii) and (iv), we get : 2x = 4 $\Rightarrow$ x = 2

Putting x = 2 in (ii), we get : 2 + y = 1 $\Rightarrow$ y = - 1

Hence, the solution is x + 2 and y = - 1

★ EQUATIONS REDUCIBLE TO LINER EQUATIONS IN TWO VARIABLES

Equations which contain the variables, only in the denominators, are called reciprocal equations. These equations can be of the following types and can be solved by the under mentioned method :

Type-I : $\frac{a}{u}+\frac{b}{v}=c$ and $\frac{a'}{u}+\frac{b'}{v}=c' \forall a,b,c,a',b',c' \in R$

Put $\frac{1}{u}=x$ and $\frac{1}{y}=v$ and find the value of x and y by any method described earlier.

Then $u=\frac{1}{x}$ and $v=\frac{1}{y}$

Type-II : au +bv = cuv and a'u + b'v = c'uv $\forall$ a,b,c,a',b',c' $\in$ R

Divide both equations by uv and equations can be converted in the form explained in (i).

Type-III : $\frac{a}{lx+my}+\frac{b}{cx+dy}=k, \frac{a'}{lk+my}+\frac{b'}{cx+dy}=k' \forall a,b,k,a',b',k' \in R$

Put $\frac{1}{lx+my}=u$ and $\frac{1}{cx+dy}=v$

Then equations are au + vc = k and a'u + b'v = k

Find the values of u and v and put in Ix + my = $\frac{1}{u}$ and cx + dy = $\frac{1}{v}$

Again solve for x and y, by any method explained earlier.

Ex.13 Solve for x and y : $\frac{3a}{x}-\frac{2b}{y}+5=0$ **and** $\frac{a}{x}+\frac{3b}{y}-2=0 (x \neq 0, y \neq 0)$

Sol. We have, $\frac{3a}{x}-\frac{2b}{y}+5=0$ and $\frac{a}{x}+\frac{3b}{y}-2=0$

Let $\frac{1}{x}=u$ and $\frac{1}{y}=v$. Then, the given equations can be written as

3au – 2bv = - 5…(i) and au +3bv = …(ii)

Multiplying (i) by 3 and (ii) by 2, we get

9au – 6bv = - 17 …(iii) and 2au + 6bv = 4 …(iv)

Adding (iii) and (iv), we get 11au = - 11 $\Rightarrow u=\frac{-1}{a}$

Put $u=\frac{-1}{a}$ in equation (ii), we get a $\left(\frac{-1}{a}\right)+3bv=2 \Rightarrow 3bv=3 \Rightarrow v=\frac{1}{b}$ But $\frac{1}{x}=u$ and $\frac{1}{y}=v$

Therefore, $\frac{1}{a}=\frac{-1}{a} \Rightarrow x=-a$ and $\frac{1}{y}=\frac{1}{b} \Rightarrow y=b$ $[\because u=\frac{-1}{a}, v=\frac{1}{b}]$

Hence the solution is x = - a and y = b.

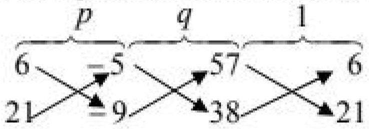
Ex.14 Solve $\frac{57}{x+y}+\frac{6}{x-y}=5$ **and** $\frac{38}{x+y}+\frac{21}{x-y}=9.$

Sol. We have, $\frac{57}{x+y}+\frac{6}{x-y}=5 \Rightarrow \frac{57}{x+y}+\frac{6}{x-y}-5=0$ and $\frac{38}{x+y}+\frac{21}{x-y}=9 \Rightarrow \frac{38}{x+y}+\frac{21}{x-y}-9=0$

Let $\frac{1}{x+y}=p$ and $\frac{1}{x-y}=q$. Then. the given equations can be written as

57p + 6q – 5 = 0 and 38p + 21q – 9 = 0

By cross-multiplication method, we have



$\therefore \quad \frac{p}{6\times(-9)-21\times(-5)}=\frac{q}{(-5)\times 38-(-9)\times 57}=\frac{1}{57\times 21-38\times 6}$

$\Rightarrow \quad \frac{p}{-54+105}+\frac{q}{-190+513}=\frac{1}{1197-228} \quad \Rightarrow \quad \frac{p}{51}=\frac{q}{323}=\frac{1}{969} \Rightarrow p=\frac{51}{969}=\frac{1}{18}$ and $q=\frac{323}{969}=\frac{1}{3}$

But $\quad \frac{1}{x+y}=p$ and $\frac{1}{x-y}=q$. therefore

$\frac{1}{x+y}=\frac{1}{19} \Rightarrow x+y=19$ …(i)

and $\quad \frac{1}{x-y}=\frac{1}{3} \Rightarrow x-y=3$ …(ii)

adding (i) and (ii) , we get $\quad 2x = 22 \Rightarrow x = 11$

Put x = 11 in (i),we get $\quad 11 + y = 19 \Rightarrow y = 8$

Hence, the solution is x = 11 and y = 8.

Ex.15 Solve for x and y : $\frac{7x-2y}{xy}=5, \frac{8x+7y}{xy}=15.$

Sol. $\frac{7x-2y}{xy}=5, \Rightarrow \frac{7}{y}-\frac{2}{x}=5$ …(i)

$\frac{8x+7y}{xy}=15 \Rightarrow \frac{8}{y}+\frac{7}{x}=15$ …(ii)

Putting $\frac{1}{y}=u$ and $\frac{1}{x}=v$, we get

7u – 2v = 5 …(iii)

8u 7v = 15 …(iv)

Multiplying (iii) by 7 and (iv) by 2 and adding we get

49u – 14v = 35

and 16u + 14v = 30

65u = 65 $\Rightarrow$ u = 1 $\Rightarrow \frac{1}{y}$ = 1 or y = 1

Substituting u = 1 in (iii) we get : 7 – 2v = 5 $\Rightarrow$ v = 1 $\Rightarrow \frac{1}{x}$ = 1 or x = 1

Hence, x = 1, y = 1.

★ APPLICATIONS OF LINEAR EQUATIONS IN TWO VARIABLES

In This section, we will study about some applications of simultaneous linear equations in solving variety of word problems related to our day-to-day life situations. The following examples are self-explanatory and will give some insight to the solution to such problems.

Type-I :Based on Articles And Their Costs / Quantities

Ex.16 7 audio cassettes and 3 video cassettes cost Rs. 1110, which 5 audio cassettes and 4 video cassettes cost Rs. 1350. Find the cost of an audio cassette and a video cassette.

Sol. Let the cost of an audio cassette and a video cassette be Rs. x and Rs. y respectively.

The cost of 7 audio cassettes and 3 video cassettes = Rs. 1110

$\Rightarrow \quad 7x + 3y = 1110$ ...(i)

The cost of 5 audio cassettes and 4 video cassettes = Rs. 1350

$\Rightarrow \quad 5x + 4y = 1350$ ...(ii)

Multiplying (i) by 4 and (ii) by 3, we get

$28x + 12y = 4440$ ...(iii)

$15x + 12y = 4050$ ..(iv)

Subtracting (iv) from (iii), we get $13x = 390 \Rightarrow x = 30$

Putting x = 30 in (i), we get $7 \times 30 + 3y = 1110 \Rightarrow 210 + 3y = 1110$

$\Rightarrow \quad 3y = 900 \Rightarrow y = 300$

Hence, the cost of an audio cassette is Rs. 30 and that of a video cassette is Rs. 300.

Type-II: Based on numbers

Ex.17 The sum of the digits of a two-digit number is 12. The number obtained by interchanging its digits exceeds the given number by 18. Find the number .

Sol. Let the digit at ten's place by x and that at unit's place be y. Then,

$x + y = 12$ ...(i)

And , the two digits number = 10x + y

Now, according to the equation,

$(10y + x) = (10x + y) + 18 \Rightarrow 9y - 9x = 18 \Rightarrow y - x = 2$ ...(ii)

Adding (i) and (ii), we get $2y = 14 \Rightarrow y = 7$

Put y = 7 in (i), we get $x + y = 12 \Rightarrow x = 5$

Hence, the enquired number is $(10 \times 5 + 7)$, i.e., 57.

Type-III: Based on Fractions

Ex.18 If we add 1 to ht numerator and subtract 1 from the denominator, a fraction reduces to 1, It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction ?

Sol. Let the required fraction be $\frac{x}{y}$. Then

$$\frac{x+1}{x-y} = 1 \Rightarrow x+1 = y-1 \Rightarrow x-y = -2 \quad \text{...(i)}$$

and $$\frac{x}{x+y} = \frac{1}{2} \Rightarrow 2x = y+1 \Rightarrow 2x - y = 1 \quad \text{...(ii)}$$

Subtracting (i) from (ii), we get x = 3

Put x = 3 in (i), we get $3 - y = -2 \Rightarrow y = 5$

Hence, the fraction is $\frac{3}{5}$

Type-IV : Based on Ages

Ex.19 Two years ago, a father was five times as old as his son. Two years later, his age will be 8 more than three times the age of the son. Find the present ages of father and son.

Sol. Let the present ages of the father and the son be x years and y years respectively .

Two years ago, Father's age = (x – 2) years and son's age = (y – 2) years

$\therefore$ $(x-2) = 5(y-2) \Rightarrow x - 5y = -8$ ...(i)

Two years later, father's age = (x + 2) years and son's age = (y + 2) years

$\therefore$ $(x+2) = 3(y+2) + 8 \Rightarrow x + 2 = 3y + 6 + 8 \Rightarrow x - 3y = 12$ ...(ii)

Subtracting (i) from (ii), we get $2y = 20 \Rightarrow y = 10$

Putting y = 10 in (ii), we get $x - 3 \times 10 = 12 \Rightarrow x = 42$

Hence , the present ages of father and son are 42 years and 1 years respectively.

Type-III: Based on Geometrical Applications

Ex.20 The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

Sol. Let the larger angle be x^0 and the smaller angle by y^0. Then,

$x + y = 180$ ...(i)

and $x = y + 18 \Rightarrow x - y = 18$ ...(ii)

Adding (i) and (ii) , we get $2x = 198 \Rightarrow x = 99$

Putting x = 99 in (i), we get $99 + y = 18 \Rightarrow y = 81$

Hence the required angles are 99^0 and 81^0.

Type-III: Based on Time, Distance and Speed.

Formulae to be used :

1. **(a) Speed = $\frac{\text{Distance}}{\text{Time}}$**

   (b) Distance = Speed × Time

   (c) Time = $\frac{\text{Distance}}{\text{Speed}}$

2. Let speed of a boat in still water = u km/h

   and speed of the current = v km/h. Then ,

   (a) Speed of a boat downstream = (u + v)km/h

   (b) Speed of a boat upstream = (u – v)km/h.

Ex.21 A man travels 370 km partly by train and partly by car. If he covers 250 km by train and the rest by car it takes him 4 hours. But, if he travels 130 km by train and rest by car, be takes 18 minutes, longer. Find the speed of the train and that of the car.

Sol. Let the speeds of the train and that of the car be x km/h and y km/h respectively .

$$\frac{250}{x} + \frac{120}{y} = 4 \qquad \left(\therefore \text{Time} = \frac{\text{Distance Speed}}{\text{Distance}}\right) \quad ...(i)$$

And if he covers 130 km by train and 240 km by car it takes 4 hours and 18 minutes.
Therefore ,

$$\frac{130}{x} + \frac{240}{y} = 4 + \frac{18}{60} \qquad \left(\therefore 18 \text{ minutes} = \frac{18}{60} \text{ hours}\right)$$

$$\frac{130}{x} + \frac{240}{y} = \frac{43}{10} \qquad \ldots\text{(ii)}$$

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$ Then, the equations (i) and (ii), can be written as

$$250u + 120v = 4 \qquad \ldots\text{(iii)}$$

and $$130u + 140v = \frac{43}{10} \qquad \ldots\text{(iv)}$$

Multiplying (iii) by 2, we get $500u + 240v = 8$ …(v)

Subtracting (iv) from (v), we get $370u = 8 - \frac{43}{10} \Rightarrow 370u = \frac{37}{10} \Rightarrow u = \frac{1}{100}$

Putting $u = \frac{1}{100}$ in (iii), we get $250 \times \frac{1}{100} + 120v = 4 \Rightarrow \frac{5}{2} + 120v = 4$

$\Rightarrow \quad 120v = 4 - \frac{5}{2} \Rightarrow v = \frac{3}{120 \times 2} = \frac{1}{80}$

but $\quad u = \frac{1}{x}$ and $v = \frac{1}{y}.$

Therefore, $\frac{1}{x} = \frac{1}{100} \Rightarrow x = 100$ and $\frac{1}{y} = \frac{1}{80} \Rightarrow y = 80$

Hence the speeds of the train and that of the car are 100 km/h and 80 km/h respectively .

Type-VII : Miscellaneous

Ex.22 8 man and 12 boys can finish is a piece of work in 10 days while 6 man and 8 boys can finish it in 14 days. Find the time taken by one men alone and that by one boy alone to finish the work .

