QUADRATIC EQUATION

BASIC CONCEPT

- Every algebraic polynomial of second degree is called a quadratic polynomial.

  For Example :

  (i) $3x^2 + 5x + 7$

  (ii) $8x^2 - 6x$

  (iii) $5x^2 - 7$

  (iv) $\sqrt{2}\,x^2 + 6x - \sqrt{3}$ and so on.

- The general form of quadratic polynomial is $ax^2 + bx + c$; where a, b, c are real numbers, $a \neq 0$ and x is variable.

  For a particular quadratic polynomial the values of a, b, c are constant and for this reason a, b and c are also called real constants. For example, in quadratic polynomial $3x^2 - 5x + 8$; 3, – 5 and 8 are constant where as x is variable.

Value of a Quadratic Polynomial

The value of a quadratic polynomial $ax^2 + bx + c$

(i) at $x = \alpha$ is $= a(\alpha)^2 + b(\alpha) + c = a\alpha^2 + b\alpha + c$

(ii) at $x = \beta$ is $a\beta^2 + b\beta + c$

(iii) at $x = 5$ is $= a(5)^2 + b(5) + c = 25a^2 + 5a + c$ and so on.

In the same way :

(i) Value of $5x^2 - 3x + 4$ at $x = 2$ is $= 5(2)^2 - 3(2) + 4$

$$= 20 - 6 + 4 = 18$$

(ii) Value of $x^2 - 8x - 15$ at $x = -1$ is $= (-1)^2 - 8(-1) - 15$

$$= 1 + 8 - 15 = -6$$

(iii) Value of $7x^2 - 4$ at $x = \frac{2}{3}$ is $= 7\left(\frac{2}{3}\right)^2 - 4$

$$= 7 \times \frac{4}{9} - 4 = \frac{28-36}{9} = \frac{8}{9}$$ and so on.

Zeros of a Quadratic Polynomial

The value of the polynomial $x^2 - 7x + 10$ at :

(i) $x = 1$ is $(1)^2 - 7 \times 1 + 10 = 1 - 7 + 10 = 4$

(ii) $x = 2$ is $(2)^2 - 7 \times 2 + 10 = 4 - 14 + 10 = 0$

(iii) $x = 3$ is $(3)^2 - 7 \times 3 + 10 = 9 - 21 + 10 = -2$

(iv) $x = 5$ is $(5)^2 - 7 \times 5 + 10 = 25 - 35 + 10 = 0$

and so on.

It is observed here that for x = 2 and x = 5; the value of polynomial $x^2 - 7x + 10$ is zero. These two values of x are called zeros of the polynomial.

Thus, if for $x = \alpha$, where α is a real number, the value of given quadratic polynomial is zero; the real number α is called zero of the quadratic polynomial.

❖ EXAMPLES ❖

Ex.1 Show that :

(i) x = 3 is a zero of quadratic polynomial $x^2 - 2x - 3$.

(ii) x = – 2 is a zero of quadratic polynomial $3x^2 + 7x + 2$.

(iii) x = 4 is not a zero of quadratic polynomial $2x^2 - 7x - 5$.

Sol. (i) The value of $x^2 - 2x - 3$ at x = 3 is $(3)^2 - 2 \times 3 - 3 = 9 - 6 - 3 = 0$

$\Rightarrow$ x = 3 is a zero of quadratic polynomial $x^2 - 2x - 3$.

(ii) The value of $3x^2 + 7x + 2$ at x = – 2 is

$3(-2)^2 + 7(-2) + 2 = 12 - 14 + 2 = 0$

$\Rightarrow$ x = – 2 is a zero of quadratic polynomial $3x^2 + 7x + 2$

(iii) The value of $2x^2 - 7x - 5$ at x = 4 is $2(4)^2 - 7(4) - 5 = 32 - 28 - 5 = -1 \neq 0$

$\Rightarrow$ x = 4 is not a zero of quadratic polynomial $2x^2 - 7x - 5$.

Ex.2 Find the value of m, if x = 2 is a zero of quadratic polynomial $3x^2 - mx + 4$.

Sol. Since, x = 2 is a zero of $3x^2 - mx + 4$

$\Rightarrow 3(2)^2 - m \times 2 + 4 = 0$

$\Rightarrow 12 - 2m + 4 = 0$, i.e., m = 8.

Quadratic Equation & Its Roots

Since, $ax^2 + bx + c$ is a quadratic polynomial, $ax^2 + bx + c = 0$ is called a quadratic equation.

(i) $-x^2 - 7x + 2 = 0$ is a quadratic equation, as $-x^2 - 7x + 2$ is a quadratic polynomial.

(ii) $5x^2 - 7x = 0$ is a quadratic equation,

(iii) $5x^2 + 2 = 0$ is a quadratic equation, but

(iv) $-7x + 2 = 0$ is not a quadratic equation.

❖ EXAMPLES ❖

Ex.3 Which of the following are quadratic equations, give reason :

(i) $x^2 - 8x + 6 = 0$

(ii) $3x^2 - 4 = 0$

(iii) $2x + \frac{5}{x} = x^2$

(iv) $x^2 + \frac{2}{x^2} = 3$

Sol. (i) Since, $x^2 - 8x + 6$ is a quadratic polynomial

$\Rightarrow x^2 - 8x + 6 = 0$ is a quadratic equation.

(ii) $3x^2 - 4 = 0$ is a quadratic equation.

(iii) $2x + \frac{5}{x} = x^3$

$\Rightarrow 2x^2 + 5 = x^3$

$\Rightarrow x^3 - 2x^2 - 5 = 0$; which is not a quadratic equation.

(iv) $x^2 + \frac{2}{x^2} = 3$

$\Rightarrow x^4 + 2 = 2x^2$

$\Rightarrow x^4 - 2x^2 + 2 = 0$; which is not a quadratic equation.

Ex.4 In each of the following, determine whether the given values are solutions (roots) of the equation or not :

(i) $3x^2 - 2x - 1 = 0$; $x = 1$

(ii) $x^2 + 6x + 5 = 0$; $x = -1$, $x = -5$

(iii) $x^2 + \sqrt{2}x - 4 = 0$; $x = \sqrt{2}$, $x = -2\sqrt{2}$

Sol. (i) $\because$ Value of $3x^2 - 2x - 1$ at $x = 1$ is $3(1)^2 - 2(1) - 1 = 3 - 2 - 1 = 0$

$\therefore$ $x = 1$ is a solution of the given equation.

Alternative method :

Substituting $x = 1$ in L.H.S. of given equation, we get

L.H.S. $= 3(1)^2 - 2 \times 1 - 1 = 3 - 2 - 1 = 0 =$ R.H.S.

L.H.S. = R.H.S. $\Rightarrow x = 1$ is a solution of the given equation.

(ii) For $x = -1$, L.H.S. $= (-1)^2 + 6(-1) + 5 = 1 - 6 + 5 = 0 =$ R.H.S.