Sol. Let one man alone can finish the work in x days and one alone can finish the work in y days. Then. the work done by one man in one day $= \frac{1}{x}$ and the work done by one boy in on e day $= \frac{1}{y}$

According to the question, $10\left(\frac{8}{x} + \frac{12}{y}\right) = 1 \Rightarrow \frac{1}{x} + \frac{3}{y} = \frac{1}{40}$ …(i)

Also, $14\left(\frac{6}{x} + \frac{8}{y}\right) = 1 \Rightarrow \frac{3}{x} + \frac{4}{y} = \frac{1}{28}$ …(ii)

Multiplying (i) by 4 and (ii) by 3, we get : $\frac{8}{x} + \frac{12}{y} = \frac{1}{10}$ …(iii) and $\frac{9}{x} + \frac{12}{y} = \frac{3}{28}$ …(iv)

Subtracting (iii) from (iv), we get $\frac{1}{x} = \frac{3}{28} - \frac{1}{10} \Rightarrow \frac{1}{x} = \frac{2}{280} \Rightarrow x = 140$

Putting x = 140 (ii), we get $\frac{3}{140} + \frac{4}{y} = \frac{1}{28} \Rightarrow \frac{4}{y} = \frac{1}{28} - \frac{3}{140}$

$\Rightarrow \quad \frac{4}{y} = \frac{5-3}{140} \Rightarrow \frac{4}{y} = \frac{2}{140} \Rightarrow y = 180$

Hence, one man alone can finish the work in 140 days and one by alone can finish the work in 280 days.

★ **SYNOPSIS**

(i) Two linear equations in the same two variables are called a pair of linear equations in two variables, or briefly, a linear pair. The most general form of a linear pair is :

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

where a1, a_2, b_1, b_2, c_1, c_2 are real numbers , such that $a_1^2 + b_1^2 \neq a_2^2 + b_2^2 \neq 0$.

(ii) A pair of linear equations in two variables can be represented, and solved , by the

(a) Graphical method (b) Algebraic method

(iii) **Graphical Method :** The graph of the pair of linear equations in two variables is represented by a pair of lines.

(a) If the pair intersects at a point, then that point is the unique common solution of the two equations. In this case, the pair is consistent.

(b) If the pair coincide, then it has infinitely many solutions – each point on the line being a solution. In this case, the pair is consistent (dependent).

(c) If two lines are parallel, then the pair has no solution, and is called inconsistent.

(iv) **Algebraic Method :** We have discussed the following methods for finding the solutions (s) of a pair of linear equations

(a) Substitution method. (b) Elimination method. (c) Cross-multiplication method.

(v) If $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, then the following situations can arise :

(a) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$: **In this case the pair of linear equations is consistent.**

(b) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$: In this case the **pair of linear equations is inconsistent.**

(c) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$: In this case the **pair of linear equations is dependent and consistent.**

(iv) There are several situations which can be mathematically represented by two equations that are not linear to start with. But we alter them so that they are reduced to a linear pair.

SOLVED NCERT EXERCISE

EXERCISE : 3 . 1

1. After tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three sears from now, I shall be three times as old as you will be". (Isn't this interesting ?) Represent this situation algebraically and graphically .

Sol. Let the present age of Aftab's daughter = x years.

and the present age of Aftab = y years (y > b)

According to the given conditions

Seven years ago, $(y - s) = 7 \times (x - 7)$ i.e., $y - 7 = 7x - 49$ i.e., $7x - y - 42 = 0$ ...(i)

Three years later, $(y + 3) = 3 \times (x + 3)$ i.e., $y + 3 = 3x + 9$ i.e., $3x - y + 6 = 0$ ...(ii)

Thus, the algebraic relations are $7x - y - 42 = 0$, $3x - y + 6 = 0$.

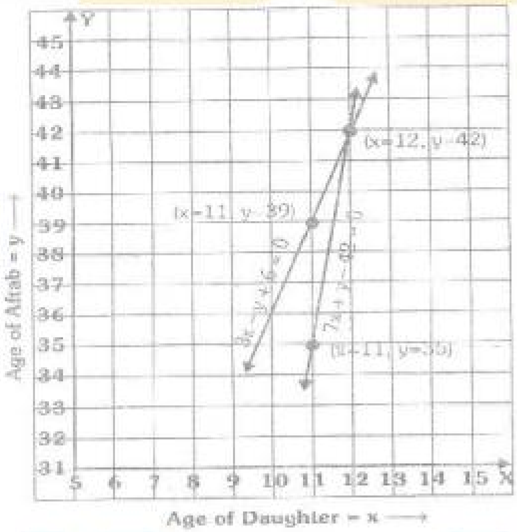
Now, we represent the problem graphically as below :

$7x - y - 42 = 0$ ...(i)

Age of Aftab's daughter = x	11	12
Age of Aftab = y = 7x – 42	35	42

$3x - y + 6 = 0$ ...(ii)

Age of Aftab's daughter = x	11	12
Age of Aftab = y = 3x + 6	39	42



From the graph, we find that x = 12 and y = 42

Thus, the present age of Aftab's daughter = 12 years

and the present age of Aftab = 42 years

2. The coach of a cricket team buys 3 bats and 6 balls for Rs. 3900. Later, she buys another bat and 3 more balls of the same kind for Rs. 1300. Represent this situation algebraically and geometrically .

Sol. [Try Yourself]

3. The cost of 2 kg of apples and 1 kg of grapes on a day found to be Rs. 160. After a month, the cost of 4 kg of apples and 2 g of grapes is Rs. 300. Represent the situation algebraically and geometrically .

Sol. [Try Yourself]

EXERCISE : 3 . 2

1. Form the pair of linear equations in the following problems, and find their solutions graphically.

(i) 10 students of class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys , find the number of boys and girls who took part in the quiz.

(ii) 5 pencils and 7 pens together cost Rs. 50, whereas 7 pencils and 5 pens together cost Rs. 46. Find the cost of one pencil and that of one pen.

Sol. (i) Let the number of boys be x and the number of girls be y.

According to the given conditions

$x + y = 10$ and $y = x + 4$

We get the required pair of linear equations as

$$x + y - 10 = 0, x - y + 4 = 0$$

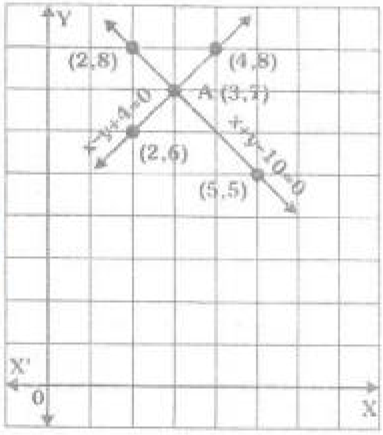
Graphical Solution

$x + y - 10 = 0$ ...(i)

x	2	5
y = 10 – x	8	5

$x - y + 4 = 0$ ...(ii)

x	2	5
y = x + 4	8	5



From the graph, we have : x = 3, y = 7 common solution of the two linear equations.

Hence, the number of boys = 3 and the number of girls = 7.

(ii) **[Try Yourself]**

2. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at point, are parallel or coincident .

(i) $5x - 4y + 8 = 0$; $7x + 6y - 9 = 0$ **(ii) $9x + 3y + 12 = 0$; $18x + 6y + 24 = 0$**

(iii) $6x - 3y + 10 = 0$; $2x - y + 9 = 0$

Sol. (i) $5x - 4y + 8 = 0$ ...(i)

$7x + 6y - 9 = 0$ ...(ii)

$$\frac{a_1}{a_2} = \frac{5}{7}, \frac{b_1}{b_2} = \frac{-4}{6} = -\frac{2}{3} \Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$\Rightarrow$ Lines represented by (i) and (ii)

Intersect at a point

[Rest Try Yourself]

3. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pairs of linear equations are consistent, or inconsistent.

(i) $3x + 2y = 5$; $2x - 3y = 7$ **(ii) $2x - 3y = 8$; $4x - 6y = 9$** **(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$; $9x - 10y = 14$**

(iv) $5x - 3y = 11$; $-10x + 6y = -22$ **(v) $\frac{4}{3}x + 2y = 8$; $2x + 3y = 12$**

Sol. (i) $3x + 2y - 5 = 0$ ...(i) $2x - 3y - 7 = 0$ ...(ii)

$$\frac{a_1}{a_2}=\frac{3}{2};\ \frac{b_1}{b_2}=-\frac{2}{3} \qquad \Rightarrow \frac{a1}{a_2}\neq\frac{b_1}{b_2}\Rightarrow$$ The equations have a unique solution.

Hence, consistent.

[Rest Try Yourself]

4. Which of the following pairs of linear equations are consistent / inconsistent ? If consistent, obtain the solution graphically :

(i) x + y = 5, 2x + 2y = 10 **(ii) x – y = 8, 3x – 3y = 16**

(iii) 2x + y – 6 = 0, 4x – 2y – 4 = 0 **(iv) 2x – 2y – 2 = 0, 4x – 4y – 5 = 0**

Sol. (i) $x + y = 5$ …(i)

$2x + 2y = 10$ …(ii)

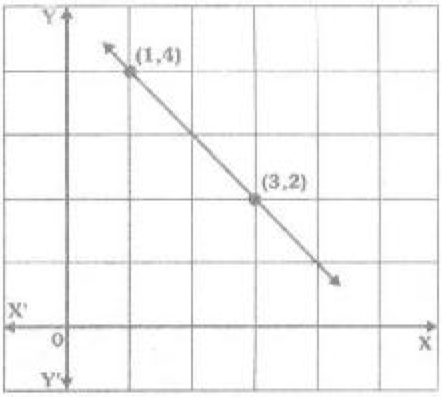
$$\frac{a_1}{a_2}=\frac{1}{2},=\frac{1}{2},\frac{c_1}{c_2}=\frac{-5}{-10}=\frac{1}{2} \quad \text{i.e.,} \quad \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

Hence, the pair of linear equations is consistent.

(i) and (ii) are same equations and hence the graph

Is coincident straight line.

x	1	3
y = 5 – x	4	2



[Rest Try Yourself]

5. Half the perimeter of a rectangular garden, whose length is 4 m more that its width, is 36 m. Find the dimensions of the garden

Sol. [Try Yourself]

6. Given the linear equation 2x + 3y – 8 = 0, write another linear equation in two variables such that the geometrical representation of the pair so formed is :

(i) Intersecting **(ii) Parallel** **(iii) Coincident lines**

Sol. (i) $2x + 3y – 8 = 0$ (Given equation)

$3x + 2y + 4 = 0$ (New equation)

Here, $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$ Hence, the graph of the two equations will be two intersecting lines.

[Rest Try Yourself]

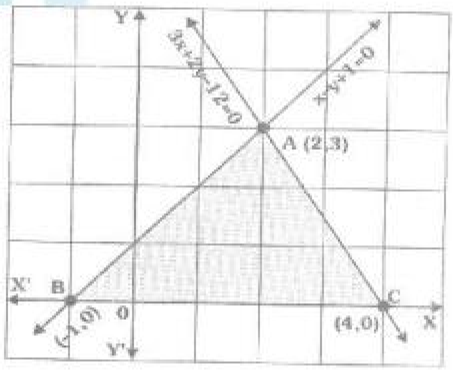
7. Draw the graphs of the equations x – y + 1 = 0 and 3x + 2y – 12 = 0. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

Sol. $x – y + 1 = 0$ …(i)

x	1	3
y = x + 1	2	4

$3x+2y – 12 = 0$ …(ii)

x	0	4
$y=\dfrac{12-3x}{2}$	6	0



The vertices of the triangle are
A (2, 3), B (-1, 0) and C (4, 0)

EXERCISE : 3.3

1. Solve the following pair of linear equations by the substitution method.

(i) $x + y = 14, x - y = 4$

(ii) $s - t = 3, \frac{s}{3} + \frac{t}{2} = 3$

(iii) $3x - y = 3, 9x - 3y = 9$

(iv) $0.2x + 0.3y = 1.3, 0.4 + 0.5y = 2.3$

(v) $\sqrt{2}x + \sqrt{3}y = 0, \sqrt{3}x - \sqrt{8}y = 0$

(vi) $\frac{3x}{2} - \frac{5y}{3} = -2, \frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

Sol. (i) $x + y = 14$ ...(i)

$x - y = 4$ ...(ii)

From (ii) $y = x - 4$ ...(iii)

Substituting y from (iii) in (i), we get

$x + x - 4 = 14 \Rightarrow 2x = 18 \Rightarrow x = 9$

Substituting $x = 9$ in (iii), we get

$y = 9 - 4 = 5$,

i.e., $y = 5$

$x = 9, y = 5$

(ii) $s - t = 3$ ...(i)

$\frac{s}{3} + \frac{t}{2} = 6$ ...(ii)

From (i) $s = t + 3$ ...(iii)

Substituting s from (iii) in (ii), we get

$\frac{t+3}{3} + \frac{t}{2} = 6 \Rightarrow 2(t+3) + 3t = 36 \Rightarrow 5t + 6 = 36 \Rightarrow t = 6$

From (iii), $s = 6 + 3 = 9$

Hence, $s = 9$, $t = 6$

[Rest Try Yourself]

2. Solve 2x + 3y = 11 and 2x – 4y = - 24 and hence find the value of 'm' for which y = mx = 3

Sol. **[Try Yourself]**

3. Form the pair of linear equations for the following problems and find their solution by substitution method.

(i) The difference between two number is 26 and one number is three times the other. Find them.

(ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

(iii) The coach of a cricket team buys 7 bats and 6 balls for Rs. 3800. Later, she buys 3 bats and 5 balls for Rs. 1750. Find the cost of each bat and each ball.

(iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is Rs. 105 and for a journey of 15 km, the charge paid is Rs. 155. what are the fixed charges and the charge per kilometer ? How much does a person have to pay for travelling a distance of 25 km ?

(v) **A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator is becomes $\frac{5}{6}$. Find the fraction.**

(vi) **Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their the present ages ?**

Hints (i) Let the two numbers be x and y (x > y). Then, x – y = 26 and x = 3y.

(ii) Let the supplementary angles by x and y (x > y) Then, x + y = 180 and x – y = 18.