$\Rightarrow x = -1$ is a solution of the given equation For $x = -5$, L.H.S. $= (-5)^2 + 6(-5) + 5 = 25 - 30 + 5 = 0 =$ R.H.S.

$\Rightarrow x = -5$ is a solution of the given equation.

(iii) For $x = \sqrt{2}$, L.H.S. $= x^2 + \sqrt{2}x - 4 = (\sqrt{2})^2 + \sqrt{2}(\sqrt{2}) - 4 = 2 + 2 - 4 = 0 =$ R.H.S.

$\therefore$ $x = \sqrt{2}$ is a solution of the given equation

For $x = -2\sqrt{2}$,

L.H.S. $= (-2\sqrt{2})^2 + \sqrt{2} \times -2\sqrt{2} - 4$

$= 4 \times 2 - 2 \times 2 - 4 = 0$ R.H.S.

$\therefore$ $x = -2\sqrt{2}$ is a solution of the given equation.

Solving a Quadratic Equation by Factorisation

Since, $3x^2 - 5x + 2$ is a quadratic polynomial; $3x^2 - 5x + 2 = 0$ is a quadratic equation.

Also, $3x^2 - 5x + 2 = 3x^2 - 3x - 2x + 2$ [Factorising]

$= 3x(x - 1) - 2(x - 1)$

$= (x - 1)(3x - 2)$

In the same way :

$3x^2 - 5x + 2 = 0 \Rightarrow 3x^2 - 3x - 2x + 2 = 0$

[Factorising L.H.S.]

$\Rightarrow (x - 1)(3x - 2) = 0$

i.e., $x - 1 = 0$ or $3x - 2 = 0$

$\Rightarrow$ $x = 1$ or $x = \frac{2}{3}$;

which is the solution of given quadratic equation.

In order to solve the given Quadratic Equation :

1. Clear the fractions and brackets, if given.
2. By transfering each term to the left hand side; express the given equation as ; $ax^2 + bx + c = 0$ or $a + bx + cx^2 = 0$
3. Factorise left hand side of the equation obtained (**the right hand side being zero**).
4. By putting each factor equal to zero; solve it.

❖ EXAMPLES ❖

Ex.5 Solve :

(i) $x^2 + 3x - 18 = 0$

(ii) $(x - 4)(5x + 2) = 0$

(iii) $2x^2 + ax - a^2 = 0$; where 'a' is a real number.

Sol. (i) $x^2 + 3x - 18 = 0 \Rightarrow x^2 + 6x - 3x - 18 = 0$

$\Rightarrow x(x + 6) - 3(x + 6) = 0$

i.e., $(x + 6)(x - 3) = 0 \Rightarrow x + 6 = 0$ or $x - 3 = 0$

$\Rightarrow x = -6$ or $x = 3$

$\therefore$ Roots of the given equation are : -6 and 3

(ii) $(x - 4)(5x + 2) = 0 \Rightarrow x - 4 = 0$ or $5x + 2 = 0$

$\Rightarrow x = 4$ or $x = -\frac{2}{5}$

(iii) $2x^2 + ax - a^2 = 0 \Rightarrow 2x^2 + 2ax - ax - a^2 = 0$

$\Rightarrow 2x(x + a) - a(x + a) = 0$

i.e., $(x + a)(2x - a) = 0 \Rightarrow x + a = 0$ or $2x - a = 0$

$\Rightarrow x = -a$ or $x = \frac{a}{2}$

Ex.6 Solve the following quadratic equations :

(i) $x^2 + 5x = 0$ (ii) $x^2 = 3x$ (iii) $x^2 = 4$

Sol. (i) $x^2 + 5x = 0$

$\Rightarrow x(x + 5) = 0$

$\Rightarrow x = 0$ or $x + 5 = 0$

$\Rightarrow x = 0$ or $x = -5$

(ii) $x^2 = 3x$

$\Rightarrow x^2 - 3x = 0$

$\Rightarrow x(x - 3) = 0$

$\Rightarrow x = 0$ or $x = 3$

(iii) $x^2 = 4 \Rightarrow x = \pm 2$

Ex.7 Solve the following quadratic equations :

(i) $7x^2 = 8 - 10x$

(ii) $3(x^2 - 4) = 5x$

(iii) $x(x + 1) + (x + 2)(x + 3) = 42$

Sol. (i) $7x^2 = 8 - 10x$

$\Rightarrow 7x^2 + 10x - 8 = 0$

$\Rightarrow 7x^2 + 14x - 4x - 8 = 0$

$\Rightarrow 7x(x + 2) - 4(x + 2) = 0$

$\Rightarrow (x + 2)(7x - 4) = 0$

$\Rightarrow x + 2 = 0$ or $7x - 4 = 0$

$\Rightarrow x = -2$ or $x = \frac{4}{7}$

(ii) $3(x^2 - 4) = 5x$

$\Rightarrow 3x^2 - 5x - 12 = 0$

$\Rightarrow 3x^2 - 9x + 4x - 12 = 0$

$\Rightarrow 3x(x - 3) + 4(x - 3) = 0$

$\Rightarrow (x - 3)(3x + 4) = 0$

$\Rightarrow x - 3 = 0$ or $3x + 4 = 0$

$\Rightarrow x = 3$ or $x = -\frac{4}{3}$

(iii) $x(x + 1) + (x + 2)(x + 3) = 42$

$\Rightarrow x^2 + x + x^2 + 3x + 2x + 6 - 42 = 0$

$\Rightarrow 2x^2 + 6x - 36 = 0$

$\Rightarrow x^2 + 3x - 18 = 0$

$\Rightarrow x^2 + 6x - 3x - 18 = 0$

$\Rightarrow x(x + 6) - 3(x + 6) = 0$

$\Rightarrow (x + 6)(x - 3) = 0$

$\Rightarrow x = -6$ or $x = 3$

Ex.8 Solve for x : $12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$

Sol. Given equation is :

$12abx^2 - 9a^2x + 8b^2x - 6ab = 0$

$\Rightarrow 3ax(4bx - 3a) + 2b(4bx - 3a) = 0$

$\Rightarrow (4bx - 3a)(3ax + 2b) = 0$

$\Rightarrow$ $4bx - 3a = 0$ or $3ax + 2b = 0$

$\Rightarrow$ $x = \frac{3a}{4b}$ or $x = -\frac{2b}{3a}$

Solving a Quadratic Equation by Completing the Square

Every quadratic equation can be converted in the form :

$(x + a)^2 - b^2 = 0$ or $(x - a)^2 - b^2 = 0$.

Steps :

1. Bring, if required, all the term of the quadratic equation to the left hand side.
2. Express the terms containing x as $x^2 + 2xy$ or $x^2 - 2xy$.
3. Add and subtract y^2 to get $x^2 + 2xy + y^2 - y^2$ or $x^2 - 2xy + y^2 - y^2$; which gives $(x + y)^2 - y^2$ or $(x - y)^2 - y^2$.

Thus,

(i) $x^2 + 8x = 0 \Rightarrow x^2 + 2x \times 4 = 0$

$\Rightarrow x^2 + 2x \times 4 + 4^2 - 4^2 = 0$

$\Rightarrow (x + 4)^2 - 16 = 0$

(ii) $x^2 - 8x = 0 \Rightarrow x^2 - 2 \times x \times 4 = 0$

$\Rightarrow x^2 - 2 \times x \times 4 + 4^2 - 4^2 = 0$

$\Rightarrow (x - 4)^2 - 16 = 0$

❖ EXAMPLES ❖

Ex.9 Find the roots of the following quadratic equations (if they exist) by the method of completing the square.

(i) $2x^2 - 7x + 3 = 0$

(ii) $4x^2 + 4\sqrt{3}x + 3 = 0$

(iii) $2x^2 + x + 4 = 0$

Sol. (i) $2x^2 - 7x + 3 = 0 \Rightarrow x^2 - \frac{7}{2}x + \frac{3}{2} = 0$

[Dividing each term by 2]

$\Rightarrow x^2 - 2 \times x \times \frac{7}{4} + \frac{3}{2} = 0$

$\Rightarrow x^2 - 2 \times x \times \frac{7}{4} + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 + \frac{3}{2} = 0$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 - \frac{49}{16} + \frac{3}{2} = 0$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 - \left(\frac{49 - 24}{16}\right) = 0$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 - \frac{25}{16} = 0$

i.e., $\left(x - \frac{7}{4}\right)^2 = \frac{25}{16} \Rightarrow x - \frac{7}{4} = \pm \frac{5}{4}$

i.e., $x - \frac{7}{4} = \frac{5}{4}$ or $x - \frac{7}{4} = -\frac{5}{4}$

$\Rightarrow x = \frac{7}{4} + \frac{5}{4}$ or $x = \frac{7}{4} - \frac{5}{4}$

$\Rightarrow x = 3$ or $x = \frac{1}{2}$

(ii) $4x^2 + 4\sqrt{3}x + 3 = 0 \Rightarrow x^2 + \sqrt{3}x + \frac{3}{4} = 0$

i.e., $x^2 + 2 \times x \times \frac{\sqrt{3}}{2} + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{3}{4} = 0$

$\Rightarrow \left(x + \frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4} + \frac{3}{4} = 0$ i.e., $\left(x + \frac{\sqrt{3}}{2}\right)^2 = 0$

$\Rightarrow x + \frac{\sqrt{3}}{2} = 0$ and $x = \frac{-\sqrt{3}}{2}$

$\therefore$ Roots are : $\frac{-\sqrt{3}}{2}$ and $\frac{-\sqrt{3}}{2}$

(iii) $2x^2 + x + 4 = 0 \Rightarrow x^2 + \frac{x}{2} + 2 = 0$

i.e., $x^2 + 2 \times x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 + 2 = 0$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 - \frac{1}{16} + 2 = 0$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 + \frac{31}{16} = 0 \quad \left[-\frac{1}{16} + 2 = \frac{-1+32}{16} = \frac{31}{16}\right]$

i.e., $\left(x + \frac{1}{4}\right)^2 = -\frac{31}{16}$

This is not possible as the square of a real number can not be negative.

Solving a Quadratic Equation by Using Quadratic Formula

Hindu Method (SriDharacharya Method):

By completing the perfect square as

$ax^2 + bx + c = 0 \quad \Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

Adding and substracting $\left(\frac{b}{2a}\right)^2$

$\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2}\right] = 0$

Which gives, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Hence the Quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ has two roots, given by

$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

Note : Every quadratic equation has two and only two roots.

❖ EXAMPLES ❖

Ex.10 Solve the following quadratic equations by using quadratic formula :

(i) $x^2 - 7x + 12 = 0$ (ii) $3x^2 - x - 10 = 0$

Sol. (i) Comparing the given equation $x^2 - 7x + 12 = 0$ with standard quadratic equation $ax^2 + bx + c = 0$; we get : $a = 1$, $b = -7$ and $c = 12$

$\therefore\ b^2 - 4ac = (-7)^2 - 4 \times 1 \times 12 = 49 - 48 = 1$

and $\sqrt{b^2 - 4ac} = \sqrt{1} = 1$

$\because\ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \frac{7 \pm 1}{2 \times 1} = \frac{7+1}{2}$ or $\frac{7-1}{2} = 4$ or 3

(ii) Comparing the given equation $3x^2 - x - 10 = 0$ with equation $ax^2 + bx + c = 0$; we get : $a = 3$, $b = -1$ and $c = -10$

$\therefore\ b^2 - 4ac = (-1)^2 - 4 \times 3 \times -10 = 1 + 120 = 121$ and $\sqrt{b^2 - 4ac} = \sqrt{121} = 11$

Hence, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \frac{1 \pm 11}{2} = \frac{1+11}{2}$ or $\frac{1-11}{2} = 6$ or -5

Ex.11 For a quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, prove that : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sol. $ax^2 + bx + c = 0$

$\Rightarrow 4a^2x^2 + 4abx + 4ac = 0$ [Multiplying by '4a']

$\Rightarrow (2ax)^2 + 2 \times 2ax \times b + b^2 - b^2 + 4ac = 0$

$\Rightarrow (2ax + b)^2 - b^2 + 4ac = 0$

$\Rightarrow (2ax + b)^2 = b^2 - 4ac$

$\Rightarrow 2ax + b = \pm \sqrt{b^2 - 4ac}$

$\Rightarrow 2ax = -b \pm \sqrt{b^2 - 4ac}$

$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Ex.12 Solve, by using quadratic formula, each of the following equations :

(i) $2x^2 + 5\sqrt{3}x + 6 = 0$ (ii) $3x^2 + 2\sqrt{5}x - 5 = 0$

Sol. (i) Comparing $2x^2 + 5\sqrt{3}x + 6 = 0$ with $ax^2 + bx + c = 0$, we get :

$a = 2$, $b = 5\sqrt{3}$ and $c = 6$

$b^2 - 4ac = (5\sqrt{3})^2 - 4 \times 2 \times 6 = 25 \times 3 - 48 = 27$

$\sqrt{b^2 - 4ac} = \sqrt{27} = \sqrt{3 \times 3 \times 3} = 3\sqrt{3}$

$\therefore\ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5\sqrt{3} \pm 3\sqrt{3}}{2 \times 4}$

$= \frac{-5\sqrt{3} + 3\sqrt{3}}{4}$ or $\frac{-5\sqrt{3} - 3\sqrt{3}}{4}$

$= \frac{-2\sqrt{3}}{4}$ or $\frac{-8\sqrt{3}}{4} = -\frac{\sqrt{3}}{2}$ or $-2\sqrt{3}$

(ii) Comparing $3x^2 + 2\sqrt{5}x - 5 = 0$ with $ax^2 + bx + c = 0$, we get :