(iii) Try Yourself

(iv) Let fixed charge be Rs x and charge per km be Rs y. Then, x + 10y = 105 and x + 15y = 155.

(v) Let $\frac{x}{y}$ be the fraction where x and y are positive integers. $\frac{x+2}{y+2}=\frac{9}{11}$ and $\frac{x+3}{y+3}=\frac{5}{6}$

(vi) Let x (in years) be the present age of Jacob's son and y (in years) be the present age of Jacob. Then, (x + 5) = 3 (x + 5) and (y – 5) = 7 (x – 5)

EXERCISE : 3 . 4

1. Solve the following pair of equations by the elimination method and the substitution method.

(i) $x + y = 5$ and $2x - 3y = 4$ **(ii) $3x + 4y = 10$ and $2x - 2y = 2$**

(iii) $3x - 5y - 4 = 0$ and $9x = 2y + 7$ **(iv) $\frac{x}{2}+\frac{2y}{3}=-1$ and $x-\frac{y}{3}=3$**

Sol. (i) Solution By Elimination Method:

$x + y = 5$ ...(i)

$2x - 3y = 4$ ...(ii)

Multiplying (i) by 3 and (ii) by 1 and adding

we get $3(x + y) + 1(2x - 3y) = 3 \times 5 + 1 \times 4$

$\Rightarrow 3x + 3y + 2x - 3y = 19$

$\Rightarrow 5x = 19 \Rightarrow x = \frac{19}{5}$

From (i), substitution $x = \frac{19}{5}$, we get

$\frac{19}{5} + y = 5 \Rightarrow y = 5 - \frac{19}{5} \Rightarrow y = \frac{6}{5}$

Hence , $x = \frac{19}{5}$, $y = \frac{6}{5}$

[Rest Try Yourself]

(i) Solution By Substitution Method:

$x + y = 5$ ...(i)

$2x - 3y = 4$ ...(ii)

From (i), $y = 5 - x$ ...(iii)

Substituting y from (iii) in (ii),

$2x - 3(5 - x) \Rightarrow 2x - 15 + 3x = 4$

$\Rightarrow 5x = 19 \Rightarrow x = \frac{19}{5}$

Then from (iii), $y = 5 - \frac{19}{5} \Rightarrow y = \frac{6}{5}$

Hence , $x = \frac{19}{5}$, $y = \frac{6}{5}$

2. Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method.

(i) **If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction ?**

(ii) **Five years ago Nuri was thrice as old as Sonu. Ten years late, Nuri will be twice as old as Sonu. How old are Nuri and Sonu ?**

(iii) **The sum of the digits of a tow-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.**

(iv) **Meena went to a bank to withdraw Rs. 2000. She asked the cashier to give her Rs. 50 and Rs. 100 notes only. Meeena got 25 notes in all. Find how many notes of Rs. 50 and Rs. 100 she received.**

(v) **A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs. 27 for a book kept for seven days, while Susy paid Rs. 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.**

Hints (i) Let the fraction be $\frac{x}{y}$. Then $\frac{x+1}{y-1}=1$; $\frac{x}{y+1}=\frac{1}{2}$

(ii) Try yourself

(iii) Let x be the digit at unit place and y be the digit at tens place of the number. So, number = $x + 10y$. Then $x + y = 9$ and $9[x + 10y] = 2[y + 10x]$.

(iv) Let x and y be the number of Rs. 50 and Rs. 100 notes respectively. Then, $x + y = 25$ and $50x + 100y = 2000$

(v) Try yourself

EXERCISE : 3 . 5

1. Which of the following pairs of linear equations has unique solution, no solution, or infinitely many solutions. In case there is a unique solution, find it by using cross multiplication method.

(i) x – 3y – 3 = 0   3x – 9y – 2 = 0
(ii) 2x + y = 0   3x + 2y = 8
(iii) 3x – 5y = 20   6x – 10y = 40
(iv) x – 3y – 7 = 0   3x – 3y – 15 = 0

Sol. (i) x – 3y – 3 = 0, 3x – 9y – 2 = 0

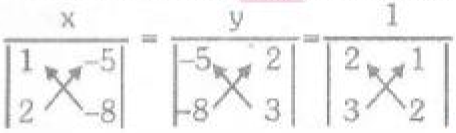
$$\frac{a_1}{a_2}=\frac{1}{3},\frac{b_1}{b_2}=\frac{-3}{-9}=\frac{1}{3},\frac{c_1}{c_2}=\frac{3}{2}\Rightarrow\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

Hence, no solution .

(ii) 2x + y = 5 ...(i) and 3x + 2y = 8 ...(ii)

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2} \qquad \left(\frac{a_1}{a_2}=\frac{2}{3},\frac{b_1}{b_2}=\frac{1}{2}\right)$$

Here, we have a unique solution. By cross multiplication, we have



$$\Rightarrow \quad \frac{x}{\{(1)(-8)-(2)(-5)\}}=\frac{y}{\{(-5)(3)-(-8)(2)\}}=\frac{1}{\{(2)(2)-(3)(1)\}}$$

$$\Rightarrow \quad \frac{x}{(-8+10)}=\frac{y}{(-15+16)}=\frac{1}{(4-3)}$$

$$\Rightarrow \quad \frac{x}{2}=\frac{y}{2}=\frac{1}{1} \qquad \Rightarrow \frac{x}{2}=\frac{1}{1} \text{ and } \frac{y}{1}=\frac{1}{1}$$

$\Rightarrow$ x = 2 and y = 1 **[Rest Try Yourself]**

2. (i) For which values of a and b does the following pair of linear equations have an infinite number of solutions ?

2x + 3y = 7

(a – b) x + (a + b) y = 3 a + b – 2

(ii) For which value of k will the following pair of linear equations have no solution ?

3x + y = 1

(2k – 1) x + (k – 1) y = 2k + 1.

Sol. (i) 2x + 3y – 7 = 0 ...(i)

(a – b) x +(a + b) y – (3a + b – 2) = 0 ...(ii)

For infinite number of solutions, we have

$$\frac{a-b}{2}=\frac{a+b}{3}=\frac{3a+b-2}{7}$$

For first and second, we have

$$\frac{a-b}{2}=\frac{a+b}{3}$$

or 3a – 3b = 2a + 2b

or a = 5b ...(i)

From second and third, we have

$$\frac{a+b}{3}=\frac{3a+b-2}{7}$$

or 7a + 7b = 9a + 3b – 6

or 4b = 2a – 6

From (i) and (ii), eliminating a,

2b = 5b – 3 $\Rightarrow$ b = 1

Substituting b = 1 in (i), we get a = 5

(ii) **[Try Yourself]**

3. Solve following pair of linear equations by the substitution and cross-multiplication methods :

$8x + 5y = 9,\ 3x + 2y = 4$

Sol. **[Try Yourself]**

4. **Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method .**

(i) **A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay Rs. 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs. 1180 as hostel charges. Find the fixed charges and the cost of food per day.**

(ii) **A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.**

(iii) **Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer , then Yash would have scored 50 marks. How many questions were there in the test ?**

(iv) **Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If travel towards each other, they meet in 1 hour. What are the speeds of the two cars ?**

(v) **The area of a rectangle gets reduced by 9 square units if its length is reduced by 5 units and breadth is increased by 3 units . If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangles.**

Sol. (i) Try Yourself

(ii) Try Yourself

Hint (iii) number of right answers = x. Number of wrong answers = y

Then, $3x - y = 40$ and $4x - 2y = 50$

Hint (iv) Speed of car i = x km/hr

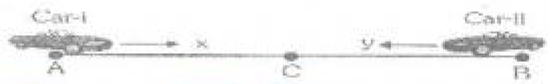
Speed of car ii = y km/hr

First case :

Two cars meet at C after 5 hrs.

$AC - BC = AB$

$\Rightarrow 5x - 5y = 100 \quad ...(i)$

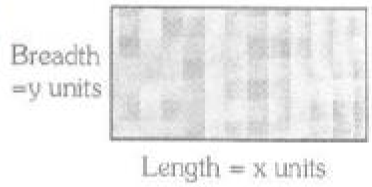


Second case :

Two cars meet at C after one hour

$x + y = 100 \quad ...(ii)$

Hint (v) In first case, area is reduced by 9 square units.



When length = x - 5 units

and breadth = y + 3 units

$\Rightarrow xy - (x-5) \times (y-3) = 9 \quad ...(i)$

In second case area increases by 67 sq. units when length = x + 3 and breadth = y + 2.

$\Rightarrow (x+3) \times (y+2) - xy = 67 \quad ...(ii)$

EXERCISE : 3 . 6

1. Solve the following pairs of equations by reducing them to a pair of linear equations :

(i) $\frac{1}{2x}+\frac{1}{3y}=2,\ \frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$

(ii) $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2,\ \frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$

(iii) $\frac{4}{x}+3y=14,\ \frac{3}{x}-4y=23$

(iv) $\frac{5}{(x-1)}+\frac{1}{(y-2)}=2,\ \frac{6}{(x-1)}-\frac{3}{(y-2)}=1$

(v) $\frac{7x-2y}{xy}=5,\ \frac{8x+7y}{xy}=15$

(iv) $6x+3y=6xy,\ 2x+4y=5xy$

(vii) $\frac{10}{(x+y)}+\frac{2}{(x-y)}=4,\ \frac{15}{(x+y)}-\frac{5}{(x-y)}=-2$

(viii) $\frac{1}{(3x+y)}+\frac{1}{(3x-y)}=\frac{3}{4},\ \frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=\frac{-1}{8}$

Sol. (i) $\frac{1}{2x}+\frac{1}{3y}=2,\ \frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$

Putting $\frac{1}{x}=u$ and $\frac{1}{y}=v$

We get $\frac{1}{2}u+\frac{1}{3}v=2,\ \frac{1}{3}u+\frac{1}{2}v=\frac{13}{6}$

Multiplying by 6 on both sides, we get

$\Rightarrow$ 3u + 2v = 12 ...(i)

2u + 3v = 13 ...(ii)

Multiplying (i) by 3 and (ii) by 2, then subtracting later from first, we get

3(3u + 2v) – 2(2u + 3v) = 3 × 12 – 2 × 13 $\Rightarrow$ 9u – 4u = 36 – 26 $\Rightarrow$ u = 2

Then substituting u =2 in (i), we get

6 + 2v = 12 $\Rightarrow$ v = 3

Now, u = 2 and v = 3

$\Rightarrow$ $\frac{1}{x}=2$ and $\frac{1}{y}=3$ $\Rightarrow$ $x=\frac{1}{2}$ and $y=\frac{1}{3}$

(ii) [Hint : Put $\frac{1}{\sqrt{x}}=u$ & $\frac{1}{\sqrt{y}}=v$],

(iii) Try Yourself

(iv) [Hint : Put $\frac{1}{x-1}=u$ and $\frac{1}{y-2}=v$ to get :

5u + v = 2 and 6u – 3v = 1]

(v) [Hint : $\frac{7x-2y}{xy} = 5,\ \frac{8x+7y}{xy} = 15$

$\Rightarrow \quad \frac{7x}{xy} - \frac{2y}{xy} = 5,\ \frac{8x}{xy} + \frac{7y}{xy} = 15$

$\Rightarrow \quad \frac{7}{y} - \frac{2}{x} = 5,\ \frac{8}{y} + \frac{7}{x} = 15$

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$, we get

7v – 2u = 5, 8v + 7u = 15

[Rest Try Yourself]

2. Formulate the following problems as a pair of linear equations, and hence find their solutions :

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current .

(ii) 2 woman and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.

(iii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by has, she takes 10 minutes longer. Find the speed of the train and the bus separately .

Sol. (i) [Hint Speed of Ritu in still water = x km/hr

Speed of current = y km/hr

Then speed downstream = (x + y) km/hr

Speed upstream = (x – y) km/hr

$\frac{20}{x+y} = 2$ and $\frac{4}{x-y} = 2$

$\Rightarrow$ x + y = 10 ...(i)

x – y = 2 ...(ii)

Hint (ii) Let 1 woman finish the work in x days and let 1 man finish the work in y days.