$a = 3$, $b = 2\sqrt{5}$ and $c = -5$

$b^2 - 4ac = (2\sqrt{5})^2 - 4 \times 3 \times - 5 = 4 \times 5 + 60 = 80$

$\sqrt{b^2 - 4ac} = \sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5}$

$\therefore\ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2\sqrt{5} \pm 4\sqrt{5}}{2 \times 3}$

$= \frac{-2\sqrt{5} + 4\sqrt{5}}{6}$ or $\frac{-2\sqrt{5} - 4\sqrt{5}}{6}$

$= \frac{2\sqrt{5}}{6}$ or $\frac{-6\sqrt{5}}{6} = \frac{\sqrt{5}}{3}$ or $-\sqrt{5}$

Ex.13 Using the quadratic formula, solve the equation :
$a^2b^2x^2 - (4b^2 - 3a^4)x - 12a^2b^2 = 0$

Sol. Comparing given equation with $Ax^2 + Bx + C = 0$, we get :
$A = a^2b^2$, $B = -(4b^4 - 3a^4)$ and $C = -12\ a^2b^2$

$\therefore\ B^2 - 4AC = (4b^4 - 3a^4)^2 - 4 \times a^2b^2 \times - 12a^2b^2$

$= 16b^8 + 9a^8 - 24a^4b^4 + 48a^4b^4$

$= 16b^8 + 9a^8 + 24a^4b^4 = (4b^4 + 3a^4)^2$

$\sqrt{B^2 - 4AC} = 4b^4 + 3a^4$

$\therefore\ x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$= \frac{(4b^4 - 3a^4) \pm (4b^2 + 3a^4)}{2 \times a^2b^2}$

$= \frac{4b^4 - 3a^4 + 4b^4 + 3a^4}{2a^2b^2}$ or $\frac{4b^4 - 3a^4 - 4b^4 - 3a^4}{2a^2b^2}$

$= \frac{8b^4}{2a^2b^2}$ or $\frac{-6a^4}{2a^2b^2} = \frac{4b^2}{a^2}$ or $\frac{-3a^2}{b^2}$

Nature or Character of the Roots of a Quadratic Equation

The nature of the roots depends on the value of $b^2 - 4ac$. $b^2 - 4ac$ is called the **discriminant** of the quadratic equation $ax^2 + bx + c = 0$ and is generally, denoted by D.

$\therefore\ D = b^2 - 4ac$

- **If D > 0**, i..e., $b^2 - 4ac > 0$, i.e., $b^2 - 4ac$ is positive ; **the roots are real and unequal**. Also,
  (i) If $b^2 - 4ac$ is a perfect square, the roots are rational and unequal.
  (ii) If $b^2 - 4ac$ is positive but not perfect square, the roots are irrational and unequal.
- If D = 0, i.e., $b^2 - 4ac = 0$; **the roots are real and equal**.
- **If D < 0**, i.e., $b^2 - 4ac < 0$; i.e., $b^2 - 4ac$ is negative; the roots are not real, i.e., **the roots are imaginary**.

❖ EXAMPLES ❖

Ex.14 Without solving, examine the nature of roots of the equations :

(i) $2x^2 + 2x + 3 = 0$ (ii) $2x^2 - 7x + 3 = 0$

(iii) $x^2 - 5x - 2 = 0$ (iv) $4x^2 - 4x + 1 = 0$

Sol. (i) Comparing $2x^2 + 2x + 3 = 0$ with $ax^2 + bx + c = 0$; we get : a = 2, b = 2 and c = 3

$D = b^2 - 4ac = (2)^2 - 4 \times 2 \times 3 = 4 - 24 = -20$; which is negative.

$\therefore$ The roots of the given equation are imaginary.

(ii) Comparing $2x^2 - 7x + 3 = 0$ with $ax^2 + bx + c = 0$; we get : a = 2, b = – 7 and c = 3

$D = b^2 - 4ac = (-7)^2 - 4 \times 2 \times 3 = 49 - 24 = 25$, which is perfect square.

$\therefore$ The roots of the given equation are rational and unequal.

(iii) Comparing $x^2 - 5x - 2 = 0$ with $ax^2 + bx + c = 0$; we get : a = 1, b = – 5 and c = – 2

$D = b^2 - 4ac = (-5)^2 - 4 \times 1 \times -2 = 25 + 8 = 33$; which is positive but not a perfect square.

$\therefore$ The roots of the given equation are irrational and unequal.

(iv) Comparing $4x^2 - 4x + 1 = 0$ with $ax^2 + bx + c = 0$; we get : a = 4, b = – 4, and c = 1

$D = b^2 - 4ac = (-4)^2 - 4 \times 4 \times 1 = 16 - 16 = 0$

$\therefore$ Roots are real and equal.

Ex.15 For what value of m, are the roots of the equation $(3m + 1) x^2 + (11 + m) x + 9 = 0$ equal ?

Sol. Comparing the given equation with $ax^2 + bx + c = 0$; we get : a = 3m + 1, b = 11 + m and c = 9

$\therefore$ Discriminant, $D = b^2 - 4ac$

$= (11 + m)^2 - 4(3m + 1) \times 9$

$= 121 + 22m + m^2 - 108m - 36$

$= m^2 - 86m + 85$

$= m^2 - 85m - m + 85$

$= m(m - 85) - 1(m - 85)$

$= (m - 85)(m - 1)$

Since the roots are equal, D = 0

$\Rightarrow (m - 85)(m - 1) = 0$

$\Rightarrow m - 85 = 0$ or $m - 1 = 0$

$\Rightarrow m = 85$ or $m = 1$

Sum and Product of the Roots

Let α and β be the two roots of the quadratic equation $ax^2 + bx + c = 0$.

Since, $ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

then, let : $\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

$\therefore$ The sum of the roots $= \alpha + \beta$

$= \frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{-b+\sqrt{b^2-4ac}-b-\sqrt{b^2-4ac}}{2a}$$

$$= \frac{-2b}{2a} = -\frac{b}{a}$$

And, the product of the roots = $\alpha . \beta$

$$= \left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)$$

$$= \frac{(-b)^2-(\sqrt{b^2-4ac})^2}{4a^2} = \frac{b^2-(b^2-4ac)}{4a^2}$$

$$= \frac{b^2-b^2+4ac}{4a^2} = \frac{4ac}{4a^2} = \frac{c}{a}$$

$\therefore$ If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$; then :

(i) The sum of the roots

$$= \alpha + \beta = -\frac{b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

(ii) The product of the roots

$$= \alpha\beta + \frac{c}{a} = \frac{\text{constant (absolute) term}}{\text{coefficient of } x^2}$$

To Construct a Quadratic Equation whose Roots are given

x^2 – (sum of roots) x + product of roots = 0

To get the quadratic equation with given roots :

(i) Find the sum of the roots.

(ii) Find the product of the roots.

(iii) Substitute the values of steps 1 and 2 in x^2 – (sum of the roots) + (product of roots) = 0 and get the required quadratic equation.