Work of 1 woman in 1 day $= \frac{1}{x}$

Work of 1 man in 1 day $= \frac{1}{y}$

Work of 2 woman and 5 men in one day $= \dfrac{2}{x} + \dfrac{5}{y} = \dfrac{5x+2y}{xy}$

The number of days required for complete work $= \dfrac{xy}{5x+2y}$

We are given that $\dfrac{xy}{5x+2y} = 4$ …(i)

Similarly, in second case $\dfrac{xy}{6x+3y} = 3$ …(ii)]

(iii) **[Try Yourself]**

EXERCISE – 1 (FOR SCHOOL / BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

Choose The Correct One

1. If A : Homogeneous system of linear equations is always consistent. R : x = 0, y = 0 ix always a solution of the homogeneous system of equations with unknowns x and y, then which of the following statement is true ?
(A) A is true and R is the correct explanation of A
(B) A is false and R is not a correct explanation of A
(C) A is true and R is false
(D) A is false and R true

2. If the pair of linear equations $x - y = 1$, $x + ky = 5$ has a unique solution $x = 2$, $y = 1$, then value of k is –
(A) -2 (B) 3 (C) -3 (D) 4

3. The pair of linear equations $2x + ky - 3 = 0$, $6x + \frac{2}{3}y + 7 = 0$ has a unique solution if –
(A) $k = \frac{2}{3}$ (B) $k \neq \frac{2}{3}$
(C) $k = \frac{2}{9}$ (D) $k \neq \frac{2}{9}$

4. The pair of linear equations $2kx + 5y = 7$, $6x - 5y = 11$ has a unique solution if –
(A) $k \neq -3$ (B) $k \neq 3$
(C) $k \neq 5$ (D) $k \neq -5$

5. The pair of equations $3x + 4y = k$, $9x + 12y = 6$ has infinitely many solutions if –
(A) $k = 2$ (B) $k = 6$
(C) $k \neq 6$ (D) $k = 3$

6. The pair of linear equations $2x + 5y = k$, $kx + 15y = 18$ has infinitely many solution if –
(A) $k = 3$ (B) $k = 6$ (C) $k = 9$ (D) $k = 18$

7. The pair of linear equations $3x + 5y = 3$, $6x + ky = 8$ do not have any solution if –
(A) $k = 5$ (B) $k = 10$ (C) $k \neq 10$ (D) $k \neq 5$

8. The pair of linear equations $3x + 7y = k$, $12x + 2ky = 4k + 1$ do not have any solution if
(A) $k = 7$ (B) $k = 14$ (C) $k = 21$ (D) $k = 28$

9. The pair of linear equations $7x - 3y = 4$, $3x + \frac{k}{7}y = 4$ is consistent only when –
(A) $k = 9$ (B) $k = -9$ (C) $k \neq -9$ (D) $k \neq 7$

10. The pair of linear equations $kx + 4y = 5$, $3x + 2y = 5$ is consistent only when –
(A) $k \neq 6$ (B) $k = 6$ (C) $k \neq 3$ (D) $k = 3$

11. The pair of linear equations $7x + ky = k$, $14x + 2y = k + 1$ has infinitely many solution if –

(A) $k = 1$ (B) $k \neq 1$ (C) $k = 2$ (D) $k = 4$

12. The pair of linear equations $13x + ky = k$, $39x + 6y = k + 4$ has infinitely many solutions if -

(A) $k = 1$ (B) $k = 2$ (C) $k = 4$ (D) $k = 6$

13. The pair of linear equations $x + y = 3$, $2x + 5y = 12$ has a unique solution $x = x_1$, $y - y_1$ then value of x_1 is –

(A) 1 (B) 2 (C) -1 (D) -2

14. The pair of linear equations $3x - 5y + 1 = 0$, $2x - y + 3 = 0$ has a unique solution $x = x_1$, $y = y_1$ then $y_1 =$

(A) 1 (B) -1 (C) -2 (D) – 4

15. The pair of linear equations $x + 2y = 5$, $7x + 3y = 13$ has a unique solution –

(A) $x = 1, y = 2$ (B) $x = 2, y = 1$

(C) $x = 3, y = 1$ (D) $x = 1, y = 3$

16. The pair of linear equations $x + 2y = 5$, $3x + 12y = 10$ has –

(A) Unique solution (B) No solution

(C) More than two solution (D) Infinitely many solutions

17. If the sum of the ages of a father and his son in years is 65 and twice the difference of their ages in years is 50, then the age of father is –

(A) 45 years (B) 40 years (C) 50 years (D) 55 years

18. A fraction becomes $\frac{4}{5}$ when 1 is added to each of the numerator and denominator. However, if we subtract 5 from each then it becomes $\frac{1}{2}$. The fraction is –

(A) $\frac{5}{8}$ (B) $\frac{5}{6}$ (C) $\frac{7}{9}$ (D) $\frac{13}{16}$

19. Three chairs and two tables cost Rs. 1850 Five chairs and three tables cost Rs. 1850. Then the total cost of one chair and table is –

(A) Rs. 800 (B) Rs. 850 (C) Rs. 900 (D) Rs.950

20. Six years hence a man's age will be three times the age of his son and three years ago he was nine times as old as his son. The present age of the man is –

(A) 28 years (B) 30 years (C) 32 years (D) 34 years

OBJECTIVE							ANSWER KEY							EXERCISE - 1
Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A	B	D	A	A	B	B	B	C	A	A	B	A	B	A
Que.	16	17	18	19	20										
Ans.	A	A	C	B	B										

EXERCISE – 2 (FOR SCHOOL / BOARD EXAMS)

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions

1. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ find for whether the following pair of liner equations are consistent or inconsistent .

(i) $x - 3y = 4$; $3x + 2y = 1$ (ii) $\frac{4}{3}x + 2y = 8$; $2x + 3y = 12$

(iii) $4x + 6y = 7$; $12x + 18y = 21$ (iv) $x - 2y = 3$; $3x - 6y = 1$

2. On comparing the ratio $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ find out whether the lines representing is following pair of linear equations intersect at a point, are parallel or coincident :

(a) (i) $2x - y = 3$; $4x - y = 5$ (ii) $x + 2y = 8$; $5x - 10y = 10$ (iii) $3x + 4y = -2$; $12x + 16y = -8$

(b) (i) $6x + 3y = 18$; $2x + y = 6$ (ii) $x - 3y = 3$; $3x - 9y = 2$ (iii) $ax - by = c_1$; $bx + ay = c_2$, where $a \neq 0, b \neq 0$

3. For the linear equations given below, write anther linear equation in two variables, such that the geometrical representation of the pair so formed is -

(i) Intersecting (ii) Parallel lines (iii) Coincident lines

(a) $2x - 3y = +$ (b) $y = 2x + 3$

4. Find the value of k for which the given system of equations has a unique solution .

(a) $(k - 3) x + 3y = k$; $kx + ky = 12$ (b) $x - ky = 2$; $3x + 2y = -5$

5. Find the value of k for which the given system of equations has no solution .

(a) $kx + 2y - 1 = 0$; $5x - 3y + 2 = 0$

(b) (i) $x + 2y = 3$; $5x + ky + 7 = 0$ (ii) $kx + 3y = k - 3$; $12x + ky = k$

6. (a) Find the value (s) of k for which the system of equations $kx - y = 2$ and $6x - 2y = 3$ has

(i) A unique solution (ii) No solution

(b)Find the value of k for which system $kx + 2y = 5$ and $3x + y = 1$ has

(i) A unique solution (ii) No solution

7. Find the value of k for which the given system of equations has an infinite number of solutions.

(a) $5x + 2y = 2k$ and $2(k + 1) x + ky = (3k + 4)$

(b) (i) $x + (k + 1) y = 5$ and $(k + 1) x + 9y = 8k - 1$ (ii) $10x + 5y - (k - 5) = 0$ and $20x + 10y - k = 0$

(c) $kx + 3y = k - 3$ and $12x + ky = k$

8. Find the value of a and b for which the given system of linear equation has an infinite number of solutions :

(a) $2x + 3y = 7$ and $(a - b) x + (a + b) y = 3a + b - 2$

(b) $(a + b) x - 2by = 5a + 2b + 1$ and $3x - y = 14$

(c) $(2a - 1) x + 3y - 5 = 0$ and $3x + (b - 1) y - 2 = 0$

Short Answer Type Questions

Based on graphical solution of system of equations :

Solve graphically each of the following pairs of equations (1–9) :

1. $x + y = 4, 2x - 3y = 3$
2. $x + y = 3, 2x + 5y - 12 = 0$
3. $\frac{4}{9}x + \frac{1}{3}y = 1, 5x + 2y = 13$
4. $2x + 3y = 4, x - y + 3 = 0$
5. $x + y = 7, 5x + 2y = 20$
6. $x + 4y = 0, 2x + 8y = 0$
7. $x + 2y = 3, 2x + 4y = 15$
8. $3x + 2y = 3, 6x + 4y = 15$
9. $2x + 3y - 5 = 0, 6x + 9y - 15 = 0$
10. Check whether the pair of equations $x + 3y = 6$, and $2x - 3y = 12$ is consistent. If so, solve graphically .
11. Show graphically that the pair of equations $2x - 3y + 7 = 0, 6x - 9y + 21 = 0$ has infinitely many solutions.
12. Show graphically that the pair of equations $8x + 5y = 9, 16x + 10y = 27$ has no solution.
13. Find whether the pair of equations $5x - 8y + 1 = 0, 3x - \frac{24}{5}y + \frac{3}{5} = 0$ has no solution, unique solution or infinitely many solutions.
14. Show graphically that the pair of equations $2x - 3y = 4, 3x - 2y = 1$ has a unique solution.
15. Show graphically that the pair of equations $3x + 4y = 6, 6x + 8y = 12$ represents coincident lines.
16. Determine by drawing graphs whether the following pair of equations has a unique solution or no :
    $2x - 3y = 6, 4x - 6y = 9$. If yes, find the solution also.
17. Determine graphically whether the pair of linear equations $3x - 5y = -1$, $2x - y = -3$ has a unique solution or not. If yes, find the solution also .
18. Solve graphically the pair of equations $x + 3y = 6$, and $3x - 5y = 18$. Hence, find the value of K if $7x + 3y = K$.
19. Solve graphically the pair of equations $2x - y = 1, x + 2y = 8$. Also find the points where the lines meet the axis of y.
20. Solve graphically the following pair of linear equations :
    $2x + 3y - 12 = 0, 2x - y - 4 = 0$. Also find the coordinates of the points where the lines meet the y-axis.
21. Solve the following pair of equations graphically : $x + y = 4, 3x - 2y - 3$
    Shade the region bounded by the lines representing the above equations and x-axis.
22. Solve the following pair of linear equations graphically : $2x + y = 8, 3x - 2y = 12$.
    From the graph, read the points where the lines meet the x-axis.
23. Solve graphically the following pair of equations : $x - y = 1, 2x + y = 8$. Shade the area bounded by these lines and the y-axis.
24. On the same axes, draw the graph of each of the following equations :
    $2y - x = 8, 5y - x = 14, y - 2x = 1$. Hence, obtain the vertices of the triangle so formed.
25. Solve graphically the pair of linear equations : $4x - 3y + 4 = 0, 4x + 3y - 20 = 0$. Find the area of the region bounded by theses lines and x-axis,

Based on substitution method :

Solve the following equations by the substitution method : (26-41)

26. $3x + 11y = 13, 8x + 13y = 2$
27. $x + 2y = 1.6, 2x + y = 1.4$
28. $11x - 8y = 27, 3x+5y = -7$
29. $0.04x + 0.02y = 5, 0.5x - 0.4y = 30$
30. $5x + 8y = -1, 6y - x = 4y - 7$

31. $12x - 16y = 20, 8x + 6y = 30$

32. $8x - 5y + 40 = 0, 7x - 2y = 0$

33. $\frac{1}{2}(9x + 10y) = 23, \frac{5x}{4} - 2y = 3$

34. $\sqrt{2}x + \sqrt{3}y = 0, \sqrt{3}x - \sqrt{8}y = 0$

35. $\frac{(3x - y)}{5} = 2y - 1, \frac{3x}{8} - \frac{y}{4} = \frac{1}{2}$

36. $3x + 15 = 4y, 3y + 17 = 2 + 3x$

37. $x + 6y = 2x - 16, 3x - 2y = 24$

38. $x = 3y - 19, y = 3x - 23$

39. $5x + 2y = 14, x + 3y = 8$

40. $x + y = 27, \frac{3}{4}x + \frac{2}{3}y = 19$

41. $\frac{x + 11}{7} + 2y = 10, \quad 3x = 8 + \frac{y + 7}{11}$

42. Solve $2x - y = 12$ and $x + 3y + 1 = 0$ and hence find the value of m for which $y = mx + 3$.

43. Solve $4x - 3y + 17 = 0$ and $5x + y + = 0$ and hence find the value of n for which $y = nx - 1$.

Based on substitution method :

Solve the following pairs of lines equations by elimination method : (44-52)

44. (a) $x + y = 5$ and $2x - 3y = 4$

(b) (i) $2x + 3y = 8$ and $4x + 6y = 7$ (ii) $11x + 15y + 23 = 0$ and $7x - 2y - 20 = 0$

(c) $78x + 91y = 39$ and $65x + 117y = 42$

45. (a) $\frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$ (b) $\frac{x}{3} + \frac{y}{4} = 11$ and $\frac{5x}{6} - \frac{y}{3} + 7 = 0$

46. (a) $ax - by = a^2 + b^2$ and $x + y = 2a$ (b) $\frac{bx}{a} - \frac{ay}{b} + a + b = 0$ and $bx - ay + 2ab = 0$

47. $ax + by - 2a + 3b = 0$ and $bx - ay - 3a - 2b = 0$

48. (a) $2(ax - by) + (a + 4b) = 0$ and $2(bx + ay) + (b - 4a) = 0$

(b) $(bx + ay) = 0$ and $(a + b)x + (a - b)y = a^2 + b^2$

49. (a) $(a + c)x - (a - c)y = 2ab$ and $(a + b)x - (a - b)y = 2ab$

(b) $(a + 2b)x + (2a - b)y = 2$ and $(a - 2b)x + (2a + b)y = 3$

50. (a) $\sqrt{2}x - \sqrt{3}y = 0$ and $\sqrt{5}x + \sqrt{2}y = 0$ (b) $\sqrt{7}x + \sqrt{11}y = 0$ and $\sqrt{3}x - \sqrt{5}y = 0$

51. $0.5x + 0.7y = 0.74$ and $0.3x + 0.5y = 0.5$

52. (a) $23x - 29y = 98$ and $29x - 23y = 110$

(b) (i) $217x + 131y = 913$ and $131x + 127y = 827$ (ii) $23x + 37y = 32$ and $37x + 23y = 88$

(c) (i) $65x - 33y = 97$ and $33x - 65y = 1$ (ii) $47x + 31y = 63$ and $31x + 47y = 15$

(iii) $99x + 101y = 499$ and $101x + 99y = 501$

Based on cross-multiplication method :

Solve each of the following pairs of equations by cross multiplication rule : (53-62)