❖ EXAMPLES ❖

Ex.16 For each quadratic equation given below, find the sum of the roots and the product of the roots :

(i) $x^2 + 3x - 6 = 0$

(ii) $2x^2 + 5\sqrt{3}x + 6 = 0$

(iii) $3x^2 + 2\sqrt{5}x - 5 = 0$

Sol. (i) Comparing $x^2 + 3x - 6 = 0$ with $ax^2 + bx + c = 0$, we get :

$a = 1$, $b = 3$ and $c = -6$

$\therefore$ The sum of the roots $= -\frac{b}{a} = -\frac{3}{2}$

And, the product of the roots $= \frac{c}{a} = \frac{-6}{1} = -6$

(ii) Comparing $2x^2 + 5\sqrt{3}x + 6 = 0$ with $ax^2 + bx + c = 0$; we get :

$a = 2$, $b = 5\sqrt{3}$ and $c = 6$

$\therefore$ The sum of the roots $= -\frac{b}{a} = -\frac{5\sqrt{3}}{2}$

And, the product of the roots $= \frac{c}{a} = \frac{6}{2} = 3$

(iii) Comparing $3x^2 + 2\sqrt{5}x - 5 = 0$ with $ax^2 + bx + c = 0$; we get :

$a = 3$, $b = 2\sqrt{5}$ and $c = -5$

$\therefore$ The sum of the roots $= -\frac{b}{a} = -\frac{2\sqrt{5}}{3}$

And, the product of the roots $= \frac{c}{a} = \frac{-5}{3}$

Ex.17 Construct the quadratic equation whose roots are given below -

(i) $3, -3$

(ii) $3 + \sqrt{3}, 3 - \sqrt{3}$

(iii) $\frac{2+\sqrt{5}}{2}, \frac{2-\sqrt{5}}{2}$

Sol. (i) Since, the sum of the roots $= (3) + (-3) = 3 - 3 = 0$

And, the product of the roots $= (3)(-3) = -9$

$\therefore$ The required quadratic equation is :

$x^2 -$ (sum of roots) $x +$ (product of roots) $= 0$

$\Rightarrow x^2 - (0)x + (-9) = 0$, i.e., $x^2 - 9 = 0$

(ii) Since, the sum of the roots

$= 3 + \sqrt{3} + 3 - \sqrt{3} = 6$

And, the product of the roots

$= (3 + \sqrt{3})(3 - \sqrt{3}) = 9 - 3 = 6$

$\therefore$ The required quadratic equation is :

$x^2 -$ (sum of roots) $x +$ (product of roots) $= 0$

$\Rightarrow x^2 - 6x + 6 = 0$

(iii) Since, the sum of the roots

$= \frac{2+\sqrt{5}}{2} + \frac{2-\sqrt{5}}{2} = \frac{2+\sqrt{5}+2-\sqrt{5}}{2} = \frac{4}{2} = 2$

And, the product of the roots

$= \left(\frac{2+\sqrt{5}}{2}\right)\left(\frac{2-\sqrt{5}}{2}\right) = \frac{4-5}{4} = -\frac{1}{4}$

$\therefore$ The required quadratic equation is :

$x^2 -$ (sum of roots) $x +$ (product of roots) $= 0$

$\Rightarrow x^2 - 2x + \left(-\frac{1}{4}\right) = 0$

$\Rightarrow x^2 - 2x - \frac{1}{4} = 0$, i.e., $4x^2 - 8x - 1 = 0$

Ex.18 If a and c are such that the quadratic equation $ax^2 - 5x + 3 = 0$ has 10 as the sum of the roots and also as the product of the roots, find a and c.

Sol. For $ax^2 - 5x + c = 0$

the sum of roots $= -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$

$= -\frac{-5}{a} = \frac{5}{a}$

and the product of roots $= \frac{\text{constant term}}{\text{coefficient of } x^2} = \frac{c}{a}$

Given : The sum of the roots = 10

$\Rightarrow \frac{5}{a} = 10$, i.e., $10a = 5 \Rightarrow a = \frac{5}{10} = \frac{1}{2}$

The product of roots = 10

$\Rightarrow \frac{c}{a} = 10 \Rightarrow c = 10a = 10 \times \frac{1}{2} = 5$

$\Rightarrow a = \frac{1}{2}$ and $c = 5$

Ex.19 If one of the roots of the quadratic equation $2x^2 + px + 4 = 0$ is 2, find the the value of p. also find the value of the other roots.

Sol. As, 2 is one of the roots, $x = 2$ will satisfy the equation $2x^2 + px + 4 = 0$

$\Rightarrow 2(2)^2 + p(2) + 4 = 0$

$\Rightarrow 8 + 2p + 4 = 0$

i.e., $2p = -12$ and $p = -6$

Substituting $p = -6$ in the equation $2x^2 + px + 4 = 0$; we get : $2x^2 - 6x + 4 = 0$

$\Rightarrow x^2 - 3x + 2 = 0$ [Dividing each term by 2]

$\Rightarrow x^2 - 2x - x + 2 = 0$

$\Rightarrow x(x - 2)(x - 1) = 0$

$\Rightarrow x - 2 = 0$ or $x - 1 = 0$

$\Rightarrow x = 2$ or $x = 1$

$\therefore$ The other (second) root is 1.

Ex.20 In the following, find the value (s) of p so that the given equation has equal roots.

(i) $3x^2 - 5x + p = 0$ (ii) $2px^2 - 8x + p = 0$

Sol. (i) Comparing $3x^2 - 5x + p = 0$ with $ax^2 + bx + c = 0$, we get : $a = 3, b = -5$ and $c = p$

Since, the roots are equal ; the discriminant $b^2 - 4ac = 0$

i.e., $\quad ^2 - 4 \times 3 \times p = 0$

$\Rightarrow 25 - 12p = 0$ and $p = \frac{25}{12} = 2\frac{1}{12}$

(ii) Comparing $2px^2 - 8x + p = 0$ with $ax^2 + bx + c = 0$; we get : $\quad a = 2p, b = -8$ and $c = p$

$b^2 - 4ac = 0$ [Given, that the roots are equal]

$\Rightarrow (-8)^2 - 4 \times 2p \times p = 0$

$\Rightarrow 64 - 8p^2 = 0$

$\Rightarrow -8p^2 = -64, p^2 = 8$ and $p = \pm\sqrt{8}$

i.e., $p = \pm 2\sqrt{2}$

Ex.21 If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then find the values of :

(i) $\alpha^2 + \beta^2$ (ii) $\alpha^3 + \beta^3$ (iii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

Sol. We know, sum of roots $(\alpha + \beta) = -\frac{b}{a}$

And, product of roots $(\alpha\beta) = \frac{c}{a}$; therefore :

(i) $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$\Rightarrow \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= \left(-\frac{b}{a}\right)^2 - 2\frac{c}{a} = \frac{b^2}{a^2} - 2\frac{c}{a} = \frac{b^2 - 2ac}{a^2}$