53. $x - 2y = 10, 4x + y = 13$

54. $5x + 3y = 35, 2x + 4y = 28$

55. $4x - 3y + 1 = 0, 2x - 5y + 11 = 0$

56. $3x + 4y = 27, 5x - 3y = 16$

57. $2x + 3y = 46, 3x + 5y = 74$

58. $2x - y + 4 = 0, x + y - 1 = 0$

59. $\frac{x}{2}-\frac{y}{3}+4=0,\ \frac{x}{2}-\frac{5y}{3}+12=0$

60. $ax+by+a=0, bx+ay+b=0$

61. $\frac{x}{a}+\frac{y}{b}=0,\ \frac{x}{a}-\frac{y}{b}=4$

62. $x+y=a+b, ax-by=a^2-b^2$

Based on equations reducible to linear equations :

Solve for x and y : (63-82)

63. $\frac{2}{x}+\frac{3}{y}=2;\ \frac{1}{x}-\frac{1}{2y}=\frac{1}{3}$

64. $\frac{4}{x}+\frac{7}{y}=29;\ \frac{3}{y}+\frac{1}{x}=11$

65. $\frac{1}{3x}-\frac{1}{7y}=\frac{2}{3};\ \frac{1}{2x}-\frac{1}{3y}=\frac{1}{6}$

66. $\frac{1}{5x}+\frac{9}{y}=4;\ \frac{3}{x}+\frac{27}{y}=24$

67. $\frac{11}{2x}-\frac{9}{2y}=-\frac{23}{2};\ \frac{3}{4x}+\frac{7}{15y}=\frac{23}{6}$

68. $x+y=2xy$; $x-y=xy$

69. $4x+3y=8xy$; $6x+5y=13xy$

70. $6x+5y=8xy$; $8x+3y=7xy$

71. $\frac{x-y}{xy}=9;\ \frac{x+y}{xy}=5$

72. $9+25xy=53x$; $27-4xy=x$

73. $\frac{16}{x+3}+\frac{3}{y-2}=5;\ \frac{8}{x+3}-\frac{1}{y-2}=0$

74. $\frac{13}{x+y}-\frac{56}{x-y}=21;\ \frac{11}{x+y}-\frac{23}{x-y}=14\frac{2}{7}$

75. $\frac{24}{2x+y}-\frac{13}{3x+2y}=2;\ \frac{26}{3x+2y}+\frac{8}{2x+y}=3$

76. $\frac{29}{x-1}-\frac{81}{y+1}=-26;\ \frac{19}{y+1}-\frac{4}{x-1}=\frac{15}{2}$

77. $\frac{2}{x-1}+\frac{y-2}{4}=2;\ \frac{3}{2(x-1)}+\frac{2(y-2)}{5}=\frac{47}{20}$

78. $\frac{x-4}{x-3}=\frac{y+4}{y+7};\ \frac{x+5}{x+2}=\frac{y-1}{y-2}$

79. $\frac{3x-2}{3y+7}=\frac{5x-1}{5y+16};\ \frac{3x-15}{x-9}=\frac{6y-5}{2y+3}$

80. $\frac{x+y+3}{x-y-3}=\frac{-3}{2};\ \frac{x-y-3}{x-y-3}=-2$

81. $\frac{x+y-1}{x-y+1}=7;\ \frac{y-x+1}{x-y+1}=35$

82. $\frac{x+2y=1}{2x-y+1}=2;\ \frac{3x-y+1}{x-y+3}=5$

SUBJECTIVE	**ANSWER KEY**	**EXERCISE -2 (X) -CBSE**

- **Very Short Answer Type Questions**

1. (i) consistent (ii) consistent (iii) consistent (iv) inconsistent
2. (a) (i) intersect at point (ii) parallel (iii) coincident (b) (i) coincident (ii) parallel (iii) intersect at a point
3. (a) (i) 3x + 5y – 7 = 0 (ii) 4x – 6y – 8 = 0 (iii) 6x – 9y = 18
(b) (i) 2x + 3y – 4 = 0 (ii) 4x – 2y + 8 = 0 (iii) 8x – 4y + 12 = 0 (many such examples may be given)
4. (a) $k \neq 6$ (b) $k \neq \frac{-2}{3}$ **5.** (a) $k = \frac{-10}{3}$ (b) (i) k = 10 (ii) k = - 6 **6.** (a) (i) $k \neq 3$ (ii) k = 3 (b)(i) $k \neq 6$(ii) k = 6
7. (a) k = 4 (b) (i) k = 2 (ii) k = 10 (c) k = 6 **8.** (a) a = 5, b = 1 (b) a = 5, b = 1 (c) a = $\frac{17}{4}$, b = $\frac{11}{5}$

- **Short Answer Type Questions**

1. x = 3, y = 1 **2.** x = 1, y = 2 **3.** x = 3, y = - 1 **4.** x = - 1, y = 2 **5.** x = 2, y = 5 **6.** Infinite number of solutions
7. No solution **8.** No solution **9.** Infinite number of solutions **10.** Yes ; x = 6, y = 0 **13.** Infinitely many solutions
16. No **17.** Yes ; x = - 2, y = - 1 **18.** x = 6, y = 0 ; K = 42 **19.** x = 2, y = 3 ; (0, - 1), (0, 4)
20. x = 3, y = 2, (0, 4), (0, - 4) **21.** x = 1, y = 3
22. x = 4, y = 0 ; The two lines meet at the x-axis at a common point (4, 0). **23.** x = 3, y = 2 **24.** (2, 5), (-4, 2), (1, 3)
25. 12 sq units. **26.** x = -3, y = 2 **27.** x = 0.4, y = 0.6 **28.** x = 1, y = - 2 **29.** x = 100, y = 50
30. x = 3, y = - 2 **31.** x = 3, y = 1 **32.** x = $\frac{80}{19}$, y = $\frac{80}{19}$ **33.** x = 4, y = 1 **34.** x = y = 0 **35.** x = 2, y = 1
36. x = 35, y = 30 **37.** x = 7, y = - 3/2 **38.** x = 11, y = 10 **39.** x = 2, y = 2 **40.** x = 12, y = 15
41. x = 3, y = 4 **42.** x = 5, y = - 2 , - 1 **43.** x = - 2, y = 3, - 2
44. (a) x = $\frac{19}{5}$, y = $\frac{6}{5}$ (b) (i) No solution (ii) x = 2, y = - 3 (c) x = $\frac{3}{13}$, y = $\frac{3}{13}$ **45.** (a) x = 2, y = -3 (b) x = 6, y = 36
46. (a) x = (a + b), y = (a – b) (b) x = - a, y = b **47.** x = 2, y = - 3 **48.** (a) x = $\frac{-1}{2}$, y = 2, (b) x = a, y = - b
49. (a) x = b, y = - b (b) x = $\frac{5b-2a}{10ab}$, y = $\frac{a+10b}{10ab}$ **50.** (a) x = 0, y = 0 (b) x = 0, y = 0 **51.** x = 0.5, y = 0.7
52. (a) x = 3, y = - 1 (b) (i) x = 3, y = 2 (ii) x = 3, y = - 1 (c) (i) x = 2, y = 1 (ii) x = 2, y = - 1 (iii) x = 3, y = 2
53. x = 4, y = - 3 **54.** x = 4, y = 5 **55.** x = 2, y = 3 **56.** x = 5, **57.** x = 8, y = 10 **58.** x = - 1, y = 2
59. x = - 4, y = 6 **60.** x = - 1, y = 0 **61.** x = 2a, y = - 2b **62.** x = a, y = b **63.** x = 2, y = 3 64. x = $\frac{1}{2}$, y = $\frac{1}{3}$
65. x = $\frac{1}{5}$, y = $\frac{1}{7}$ **66.** x = $\frac{1}{5}$, y = 3 67. x = $\frac{1}{2}$, y = $\frac{1}{5}$ **68.** x = 2, = $\frac{2}{3}$ **69.** x = $\frac{1}{2}$, y = 2 **70.** x = 1, y = 2
71. x = - $\frac{1}{2}$, y = $\frac{1}{7}$ **72.** x = 3, y = 2 **73.** x = 5, y = 3 **74.** x = - 3, y = 4 **75.** x = 3, y = 2 **76.** x = 3, y = 1
77. x = 3, y = 6 **78.** x = 7, y = 5 **79.** x = 3, y = 2 **80.** x = 3, y = 2 **81.** x = $\frac{2}{9}$, y = $\frac{7}{6}$ **82.** x = 13, y = 10

EXERCISE – 3 (FOR SCHOOL / BOARD EXAMS)

APPLICATIONS TO WORD PROBLEMS

Based On Articles And Their Costs :

Q.1 4 charier and 3 tables cost Rs 2100 and 5 chairs and 2 tables cost Rs. 1750. Find the cost of a chair and a table separately .

Q.2 37 pens and 53 pencils together cost Rs. 320, while 53 pens and 37 pencils together cost Rs 40. Find the cost of a en and that of a pencil.

Q.3 4 tables and 3 chairs together cost Rs 2250 and 3 tables and 4 chairs cost Rs 1950. Find the cost of 2 chairs and 1 table.

Q.4 A and B each have certain number of oranges. A says to A says to B. "if you give me 10 of your oranges, I will have twice the number of oranges left with you. "B relies," if you give me 10 of your oranges, If will have the same number of oranges as left with you." Find the number of oranges with A and B separately.

Q.5 A and B each have a certain number of mangoes. A says to B, " if you give 30 of your mangoes, I will have twice as many as left with you." B replies, "if you give me 10, I will have thrice as many as left with you." How many mangoes does each have ?

Q.6 One says, " give me a hundred , friend ! I shall then become twice as rich as you, " The other replies," If you give me ten, I shall be six times as rich as you." Tell me what is the amount of their respective capital ?

Q.7 Reena has pens and pencils which together are 40 in number. If she has 5 more pencils and 5 less pens, then number of pencils would become 4 times the number of pens. Find the original number of pens and pencils.

Q.8 A man has only 20 paisa coins and 25 paisa coins in his purse. If he has 50 coins in all totaling Rs 11.25, how many cons of each kind does he have :

Q.9 A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs 27 for a book kept for seven days, while Susy paid Rs 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Based on numbers :

Q.10 Sum of two numbers is 35 and their difference is 13. Find the numbers .

Q.11. The sum of two number is 8. If their sum is 4 times their difference. Find the number.

Q.12. The sum of two numbers is 1000 and the difference between their squares is 256000. Find the numbers .

Q.13 In a two digit number, the unit's digit is twice the ten's digit . If 27 is added to the number. the digits interchange their paces. Find the number.

Q.14. In a two digit number, the ten's digit is three times the unit's digit . When the number is decreased by 54, the digits are reversed. Find the number.

Q.15 The sum of the digits of a two digit number is 15. The number obtained by reversing the order of digits of the given number exceeds the given number by 9. Find the given number .

Q.16 The sum of the digits of a two digit number is 8 and the difference between the number and that formed by reversing the digit is 18. Find the number.

Q.17. The sum of a two digit number and the number formed by interchanging its digits is 110. If 10 is subtracted from the first number, the new number is 4 more than 5 times the sum of its digits in the first number. Find the first number .

Q.18 The sum of a two digit number and the number formed by interchanging its digits is 132. If 12 is added to the number, the new number becomes 5 times the sum of the digits. Find the number.

Q.19 The sum of a two digit number and the number obtained by reversing the order of its digits is 121, and the two digits differ by 3. Find the number.

Q.20 A two digit number is 3 more than 4 times the sum of digits. If 18 is added to the number, the digits are reversed. Find the number.

Based On Fractions :

Q.21 A fraction becomes $\frac{4}{5}$, if 1 is added to both numerator and denominator. If however, 5 is subtracted from both numerator and denominator, the fraction becomes $\frac{1}{2}$. What is the fraction ?

Q.22 A fraction is such that if the numerator is multiplied by 3 and the denominator is reduced by 3, we get $\frac{18}{11}$. But , if the numerator is increased by 8 and the denominator is doubled, we get $\frac{2}{5}$. Find the fraction ?

Q.23 The denominator of a fraction is 4 more than twice the numerator. When both the numerator and denominator are decreased by 6, then the denominator becomes 12 times the numerator . Determine the fraction.

Q.24 The numerator of a fraction is 4 less than the denominator. If the numerator is decreased by 2 and denominator is increased by 1, then the denominator is eight times the numerator. Find the fraction.

Q.25 The sum of the numerator and denominator of a fraction is 4 more than twice the numerator . If the numerator and denominator are increased by 3, they are in the ratio 2 : 3. Determine the fraction .

Q.26. The sum of the numerator and denominator of a fraction is 3 less than twice the denominator. If the numerator and denominator by 1, the numerator becomes half the denominator. Determine the fraction.

Based On ages :

Q.27. If twice the son's age in years is added to the father's age, the sum is 70. But if twice the father's age is added to the son's age, the sum is 95. Find the ages of father and son .

Q.28. A father is three times as old as his son. After twelve years, his age will be twice as that of his son then. Find their present ages.

Q.29. I am three times as old as my son. Five years later, I shall be two and a half times as old as my son. How old am I and how old is my son ?

Q.30 Ten years ago, father was twelve times as old as his son and ten years hence, he will be twice as old as his son will be. Find their present ages.

Q.31 Five years hence, father's age will be three times the age of his son. Five years ago, father was seven times as old as his son. Find their present ages.

Q.32 The present age of a father is three years more than three times the age of the son. Three years hence, father's age will be 10 years more than twice the age the son. Determine their present ages.

Q.33 A and B are friends and their ages differ by 2 years. A's father D is twice as old as A and B is twice as old as his sister C. The age of D and C differ by 40 years. Find the ages of A and B.