(ii) $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$

$= \left(-\frac{b}{a}\right)\left(\frac{b^2 - 2ac}{a^2} - \frac{c}{a}\right)$

$= \left(-\frac{b}{a}\right)\left(\frac{b^2 - 2ac - ac}{a^2}\right) = -\frac{b(b^2 - 3ac)}{a^3}$

(iii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$

$= \frac{\frac{b^2 - 2ac}{a^2}}{\frac{c}{a}} = \frac{b^2 - 2ac}{a^2} \times \frac{a}{c} = \frac{b^2 - 2ac}{ca}$

Relation between Roots and Coefficients :

If roots of quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ are α and β then :

(i) $(\alpha - \beta) = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta} = \pm\frac{\sqrt{b^2 - 4ac}}{a} = \frac{\pm\sqrt{D}}{a}$

(ii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \frac{b^2 - 2ac}{a^2}$

(iii) $\alpha^2 - \beta^2 = (\alpha + \beta)\sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

$= -\frac{b\sqrt{b^2 - 4ac}}{a^2} = \frac{\pm\sqrt{D}}{a}$

(iv) $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$.

$$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=-\frac{b(b^2-3ac)}{a^3}$$

(v) $\alpha^3-\beta^3=(\alpha-\beta)(\alpha^2+\beta^2+\alpha\beta)$.

$$\alpha^3-\beta^3=(\alpha-\beta)^3+3\alpha\beta(\alpha-\beta)$$

$$=\sqrt{(\alpha+\beta)^2-4\alpha\beta}\,\{(\alpha+\beta)^2-\alpha\beta\}$$

$$=\frac{(b^2-ac)\sqrt{b^2-4ac}}{a^3}$$

(vi) $\alpha^4+\beta^4=\{(\alpha+\beta)^2-2\alpha\beta\}^2-2\alpha^2\beta^2 = \left(\frac{b^2-2ac}{a^2}\right)^2-2\,\frac{c^2}{a^2}$

(vii) $\alpha^4-\beta^4=(\alpha^2-\beta^2)(\alpha^2+\beta^2)$

$$=\frac{-b(b^2-2ac)\sqrt{b^2-4ac}}{a^4}$$

(viii) $\alpha^2+\alpha\beta+\beta^2=(\alpha+\beta)^2-\alpha\beta$

(ix) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$

(x) $\alpha^2\beta+\beta^2\alpha=\alpha\beta(\alpha+\beta)$

(xi) $\left(\frac{\alpha}{\beta}\right)^2+\left(\frac{\beta}{\alpha}\right)^2=\frac{\alpha^4+\beta^4}{\alpha^2\beta^2}=\frac{(\alpha^2+\beta^2)^2-2\alpha^2\beta^2}{\alpha^2\beta^2}$

Equations Reducible to Quadratic Equations

Type 1 : Equations of the form $ax^4+bx^2+c=0$,

Method : Substitude $x^2=y$ and solve.

❖ EXAMPLES ❖

Ex.21 Solve the following equations :

(i) $x^4-26x^2+25=0$

(ii) $z^4-10z^2+9=0$

Sol. (i) Substituting $x^2=y$:

$x^4-26x^2+25=0$

$\Rightarrow y^2-26y+25=0$

i.e., $y^2-25y-y+25=0$

$\Rightarrow y(y-25)-1(y-25)=0$

i.e., $(y-25)(y-1)=0$

$\Rightarrow y-25=0$ or $y-1=0$

i.e., $y=25$ or $y=1$

$y=25\Rightarrow x^2=25$ | $y=1\Rightarrow x^2=1$

$\Rightarrow x=\pm 5$ | $\Rightarrow x=\pm 1$

$\therefore$ Roots of the given equation are : $\pm 5, \pm 1$

(ii) Substituting $z^2=x$

$z^4-10z^2+9=0\Rightarrow x^2-10x+9=0$

i.e., $x^2 - 9x - x + 9 = 0 \Rightarrow x(x - 9) - 1(x - 9) = 0$

i.e., $(x - 9)(x - 1) = 0 \Rightarrow x - 9 = 0$ or $x - 1 = 0$

$x = 9 \Rightarrow z^2 = 9$ | $x = 1 \Rightarrow z^2 = 1$

$\Rightarrow z = \pm 3$ | $\Rightarrow z = \pm 1$

$\therefore$ Solution of the given equation is : $\pm 3, \pm 1$.

Type 2 : Equation of the form : $px + \frac{q}{x} = r$

Method : (i) Multiply each term by x.

(ii) Solve the quadratic equation obtained to get the non-zero value(s) of x.

Ex.22 Solve :

(i) $x + \frac{5}{x} = 6$ (ii) $3y + \frac{5}{16y} = 2$

Sol. (i) $x + \frac{5}{x} = 6$

$\Rightarrow x^2 + 5 = 6x$ [Multiplying each term by x]

$\Rightarrow x^2 - 6x + 5 = 0$

$\Rightarrow x^2 - 5x - x + 5 = 0$

i.e., $x(x - 5) - 1(x - 5) = 0$

$\Rightarrow (x - 5)(x - 1) = 0$ i.e., $x - 5 = 0$ or $x - 1 = 0$

$\Rightarrow x = 5$ or $x = 1$.

$\therefore$ Required solution is 5, 1

(ii) $3y + \frac{5}{16y} = 2$

$\Rightarrow 3y \times 16y + 5 = 2 \times 16y$

$\Rightarrow 48y^2 - 32y + 5 = 0$

$\Rightarrow 48y^2 - 12y - 20y + 5 = 0$

i.e., $12y(4y - 1) - 5(4y - 1) = 0$

$\Rightarrow (4y - 1)(12y - 5) = 0$

i.e., $4y - 1 = 0$ or $12y - 5 = 0$

$\Rightarrow 4y = 1$ or $12y = 5$ i.e., $y = \frac{1}{4}$ or $y = \frac{5}{12}$

$\therefore$ Required solutions is : $\frac{1}{4}, \frac{5}{12}$

Type 3 :

Equations involving one radical : $\sqrt{a - x^2} = bx + c$

Method :

1. Square both the sides to get : $a - x^2 = (bx + c)^2$
2. Now simplify it to get a quadratic equation.
3. Solve the quadratic equation obtained.

Ex.23 Solve :

(i) $\sqrt{x} + 2x = 1$ (ii) $\sqrt{3x^2 - 2} + 1 = 2x$

(iii) $\sqrt{2x^2 + 9} + x = 13$

Sol. (i) $\sqrt{x} + 2x = 1$

$\Rightarrow \sqrt{x} = 1 - 2x$ i.e., $x = (1 - 2x)^2$

$\Rightarrow x = 1 + 4x^2 - 4x$ i.e., $1 + 4x^2 - 4x - x = 0$

$\Rightarrow 4x^2 - 5x + 1 = 0$ i.e., $4x^2 - 4x - x + 1 = 0$

$\Rightarrow 4x(x - 1) - 1(x - 1) = 0$

i.e.,$(x - 1)(4x - 1) = 0 \Rightarrow x - 1 = 0$

or $4x - 1 = 0$

i.e.,$x = 1$ or $x = \frac{1}{4}$

Remember :

In the case of equations involving radical (s); it is necessary to check each value of variable x, or y, or z obtained, that value (s) satisfy the given equation or not :

Checking :

For $x = 1$: L.H.S. $= \sqrt{x} + 2x$

$= \sqrt{1} + 2 \times 1 = 1 + 2 = 3$

R.H.S. $= 1 \Rightarrow$ L.H.S. $\neq$ R.H.S.