Q.34 A is elder to B by 2 years. A's father F is twice as old as A and B is twice as old his sister S. If the ages of the father and sister differ by 40 years, find the age of A.

Q.35 Father's age is three times the sum of ages of his two children . After 5 years his age will be twice the sum of ages of two children. Find the age of father.

Based On Time, Distance And Speed :

Q.36 Points A and B are 90 km. apart from each other on a highway. A car starts from A and another from B at the same time. If they go in the same direction, they meet in 9 hours and if they go in opposite directions, they meet in 9/7 hours. Find their speeds.

Q.37 Points A and B are 70 km. apart on a highway. A car starts from A and another from B simultaneously. If they ravel in the same direction, they meet in 7 hours but if they travel towards each other they meet in one hour. Find the speeds of the two cars.

Q.38 Points A and B are 80 km. apart from each other on a highway. A car starts from A and another from B at the same time. If they move in the same direction, they meet in 8 hours and if move in opposite directions, they meet in one hour and twenty minutes. Find the speeds of the two cars.

Q.39 Rahul travels 600 km to his home partly by train and partly by car. He takes 8 hours if he travels 120 km by train and rest by car. He takes 20 minutes longer if he travels 200 km by train and the rest by car. Find the speed of the train and the car.

Q.40 A man travels 370 km partly by train and partly by car. If he covers 250 km by train and the rest by car, it takes him 4 hours. But , if he travels 130 km by train and the rest by car, he takes 18 minutes longer. Find the speed of the train and that of the car.

Q.41 A boat covers 32 m upstream and 36 km downstream in 7 hours. Also, it covers 40 km upstream and 48 km downstream in 9 hours. Find the speed of he boat in still water and that of the stream.

Q.42 A sailor goes 8 km downstream in 40 minutes and returns in 1 hour. Determine the speed of the sailor in still water and the speed of the current .

Q.43 The boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours. It can go 40 km upstream and 55 km downstream. Determine the speed of stream and that of the boat in still water.

Q.44 A boat goes 24 km upstream and 28 km downstream in 6 hrs. It goes 30 km upstream and 21 km downstream in $6^{1/2}$ hrs. Find the speed of the boat in still water and also speed of the stream.

Q.45 X takes 3 hours more than Y to walk 30 km, But, if X doubles his pace, he is ahead of Y by $1^{1/2}$ hours. Find their speed of walking.

Q.46 While covering a distance of 30 km. Ajeet takes 2 hours more than Amit. If Ajeet doubles his speed, he would take 1 hour less than Amit. Find their speeds of walking .

Q.47 A man walks a certain distance with certain speed. If he walks $\frac{1}{2}$ km an hour faster, he takes 1 hour less. But, if he walks 1 km an hour slower, he takes 3 more hours. Find the distance covered by the man and his original rate of walking.

Q.48 A train covered a certain distances at a uniform speed. If the train would have been 6 km/h faster, it would have taken 4 hours less than the scheduled time. And, if the train were slower by 6 km/h, it would have taken 6 hours more than the scheduled time. Find the length of the journey .

Based on geometrical applications :

Q.49 In a $\Delta ABC, \angle C = 3\angle B = 2(\angle A + \angle B)$. Find the three angles.

Q.50 Find the four angles of a cyclic quadrilateral ABCD in which $\angle A = (2x-1)^0, \angle B = (y+5)^0, \angle C = (2y+15)^0$ and $\angle D = (4x-7)^0$.

Q.51 In a $\Delta ABC, \angle A = x^0, \angle B = 3x^0$ and $\angle C = y^0$. If 3y – 5x = 30, prove that the triangle is right angled.

Q.52 The area of a rectangle gets reduced by 9 square units if its length is reduced by 5 units and the breadth is increased by 3 units. If we increase the length by 3 units and breadth by 2 units, the area is increased by 67 square units. Find the length and breadth of the rectangle .

Q.53 If in a rectangle, the length is increased and breadth reduced each by 2 units, the area is reduced by 28 square units. If, however the length is reduced by unit and the breadth increased by 2 units, the area increases by 33 square units. Find the area of the rectangle .

Q.54 In a rectangle, if the length is increased by 3 meters and breadth is decreased by 4 metres, the area of the rectangle is reduced by 67 square metres. If length is reduced by 1 metre and breadth is increased by 4 metres, the area is increased by 89 sq. metres. Find the dimensions of the rectangle .

Miscellaneous problems :

Q.55 A man starts his job with a certain monthly salary and earns a fixed increment every year. If his salary was Rs.1500 after 4 years of service and Rs. 1800 after 10 years of service, what was his starting salary and what is the annual increment ?

Q.56 A railway half ticket costs half the full fare and the reservation charge is the same o half ticket as on full ticket. One reserved first class ticket from Mumbai to Ahmedabad costs Rs 216 and one full and one half reserved first class tickets cost Rs. 327. What is the basic first class full fare and what is the reservation charge ?

Q.57 Meena went to a bank to withdraw Rs. 2000. She asked the cashier to given her Rs. 50 and Rs. 100 notes only. Meena got 25 notes in all. Find how many notes of Rs. 50 and Rs. 100 she received.

Q.58 Yash scored 40 marks in test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test ?

Q.59 The incomes of X and Y are in the ratio of 8 : 7 and their expenditures are in the ratio 19 : 16. If each saves Rs. 1250, find their incomes.

Q.60 The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them saves Rs. 200 per month, find their monthly incomes.

Q.61 8 men and 12 boys can finish a piece of work in 10 days while 6 men and 8 boys can finish it in 14 days. Find the time taken by one man alone and that by one boy alone to finish the work.

Q.62 2 men and 7 boys can do a piece of work in 4 days. The same work is done in 3 days by 4 men and 4 boys. How long would it take one man and one boy to do it ?

Q.63 2 women and 5 men can together finish a piece of embroidery in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman along to finish the embroidery, and that taken by 1 man alone.

Q.64 Students of a class are made to stand in rows. If one student is extra in a row, there would be 2 rows less. If one student is less in a row, there would be 3 rows more. Find the number of students in the class.

Q.65 The students of a class are made to stand in rows. If 3 students are extra in raw, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

SUBJECTIVE **ANSWER KEY** **EXERCISE -3(X)-SBSE**

1. Cost of a chair = Rs. 15, Cost of a table = Rs. 500 **2.** Cost of a pen = Rs. 6.50, Cost of a pencil = Rs. 1.50
3. Rs. 150 **4.** A : 70 oranges, B : 50 oranges **5.** A : 34 mangoes, B : 62 mangoes
6. Rs. 40 , Rs. 170 **7.** Number of pens = 13, Number of pencils = 27 **8.** 25 coins of each kind.
9. Rs. 15, Rs. 3 **10.** 24, 11 **11.** 5, 3 **12.** 628, 372 **13.** 36 **14.** 93 **15.** 78 **16.** 53
17. 64 **18.** 48 **19.** 47 or 74 **20.** 35 **21.** $\frac{7}{9}$ **22.** $\frac{12}{25}$ **23.** $\frac{7}{18}$ **24.** $\frac{3}{7}$ **25.** $\frac{5}{9}$
26. $\frac{4}{7}$ **27.** Father's age = 40 years, Son's age = 15 years. **28.** Father's age = 36 years, Son's age = 12 years
29. My present age is 45 years and my son's present age is 15 fears.
30. Father's age = 34 years. Son's age = 12 years. **31.** Father's age = 40 years, Son's age = 10 years.
32. Father's age = 33 years, Son's age = 10 years.
33. A's age $= 27\frac{1}{3}$ years, B's age $= 29\frac{1}{3}$ years or A's age = 26 years, B's age = 24 years.
34. 26 years **35.** Father's age = 45 years **36.** 40 km/h & 30 km/hr
37. 40 km/h & 30 km/hr **38.** 35 km/h & 25 km/hr
39. Speed of train = 6 km/h & Speed of car = 80 km/hr **40.** Speed of train = 100 km/h & Speed of car = 80km/hr
41. Speed of boat = 10 km/h & Speed of steam = 2 km/hr **42.** Speed of sailor = 10 km/h & Speed of current =2km/hr
43. Speed of boat = 8 km/h & Speed of stream = 3 km/hr **44.** Speed of boat = 10 km/h & Speed of stream = 4 km/hr
45. X's speed $= \frac{10}{3}$ km/hr, Y's speed = 5 km/hr **46.** Ajeet's speed = 5 km/h & Amit's speed = 7.5 km/hr
47. Distance = 36 km, original speed = 4 km/hr **48.** 720 km
49. $\angle A = 20^0, \angle B = 40^0, \angle C = 120^0$ **50.** $\angle A = 65^0, \angle B = 55^0, \angle C = 115^0, \angle D = 15^0$
52. Length = 17 units breadth = 9 units **53.** 253 Sq. units
54. Length = 28 m, Bredth = 19 m
55. Starting salary = Rs. 1300, Annual increment = Rs. 50 **56.** Fare = Rs. 21, Reservation charge = Rs. 6
57. 10, 15 **58.** 20 **59.** X;s income = Rs. 6000, Y's income = Rs. 5250 **60.** Rs. 1800, Rs. 1400
61. Man : 140 days, Boy : 280 days. **62.** Man : 15 days, Boy : 60 days. **63.** Woman : 36 days, Man : 18 days
68. 60 **65.** 36

EXERCISE -4 (FOR SCHOOL / BOARD EXAMS)

PREVIOUS YEARS BOARD (SBSE) EMERSIONS

Short Answer Type – I

1. Find the value of k for which the following system of linear equations has infinite number of solutions
x + (k + 1) y = 5 ; (k + 1) x + 9y = 8k – 1 **[AI-2003]**

2. Find the value of k so that the system of linear equations will have infinite number of solutions :
x + (k + 2) y = 4 ; (2k – 1) x + 25y = 6k + 2 **[foreign-2003]**

3. Solve the following system of linear equations :
2(ax – by) + (a + 4b) = 0, 2(bx + ay) + (b – 4a) = 0.

OR

Two years ago, a father was five times as old as his son. Two years later , his age will be 8 more than three times the age of the son. Find the present ages of father and son. **[Delhi-2003]**

4. Solve the following system of linear equations : 6(ax + by) = 3a + 2b ; 6(bx – ay) = 3b – 2a .

OR

The sum of the digits of a two digit number is 15. The number obtained by interchanging the digits exceeds the given number by 9. Find the number **[AI-2004]**

5. Solve the following system of linear equations : 3(bx + ay) = a - +b, 3(ax – by) = - (6a + b).

OR

If 1 is added to each of numerator and denominator of a fraction, it becomes 2/3. However, if 1 is subtracted form each of numerator and denominator it becomes 3/5. Find the fraction. **[Foreign-2003]**

6. solve for x and y : $\frac{4}{x}+3y=14,\ \frac{3}{x}-4y=23$

OR

Solve for x and y : $\frac{b}{a}x+\frac{b}{b}y=a^2+b^2,\ x+y=2ab.$ **[Delhi-2004C]**

7. If (x – 4) is a factor of $x^3 + ax^2 + 2bx – 24$ and a – b = 8, find the values of a and b. **[Delhi-2004C]**

8. If (x + 3) is a factor of $x^3 + ax^2 – bx + 6$ and a + b = 7, find the values of a and b **[Delhi-2004C]**

9. If (x + 2) is a factor of $x^3 + ax^2 + 4bx + 12$ and a + b = - 4, find the values of a and b. **[Delhi-2004C]**

10. Solve for x and y : $\frac{2}{x}+\frac{3}{y}=13,\ \frac{5}{x}-\frac{4}{y}=-2, x, y\neq 0$

OR

Solve for x and y : ax + by – a + b = 0, bx – ay – a – b = 0 **[AI-2004C]**

11. If (x – 2) is a factor of x3+ax2+bx+18 and a – b = 7, find a and b. **[AI-2004C]**

12. Solve the following system of linear equations : ax + by = a – b, bx – ay = a + b. **[Delhi-2004C]**

13. Solve for x and y : $\frac{x}{a}+\frac{y}{b}=2,\ ax-by=a^2-b^2$

OR

A two digit number is four times the sum of its digits and twice the product of the digits. Find the number.
[AI-2005]

14. Solve for x and y : $\frac{x}{a}-\frac{y}{b}=a-b,\ ax+by=a^3+b^3.$

OR

A number consisting of two digit, is equal to 7 times the sum of its digits. When 27 is subtracted from the number , the digit interchange places. Find the number . **[Foreign-2005]**

15. Solve for x and y : $\frac{2a}{x}+\frac{3b}{y}+1=0;\ \frac{3a}{x}-\frac{b}{y}-4=0$ **[Delhi -2005]**

16. Solve for x and y : $\frac{3a}{x}-\frac{2b}{y}+=0;\ \frac{a}{x}+\frac{3b}{y}-2=0$ **[AI-2005C]**

17. Solve for x and y ; 47x + 31y = 63, 31x + 47 = 15

OR

Solve for x and y : $\frac{ax}{b}-\frac{by}{a}=a+b; ax-by=2ab$ **[Delhi -2006]**

18. Solve the system of equations :

$\frac{bx}{a}-\frac{ay}{b}+a+b=0$ and $bx-ay+2ab=0$

OR

The sum of ht digits of a two digit number is 12. the number obtained by interchanging the two digits exceeds the given number by 18. Find the number **[AI-2006]**