Hence $x = 1$ does not statisfy the given equation so $x = 1$ is not a solution of it.

For $x = \frac{1}{4}$ L.H.S. $= \sqrt{x} + 2x = \sqrt{\frac{1}{4}} + 2 \cdot \frac{1}{4}$

$= \frac{1}{2} + \frac{1}{2} = 1 =$ R.H.S.

Hence, $x = \frac{1}{4}$ satisfies the given equation and so $x = \frac{1}{4}$ is a solution of it.

(ii) $\sqrt{3x^2 - 2} + 1 = 2x$

$\Rightarrow \sqrt{3x^2 - 2} = 2x - 1$ i.e., $(3x^2 - 2) = (2x - 1)^2$

$\Rightarrow 3x^2 - 2 = 4x^2 - 4x + 1$

i.e.,$4x^2 - 3x^2 - 4x + 1 + 2 = 0$

$\Rightarrow x^2 - 4x + 3 = 0$ i.e.,$x^2 - 3x - x + 3 = 0$

$\Rightarrow x(x - 3) - 1(x - 3) = 0$

i.e.,$(x - 3)(x - 1) = 0 \Rightarrow x - 3 = 0$ or $x - 1 = 0$

i.e.,$x = 3$ or $x = 1$

On checking, we find that both the values of x obtained satisfy the given equation.

Required solution = 3, 1

(iv) $\sqrt{2x + 9} + x = 13$

$\Rightarrow \sqrt{2x + 9} = 13 - x$ i.e.,$2x + 9 = (13 - x)^2$

$\Rightarrow 2x + 9 = 169 + x^2 - 26x$

i.e.,$x^2 - 26x - 2x + 169 - 9 = 0$

$\Rightarrow x^2 - 28x + 160 = 0$ i.e., $x^2 - 20x - 8x + 160 = 0$

$\Rightarrow x(x - 20) - 8(x - 20) = 0$

i.e.,$(x - 20)(x - 8) = 0 \Rightarrow x = 20$ or $x = 8$

On checking, we find that only $x = 8$ satisfies the given equation.

$\therefore$ Required solution = 8

Type 4 : Equation involving two radicals, such as:

$$\sqrt{ax+b} + \sqrt{cx+d} = e$$

or $\sqrt{ax+b} - \sqrt{cx+d} = f$

Ex.24 Solve :

(i) $\sqrt{4x-3} + \sqrt{2x+3} = 6$

(ii) $\sqrt{2x+9} - \sqrt{x-4} = 3$

Sol. (i) $\sqrt{4x-3} + \sqrt{2x+3} = 6$

$\Rightarrow \sqrt{4x-3} = 6 - \sqrt{2x+3}$

i.e., $4x - 3 = (6 - \sqrt{2x+3})^2$

$\Rightarrow 4x - 3 = 36 + (2x + 3) - 2 \times 6\sqrt{2x+3}$

i.e., $12\sqrt{2x+3} = 36 + 2x + 3 - 4x + 3$

$\Rightarrow 12\sqrt{2x+3} = 42 - 2x$

i.e., $6\sqrt{2x+3} = 21 - x$ [Dividing each term by 2]

$\Rightarrow 36(2x + 3) = (21 - x)^2$ [Squaring both the sides]

$\Rightarrow 72x + 108 = 441 + x^2 - 42x$

$\Rightarrow x^2 - 42x - 72x + 441 - 108 = 0$

i.e., $x^2 - 114x + 333 = 0$

$\Rightarrow x^2 - 111x - 3x + 333 = 0$

i.e., $x(x - 111) - 3(x - 111) = 0$

$\Rightarrow (x - 111)(x - 3) = 0$

i.e., $x - 111 = 0$ or $x - 3 = 0$

$\Rightarrow x = 111$ or $x = 3$

Checking :

For $x = 111$:

L.H.S. $= \sqrt{4\times111-3} + \sqrt{2\times111+3}$

$= \sqrt{441} + \sqrt{225} = 21 + 15 = 36$

But R.H.S. = 6

$\Rightarrow$ L.H.S. $\neq$ R.H.S. and so $x = 111$ is not a solution of the given equation.

For $x = 3$

L.H.S. $= \sqrt{4\times3-3} + \sqrt{2\times3+3} = \sqrt{9} + \sqrt{9}$ $= 3 + 3 = 6 =$ R.H.S.

$\Rightarrow x = 3$ is a solution of the given equation.

(ii) $\sqrt{2x+9} - \sqrt{x-4} = 3$

$\Rightarrow \sqrt{2x+9} = 3 + \sqrt{x-4}$

i.e., $2x + 9 = 9 + x - 4 + 6\sqrt{x-4}$ [Squaring]

$\Rightarrow 2x + 9 = 9 + x - 4 + 6\sqrt{x-4}$

$\Rightarrow x + 4 = 6\sqrt{x-4}$

$\Rightarrow (x + 4)^2 = 36(x - 4)$

i.e., $x^2 + 8x + 16 = 36x - 144$

$\Rightarrow x^2 - 28x + 160 = 0$

i.e., $x^2 - 20x - 8x + 160 = 0$

$\Rightarrow x(x - 20) - 8(x - 20) = 0$

i.e., $(x - 20)(x - 8) = 0$

$\Rightarrow x - 20 = 0$ or $x - 8 = 0$

i.e., $x = 20$ or $x = 8$.

On checking, we find that both the values of x obtained satisfy the given equation.