19. Solve the system of equations for x ; $\frac{b^2x}{a}-\frac{a^2y}{b}=ab(a+b)$ and $b^2x-a^2y=2a^2b^2$

OR

A man sold a table and a chair together for Rs. 850 at a loss of 10% on the table and a gain of 10% on the chair. By selling them together for Rs. 950, he would have made a gain of 10% on the table and loss of 10% on the chair. Find the cost price of each . **[Foreign-2006]**

20. Solve the following equations for x and y : $mx-ny=m^2+n^2,\ x+y=2m$

OR

Abdul travelled 300 km by train and 200 km by taxi, it took him 5 hours 30 minutes. But if he travels 2650 km by train and 240 km taxi he takes 6 minutes longer. Find the speed of the train and that of the taxi **[Delhi-2006C]**

21. Solve the following equations for x and y : $\frac{a^2}{x}-\frac{b^2}{y}=0$; $\frac{a^2b}{x}+\frac{b^2a}{y}=a+bx, y\neq 0.$

OR

The sum of the numerator and the denominator of a fraction is 12. If the denominator is increased by 3, the fraction becomes $\frac{1}{2}$. Find the fraction. **[AI-2006C]**

22. Solve for x and y : $x+\frac{6}{y}=6,\ 3x-\frac{8}{y}=5$

OR

Solve for x and y : $\frac{x+1}{2}+\frac{y-1}{3}=8$; $\frac{x-1}{3}+\frac{y+1}{2}=9$ **[Delhi -2007]**

23. Solve for x and y : $8x-9y=6xy$; $10x+6y=19xy$

OR

Solve for x and y : $4x+\frac{y}{3}=\frac{8}{3}; \frac{x}{2}+\frac{3y}{4}=-\frac{5}{2}$ **[AI-2007]**

24. Find the value of k so that the following system of equations has no solution :
$3x-y=5$; $6x-2y-k=0$ **[Delhi-2008]**

25. Find the value of k so that the following system of equations has infinite solutions :
$3x-y-5=0$; $6x-2y+k=0$ **[Delhi-2008]**

26. Find the value (s) of k for which the pair of linear equations $kx+3y=k-2$ and $12x+ky=k$ has no solution **[Delhi-2008]**

27. Find the number of solutions of the following pair of linear equations :
$x+2y-8=0$; $2x+4y=16$ **[AI-2009]**

28. Write whether the following pair of linear equations is consistent or not :
$x+y=14$; $x-y=4$ **[Foreign-2009]**

29. Without drawing the graph find out whether the lines representing the following pair of liner equations intersect at a point, are parallel or coincident : $9x-10y=21$; $\frac{3}{2}x-\frac{5}{3}y=\frac{7}{2}$ **[Foreign-2009]**

30. Without drawing the graph find out whether the lines representing the following pair of linear equations intersect at a point , are parallel or coincident : $48x-7y=24$; $\frac{9}{5}x-\frac{7}{10}y=\frac{9}{10}$ **[Foreign-2009]**

31. Without drawing the graph, find out whether the lines representing the following pair of linear equations intersect at a point, are parallel or coincident : $5x + 3y - 6 = ; \frac{9}{5}x + 3y = 6$ **[Foreign-2009]**

SHORT ANSWER TYPE –II

1. Solve the following system of linear equations graphically : $2x - 3y = 1$, $3x - 4y = 1$ Does the point (3, 2) lie on any of the lies ? Write its equation **[Delhi-2003]**

2. Solve for x and y : $\frac{4}{x} + 5y = 7$, $\frac{3}{x} + 4y = 5$

OR

Father's age is three times the sum of ages of his two children . After 5 years his age will be twice the sum of age of two children. Find the age of father . **[Delhi-2003]**

3. Solve the following system of liner equations graphically : $3x - 5y = 19$, $3y - 7x + 1 = 0$ Does the point (4, 9) lie on any of the lines ? Write its equations **[AI-2003]**

4. Solve the following system of linear equations graphically : $2x + y = 10$, $4x - y = 8$. Does the point (1, -4) lie on any of the lines ? Write its equation. **[Foreign-2003]**

5. The sum of numerator and denominator of a fraction is 8. is added to both the numerator and denominator the fraction becomes 3/4. Find the fraction. **[AI-2003]**

6. Solve the following system of equations : $\frac{a}{x} - \frac{b}{y} = 0, \frac{ab^2}{x} + \frac{a^2b}{y} = a^2 + b^2$; $x, y \neq 0$.

OR

5 years hence the age of a father shall be three times the age of his son while 5 years earlier the age of the father was 7 times the age of his son. Find their present ages. **[Foreign-2003]**

7. Solve the following system of linear equations graphically : $4x - 5y - 20 = 0$, $3x + 5y - 15 = 0$. Determine the vertices of the triangle formed by the lines, representing the above equations, and the y-axis. **[Delhi-2004]**

8. Solve the following system of linear equations graphically : $5x - 6y + 30 = 0$, $5x + 4y - 20 = 0$. Also find the vertices of the triangle formed by the above two lines and x-axis. **[AI-2004]**

9. Solve the following system of linear equations graphically : $2x + y + 6 = 0$, $3x - 2y - 12 = 0$. Also find the vertices of the triangle formed by the lines representing the above equations and x-axis. **[Foreign-2004]**

10. Solve the following system of linear equations graphically : $2x + 3y = 4$, $3x - y = -5$. Shade the region bounded by the above lines and the x-axis. **[Delhi-2004C]**

11. Solve the following system of linear equations graphically : $3x + y = 1 = 0$, $2x - 3y + 8 = 0$ Shade the region bounded by the lines and the x-axis. **[AI-2004C]**

12. The monthly incomes of A and B are in the ratio of 9 : 7 and their monthly expenditures are in the ratio of 4 : 3 If each saves Rs. 1600 per month, find the monthly incomes of each. **[AI-2004C]**

13. Solve the following system of equations graphically : $x + 2y = 5$, $2x - 3y = -4$. Also find the points where the lines meet the x-axis. **[Delhi-2005]**

14. Solve the following system of equations graphically : $2x - y = 4$; $3y - x = 3$. Find the points where the lines meet the y-axis. **[AI-2005]**

15. Solve the following system of equations graphically : $3x - y = 3$, $x - 2y = -4$. Shade the are of the region bounded by the lines and x-axis. **[Delhi-2005C]**

16. Draw the graphs of the equations : $4x - y - 8 = 0$ and $2x - 3y + 6 = 0$. Also determine the vertices of the triangle formed by the lines and x-axis. **[Delhi-2006]**

17. Draw the graphs of the following equations : $3x - 4y + 6 = 0$; $3x + y - 9 = 0$. Also determine the co-ordinates of the vertices of the triangle formed by these lines and the x-axis. **[AI-2006]**

18. Draw the graphs of the equations : $4x - 3y - 6 = 0$; $x + 3y - 9 = 0$. Determine the co-ordinates of the vertices of the triangle formed by the lines and the y-axis. **[Foreign-2006]**

19. Solve the following system of linear equations graphically : $3x - 2y - 1 = 0$; $2x - 3y + 6 = 0$. Shade the region bounded by the lines and x-axis. **[Delhi-2006C]**

20. Solve the following system of equations graphically for x and y : $3x + 2y = 12$; $5x - 2y = 4$. Find the co-ordinates of the points where the lines meet the y-axis. **[AI-2006C]**

21. Solve the following system of equations graphically . $2x + 3y = 8$; $x + 4y = 9$. **[Delhi-2007]**

22. Solve the following system of linear equations graphically. $2x + 3y = 12$; $2y - 1 = x$ **[AI-2007]**

23. Represent the following system of linear equations graphically. From the graph, find the points where the lines intersect y-axis. $3x + y - = 0$; $2x - y - 5 = 0$. **[Delhi-2008]**

24. Solve for x and y ; $(a - b)\,x + (a + b)\,y = a^2 - 2ab - b^2$; $(a + b)(x + y) = a^2 + b^2$.

OR

Solve for x and y : $37x + 43y = 123$; $43x + 37y = 117$. **[AI-2008]**

25. Represent the following pair of equations graphically and write and co-ordinates of points where the lines intersect y-axis : $x + 3y = 16$; $2x - 3y = 12$. **[Foreign-2008]**

26. Solve the following pair of equations : $\frac{5}{x-1} + \frac{1}{y-2} = 2; \frac{6}{x-1} - \frac{3}{y-2} = 1$ **[Delhi-2009]**

27. Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds. They meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars ? **[Delhi-2009]**

28. Solve the following pair of equations : $\frac{10}{x+y} + \frac{2}{x-y} = 4; \frac{15}{x+y} - \frac{2}{x-y} = -2$ **[Delhi-2009]**

29. Solve for x and y : $\frac{ax}{b} - \frac{by}{a} = a + b; ax - by = 2ab.$ **[AI -2009]**

SUBJECTIVE	ANSWER KEY	EXERCISE-4 (X) CBSE

- **Short Answer Type-I**

1. k = 2 **2.** k = 3 **3.** x = -1/2, y = 2 or 42yrs, 10yrs **4.** x = 1/2, y = 1/3 or 78 **5.** x = -2, y = 1/3 or 7/11
6. x = 1/5, y = -2 or x = ab, y = ab **7.** a = 1, b = - 7 **8.** a = 0, b = 7 **9.** a = -3, b = -1
10. x = 1/2, y = 1/3 or x = 1, y = - 1 **11.** a = -2, b = - 9 **12.** x = 1, y = - 1 **13.** x = a, y = b or 36
14. $x = a^2$, $y = b^2$ or 63 **15.** x = a, y = - b **16.** x = - a, y = b **17.** x = 2, y = -1 or x = b, y = - a
18. x = - a, y = b or 57 **19.** $x = a^2$, $y = -b^2$ or cast of table = Rs. 700, cost of chair = Rs 200
20. x = m + n, y = m – n or speed of train = 100 km/h. Speed of taxi = 80 km/h **21.** $x = a^2$, $y = b^2$ or $\frac{5}{7}$
22. $x = -\frac{14}{5}$, $y = \frac{1}{13}$ or x = 7, y = 13 **23.** x = 3/2, y = 2/3 or x = 1, y = - 4 **24.** $k \neq 10$ **25.** k = - 10 **26.** $k = \pm 6$
27. Infinite number of solutions **28.** Consistent **29.** Coincident lines **30.** Parallel **31.** Unique solution.

- **Short Answer Type-II**

1. (-1, -1) ; Yes ; 3x – 4y = 1 **2.** x = 1/3, y = - or 45 years **3.** (-2, -5) ; Yes ; 3y – 7x + 1 = 0 **4.** (3, 4) ; yes ; 4x – y = 8
5. 3/5 **6.** (a, b) or 40, 10 years **7.** (0, -4), (5, 0), (0, 3) **8.** (-6, 0), (0, 5), (4,0) **9.** (-3, 0), (0, -6), (4, 0)
12. A's = Rs. 14400, B's = Rs. 11200 **13.** (5, 0), (-2, 0) **14.** (0, -4), (0, 1) **16.** (-3, 0), (2, 0), (3, 4) **17.** (-2, 0), (2, 3), (3, 0)
18. (0, 3), (3, 2), (0, -2) **19.** x = 3, y = 4 **20.** x = 2, y = 3 **21.** x = 1, y = 2 **22.** x = 3, y = 2 **23.** (0, 5) and (0, -5)
24. x = a + b, $y = \frac{-2ab}{a+b}$ or x = 1, y = 2 **25.** (0, 2) and (0, -4) **26.** x = 4, y = 5 **27.** 60 km/h ; 40 km/h
28. x = 3 ; y = 2 **29.** x = b, y = - a

EXERCISE -5 (FOR OLYMPIADS)

Choose The Correct One

1. The number of solutions of the equation $2x + y = 40$, where both x and y are positive integers and $x \leq y$ is :
(A) 7 (B) 13 (C) 14 (D) 18

2. A confused bank teller transposed the rupees and paise when he cashed a cherub for Mansi, giving her rupees instead of paisa and paise instead of rupees. After buying a toffee for 50 paise, Mansi noticed that she was left with exactly three times as much as the amount on the cheque. Which of the following is a valid statement about the cheque amount ?
(A) Over Rs. 4 but less than Rs. 5 (B) Over Rs. 13 but less than Rs. 14
(C) Over Rs. 7 but less than Rs. 8 (D) Over Rs. 18 but less than Rs. 19

3. John inherited \$25000 and invested part of it in a money market account, part in municipal bounds, and part in a mutual fund. After one year, he received a total of \$ 1620 in simple interest from the three investments. The money market paid 6% annually, the bonds paid 7% annually, and the mutual funds paid 8% annually. There was \$ 6000 more invested in the bonds than the mutual funds. The amount John invested in each category are in the ratio :
(A) 15 : 8 : 2 (B) 11 : 13 : 1 (C) 2 : 2 : 1 (D) None of these

4. Which one of the following conditions must p,q and r satisfy so that the following system of linear simultaneous equations has at least one solution , such that $p + q + r \neq 0$?

$$x + 2y - 3z = p \; ; \; 2x + 6y - 11z = q \; ; \; x - 2y + 7z = r$$

(A) $5p - 2q - r = 0$ (B) $5p + 2q + r = 0$ (C) $5p + 2q - r = 0$ (D) $5p - 2q + r = 0$

5. If x and y are integers, then the equation $5x + 19y = 64$ has :
(A) No solution for $x < 300$ and $y < 0$ (B) No solution for $x > 250$ and $y > -100$
(C) A solution for $250 < x < 300$ (D) A solution for $-59 < y < -56$

6. The number of solutions of the equation $2x + y = 40$, where both x and y are positive integers and $x \leq y$ is :
(A) 7 (B) 13 (C) 14 (D) 18

7. Study the question and statements given below : Decide whether any information provided in the statement (s) is redundant and / or can be dispensed with, to answer it.
If 7 is added to numerator and denominator each of fraction a/b. will the new fraction be less than the original one ? (Assume both a and b to be positive)
Statement-I : $a = 73$, $b = 103$
Statement-II : The average of a and b is less than b.
Statement-III : $a - 5$ is greater than $b - 5$.
(A) II and either I or III (B) Only I or III (C) Any two of them (D) Any one of them

8. A cyclist drove 1 km, with the wind in his back , in 3 min and drove the same way back, against the wind in 4 min. If we assume that the cyclist always puts constant force on the pedals, how much time would it take to drive 1 km without wind ?
(A) $2\frac{1}{3}$ min. (B) $3\frac{3}{7}$ min. (C) $2\frac{3}{7}$ min. (D) $3\frac{7}{12}$ min.

9. A person buys 18 local tickets for Rs. 110. Each first class ticket costs Rs. 10 and each second class ticket costs Rs. 3. What will another lot of 18 tickets in which the number of first class and second class tickets are interchanged cost ?
(A) Rs. 112 (B) Rs. 118 (C) Rs. 121 (D) Rs. 124

10. Rajesh walks to and fro to a shopping mall. He spends 30 min. shopping. If he walks at a speed of 10 km/h, he returns to home at 19:00h. It he walks at 15 km/h. he returns at 18:30 h. How fast must he walk in order to return home at 18:15 h ?
(A) 17 km/h (B) 17.5 km/h (C) 18 km/h (D) 20 km/h

11. A single reservoir supplies the petrol to the whole city, while the reservoir is fed by a single pipeline filling the reservoir with the stream of uniform volume. When the reservoir is full and if 40000 liters of petrol is used daily, the supply fails in 90 days. If 32000 liters of petrol is used daily, the supply fails in 60 days. How much petrol can be used daily without the supply ever failing ?
(A) 64000 litres (B) 56000 litres (C) 78000 litres (D) 60000 litres

12. Two horses start trotting towards each other, one from A to B and another from B to A . They cross each other after one hour and the first horse reaches B, 5/6 hours before the second hoarse reaches A. If the distance between A and B is 50 km. What is the speed of the slower hours ?
(A) 30 km/h (B) 15 km/h (C) 25 km/h (D) 20 km/h

13. A man row downstream at 12 km/h and upstream at 8 km/h. What is the speed of man in still water ?
(A) 12 km/h (B) 10 km/h (C) 8 km/h (D) 9 km/h

14. A motor boat takes 12 hours to go downstream and it takes 24 hours to return the same distance. What is the time taken by boat in still water ?
(A) 15 h (B) 16 h (C) 8 h (D) 20 h

15. Equation $xy^2 + xy^2 = 2xy$, x, $y \neq 0$ is
(A) Linear (B) Quadratic (C) Cubic (D) Not an equation

16. Sum of two integers is 88. If the greater is divided by the smaller, the quotient is 5 and the remainder is 10. the greater integer is :
(A) 13 (B) 75 (C) 65 (D) 23

17. The length of the sides of a triangle are $3x+2y, 4x+\frac{4}{3}y$ and $3(x+1)+\frac{3}{2}(y-1)$. If the triangle is equilateral , then its side is
(A) 8 (B) 10 (C) 12 (D) 16

18. The largest angle of a triangle is twice the sum of the other two. The smaller angle is one fourth of the largest. The largest angle is :
(A) 90^0 (B) 60^0 (C) 120^0 (D) None of these

19. In town, $\frac{2}{3}$ of men are married to $\frac{3}{7}$ of the women . In the town total population is more than 1000. If all marriages happen within the town. The smallest possible number of total population is (assume there are only adults in the town):
(A) 1012 (B) 1035 (C) 1058 (D) None of these

20. The solution of the equations : $\frac{x}{4}=\frac{y}{3}=\frac{z}{2}, \quad 7x+8y+5z=62$ is :
(A) (4, 3, 2) (B) (2, 3, 4) (C) (3, 4, 2) (D) (4, 2, 3)

21. If $\frac{1}{3}(x+y)2z=21, \ 3x-\frac{1}{2}(y+z)=65, \quad x+\frac{1}{2}(x+y-z)=38$, then its solution is :
(A) (24, 9, 5) (B) (2, 9, 5) (C) (4, 9, 5) (D) (5, 24, 9)

22. The solution of the equations : $\frac{xy}{y-x}=110, \ \frac{yz}{z-y}=132, \ \frac{zx}{z+x}=\frac{60}{11}$ is :
(A) (12, 11, 10) (B) (10, 11, 12) (C) (11, 10, 12) (D) (12, 10, 11)

23. Four men earn as much in a day as 7 women. 1 women earns as much as 2 boys. If 6 men, 10 women and 14 boys work together for 8 days to earn Rs. 2200, then what will be the earning of 8 men and 6 women working together is for 10 days ?
(A) Rs. 2000 (B) Rs. 1800 (C) Rs. 2400 (D) None of these

24. The point of intersection of the straight lines $2x - y + 3 = 0$, $3x - 7y + 10 = 0$ lies in :
(A) I quadrant (B) II quadrant (C) III quadrant (D) IV quadrant

25. A right-angled triangle is formed by the straight line : $4x + 3y = 12$ with both the axis. Then length of perpendicular from the origin to the hypotenuse is :
(A) 3.5 units (B) 2.4 units (C) 4.2 units (D) None of these

OBJECTIVE							ANSWER KEY							EXERCISE - 5
Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	D	A	A	C	B	C	B	D	D	B	D	B	B	A
Que.	16	17	18	19	20	21	22	23	24	25					
Ans.	B	C	C	A	A	A	B	A	B	B					

COMPETITION WINDOW

LINEAR IN EQUATIONS

Inequations : A statement involving variable (s) and the sign of inequality viz, $<, >, \leq$ or $\geq$ is called an inequation or an inequality.

An inequation may contain one or more variables. Also, is may be linear or quadratic or cubic etc.

E.g. (i) $3x - 2 < 0$ (ii) $2x + 3y < 1$ (iii) $x2 - 5x + 4 \leq 0$

Linear Inequations In One Variable : Let a be a non-zero real number and x be a variable. Then inequations of the form $ax + b < 0$, $ax + b \leq 0$, $ax + b > 0$ and $ax + b \geq 0$ are known as linear equations in one variable x.

E.g. $9x - 15 > 0$, $5x - 4 \geq 0$, $3x + 2 < 0$, $2x - 3 \leq 0$

Solving Linear Inequations In One Variable : In the process of solving an inequation, we use mathematical simplifications which are governed by the following rules :

Rule-I : Same number may be added (or subtracted from) both sides of an inequation without changing the sign of inequality

Rule-II: Both sides of an inequation can be multiplied (or divided) by the same positive real number without changing the sign of inequality. However the sign of inequality is reversed when both sides of an inequation are multiplied (or divided) by a negative number.

Rule-III : Any tern of an inequation may be taken to the other side with it's sign changed without affecting the sign of inequality

Ex. Solve : $5x - 3 < 3x + 1$, when (i) x is a real number (ii) x is an integer (iii) x is a natural number.

Sol. We have

$5x - 3 < 3x + 1$

$\Rightarrow$ $5x - 3x < 3x + 1$ [Transposing 3x on LHS and – 3 on RHS]

$\Rightarrow$ $2x < 4 \Rightarrow \frac{2x}{2} < \frac{4}{2} \Rightarrow x < 2$

(i) If $x \in R$, then $x < 2 \Rightarrow x \in (-\infty, 2)$

(ii) If $x \in Z$, then $x < 2 \Rightarrow x = 1, 0, -1, -2, -3, -4, \ldots\ldots\ldots$

(iii) If $x \in N$, then $x < 2 \Rightarrow x = 1$

Ex. Solve ; $\frac{2x+4}{x-1} \geq 5$

Sol. We have,

$\frac{2x+4}{x-1} \geq 5 \quad \Rightarrow \quad \frac{2x+4}{x-1} - 5 \geq 0 \Rightarrow \frac{-3x+9}{x-1} \geq 0$

$\Rightarrow$ $\frac{3x-9}{x-1} \leq 0$ [Multiplying both sides by – 1]

$\Rightarrow$ $\frac{x-3}{x-1} \leq 0$ [Dividing both sides by 3]

$\Rightarrow$ $1 < x \leq 3 \Rightarrow x \in (1, \ 3)$

EXERCISE-6 (FOR IIT-JEE/AIEEE)

Choose The Correct One

1. Solve : $3x-7>x+1, x \in R$:

(A) $(4,\infty)$ (B) $[4,\infty)$

(C) $(-\infty,\ 4]$ (D) $(-\infty,\ 4]$

2. Solve : $\frac{x}{5}<\frac{3x-2}{4}-\frac{5x-3}{5}, x \in R$:

(A) $(2/9,\ \infty)$ (B) $[2/9,\ \infty)$

(C) $(-\infty,\ 2.9)$ (D) $(-\infty,\ 2.9)$

3. Solve : $\frac{2(x-1)}{5}\le\frac{3(2+x)}{7}, x \in R$:

(A) $(44,\ \infty)$ (B) $[44,\ \infty)$

(C) $[-44,\ \infty)$ (D) $(-44,\ \infty)$

4. Solve : $\frac{5x}{2}+\frac{3x}{4}\ge\frac{39}{4}, x \in R$:

(A) $(-3,\ \infty)$ (B) $(3,\ \infty)$

(C) $(-\infty,\ 3)$ (D) $[3,\ \infty)$

5. Solve : $\frac{x-1}{3}+4<\frac{x-5}{5}-2, x \in R$:

(A) $(50,\ \infty)$ (B) $(-\infty,\ -50)$

(C) $[50,\ \infty)$ (D) None of these

6. Solve : $\frac{2x+3}{4}-3<\frac{x-4}{3}-2, x \in R$:

(A) $(13/2,\ \infty)$ (B) $(-\infty,\ -13/2)$

(C) $(-13/2,\ \infty)$ (D) $[13/2,\ \infty)$

7. Solve : $\frac{5-2x}{3}<\frac{x}{6}-5, x \in R$:

(A) $(8,\ \infty)$ (B) $[8,\ \infty)$ (C) $(-\infty,\ -8)$ (D) $(-\infty,\ 8)$

8. Solve : $\frac{4+2x}{3}\ge\frac{x}{2}-3, x \in R$:

(A) $(26,\ \infty)$ (B) $(-\infty,\ 26]$ (C) $[-26,\ \infty)$ (D) $(-\infty,\ -26)$

9. Solve : $\frac{2x+3}{5}-2<\frac{3(x-2)}{5}, x \in R$:

(A) $(-1,\ \infty)$ (B) $[1,\ \infty)$ (C) $(-\infty,\ -1)$ (D) $(-\infty,\ 1)$

10. Solve : $x-2\le\frac{5x+8}{3}, x \in R$

(A) $[-7,\ \infty)$ (B) $(7,\ \infty)$ (C) $(-\infty,\ 7)$ (D) $(-\infty,\ 7)$

11. Solve : $\frac{6x-5}{4x+1}<0, x \in R$:

(A) $(-1/4, 5/6)$ (B) $[-1/4, 5/6]$ (C) $(-\infty, -1/4)$ (D) $(5/6, \infty)$

12. Solve : $\dfrac{2x-3}{3x-7} > 0, x \in R$

(A) $[3/2, 7/3]$ (B) $(3/2, 7/3)$

(C) $(-\infty, 3/2) \cup (7/3, \infty)$ (D) None of these

13. Solve : $\dfrac{3}{x-2} < 1, x \in R$:

(A) $(2, 5)$ (B) $[-\infty, 2) \cup (5, \infty)$

(C) $(-\infty, -2) \cup (5, \infty)$ (D) None of these

14. Solve : $\dfrac{1}{x-1} \le 2, x \in R$:

(A) $(-1, 3/2]$ (B) $(-\infty, -1) \cup (3/2, \infty)$

(C) $(-1, 3/2)$ (D) $(-\infty, 1) \cup [3/2, \infty)$

15. Solve : $\dfrac{4x+3}{2x-5} < 6, x \in R$

(A) $(5/2, 33/8)$ (B) $(-\infty, -5) \cup (4, \infty)$

(C) $(-5/2, 33/8)$ (D) $(-\infty, 5/2) \cup (33/8, \infty)$

16. Solve : $\dfrac{5x-6}{x+6} < 1, x \in R$:

(A) $(-6, -3)$ (B) $(6, \infty)$ (C) $(-6, 3)$ (D) None of these

17. Solve : $\dfrac{5x+8}{4-x} < 2, x \in R$:

(A) $[0,, 4)$ (B) $[4, \infty)$ (C) $(-\infty, 4)$ (D) $(-\infty, 0) \cup (4, \infty)$

18. Solve : $\dfrac{x-1}{x+3} > 2, x \in R$:

(A) $(7, 3)$ (B) $[7, 3]$ (C) $(-7, -3)$ (D) $[-7, -3]$

19. Solve : $\dfrac{7x-5}{8x+3} > 4, x \in R$:

(A) $(17/25, 3/8)$ (B) $(17/25, 3/8]$ (C) $(-17/25, -3/8)$ (D) $[17/25, 3/8]$

20. $\dfrac{2x-3}{3x-7} > 0, x \in R$

(A) $(-5, 5)$ (B) $[-5, 5]$ (C) $(-\infty, -5) \cup (5, \infty)$ (D) None of these

OBJECTIVE							ANSWER KEY							EXERCISE -6
Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A	C	C	D	B	B	A	C	A	A	A	C	B	D	D
Que.	16	17	18	19	20										
Ans.	C	D	C	C	C										