$\therefore$ Required solution = 20, 8

Type 5 : Equation of the forms :

(i) $a\left(x^2+\frac{1}{x^2}\right) + b\left(x+\frac{1}{x}\right) + c = 0$

(ii) $a\left(x^2-\frac{1}{x^2}\right) + b\left(x-\frac{1}{x}\right) + c = 0$

Method :

1. For form (i), take $x + \frac{1}{x} = y \Rightarrow x^2 + \frac{1}{x^2} = y^2 - 2$

   For form (ii), take $x - \frac{1}{x} = y \Rightarrow x^2 + \frac{1}{x^2} = y^2 + 2$

2. On substituting the value of $x^2 + \frac{1}{x^2}$; the given equation will reduce into a quadratic equation.

3. On solving the equation obtained, we shall be getting the value (s) of y.

4. For each value of y, get the value (s) of x by taking $x + \frac{1}{x} = y$ or $x - \frac{1}{x} = y$, as required.

Ex.25 Solve the following equations by reducing them into quadratic equations :

(i) $2\left(x^2+\frac{1}{x^2}\right) - 9\left(x+\frac{1}{x}\right) + 14 = 0$

(ii) $8\left(x^2+\frac{1}{x^2}\right) - 42\left(x-\frac{1}{x}\right) + 29 = 0$

Sol.(i) Let $x + \frac{1}{x} = y \quad \Rightarrow \left(x+\frac{1}{x}\right)^2 = y^2$

i.e., $x^2 + \frac{1}{x^2} + 2 \times x \times \frac{1}{x} = y^2$

$\Rightarrow x^2 + \frac{1}{x^2} = y^2 - 2$

$\therefore 2\left(x^2+\frac{1}{x^2}\right) - 9\left(x+\frac{1}{x}\right) + 14 = 0$

$\Rightarrow 2(y^2 - 2) - 9y + 14 = 0$

$\Rightarrow 2y^2 - 4 - 9y + 14 = 0$ i.e., $2y^2 - 9y + 10 = 0$

$\Rightarrow 2y^2 - 5y - 4y + 10 = 0$

i.e., $y(2y - 5) - 2(2y - 5) = 0$

$\Rightarrow$ (2y – 5) (y – 2) = 0, i.e., 2y – 5 = 0 or y – 2 = 0

$\Rightarrow y = \frac{5}{2}$ or $y = 2$

When $y = \frac{5}{2}$

$\Rightarrow x + \frac{1}{x} = \frac{5}{2}$, i.e., $2x^2 - 5x + 2 = 0$

$\Rightarrow 2x^2 - 4x - x + 2 = 0$, i.e., $(2x - 1)(x - 2) = 0$

$\Rightarrow x = \frac{1}{2}$ or $x = 2$

When y = 2

$\Rightarrow x + \frac{1}{x} = 2$, i.e., $x^2 - 2x + 1 = 0$

$\Rightarrow (x - 1)^2 = 0$ i.e., $x - 1 = 0$

$\therefore$ Required solution = $\frac{1}{2}$, 2 , 1

(ii) Let $x - \frac{1}{x} = y \quad \Rightarrow \left(x - \frac{1}{x}\right)^2 = y^2$

i.e., $x^2 + \frac{1}{x^2} - 2 = y^2 \quad \Rightarrow x^2 + \frac{1}{x^2} = y^2 + 2$

$\therefore 8\left(x^2 + \frac{1}{x^2}\right) - 42\left(x - \frac{1}{x}\right) + 29 = 0$

$\Rightarrow 8(y^2 + 2) - 42y + 29 = 0$

$\Rightarrow 8y^2 + 16 - 42y + 29 = 0$

i.e., $8y^2 - 42y + 45 = 0$

$\Rightarrow 8y^2 - 12y - 30y + 45 = 0$

i.e., $4y(2y - 3) - 15(2y - 3) = 0$

$\Rightarrow (2y - 3)(4y - 15) = 0$

i.e., $2y - 3 = 0$ or $4y - 15 = 0$

$\Rightarrow y = \frac{3}{2}$ or $y = \frac{15}{4}$

When $y = \frac{3}{2}$

$\Rightarrow x - \frac{1}{x} = \frac{3}{2}$ i.e., $2x^2 - 3x - 2 = 0$

$\Rightarrow 2x^2 - 4x + x - 2 = 0$ i.e., $(2x + 1)(x - 2) = 0$

$\Rightarrow x = -\frac{1}{2}$ or $x = 2$

When $y = \frac{15}{4}$

$\Rightarrow x - \frac{1}{x} = \frac{15}{4}$ i.e., $4x^2 - 15x - 4 = 0$

$\Rightarrow 4x^2 - 16x + x - 4 = 0$ i.e., $(4x + 1)(x - 4) = 0$

$\Rightarrow x = -\frac{1}{4}$ or $x = 4$

$\therefore$ Required solution = $\frac{1}{2}$, 2, $-\frac{1}{4}$, 4

Ex.26 Solve for x :

$\sqrt{x^2-36} - (x-6) = 2\sqrt{x^2-15x+54}$

Sol. Since, $x^2 - 36 = (x-6)(x+6)$, $x - 6 = \sqrt{(x-6)^2}$

and $x^2 - 15x + 54 = x^2 - 9x - 6x + 54 = (x-9)(x-6)$

$\therefore \sqrt{x^2-36} - (x-6) = 2\sqrt{(x-9)(x-6)}$

$\Rightarrow \sqrt{(x-6)(x+6)} - \sqrt{(x-6)^2} - 2\sqrt{(x-9)(x-6)} = 0$

$\Rightarrow \sqrt{x-6}\,(\sqrt{x+6} - \sqrt{x-6} - 2\sqrt{x-9}) = 0$

$\Rightarrow \sqrt{x-6}$ or $\sqrt{x+6} - \sqrt{x-6} - 2\sqrt{x-9} = 0$

$\sqrt{x-6} = 0 \Rightarrow x - 6 = 0 \Rightarrow x = 6$

$\sqrt{x+6} - \sqrt{x-6} - 2\sqrt{x-9} = 0$

$\Rightarrow \sqrt{x+6} - \sqrt{x-6} = 2\sqrt{x-9}$

$\Rightarrow (\sqrt{x+6} - \sqrt{x-6})^2 = (2\sqrt{x-9})^2$

i.e., $x + 6 + x - 6 - 2\sqrt{(x+6)(x-6)} = 4(x-9)$

$\Rightarrow 2x - 2\sqrt{x^2-36} = 4x - 36,$

i.e., $x - \sqrt{x^2-36} = 2x - 18$

$\Rightarrow 18 - x = \sqrt{x^2-36}$, i.e., $(18-x)^2 = x^2 - 36$

$\Rightarrow 324 + x^2 - 36x = x^2 - 36$, i.e, $36x = 360$

$\Rightarrow x = 10$

On checking, we find x = 6 and x =10 both satisfy the given equation.

$\therefore$ Required solution = 6, 10

Ex.27 Solve for x :

$$\sqrt{\frac{2x^2+x+2}{x^2+3x+1}} + 2\sqrt{\frac{x^2+3x+1}{2x^2+x+2}} - 3 = 0$$

Sol. Let $\sqrt{\frac{2x^2+x+2}{x^2+3x+1}} = y \Rightarrow \sqrt{\frac{x^2+3x+1}{2x^2+x+2}} = \frac{1}{y}$

$\therefore$ Given equation reduces to $y + 2 \times \frac{1}{y} - 3 = 0$

$\Rightarrow y^2 - 3y + 2 = 0$ i.e., $(y-2)(y-1) = 0$

$\Rightarrow y = 2$ or $y = 1$

When $y = 2 \Rightarrow \sqrt{\frac{2x^2+x+2}{x^2+3x+1}} = 2$

$\Rightarrow \frac{2x^2+x+2}{x^2+3x+1} = 4$ [Squaring]

i.e., $4x^2 + 12x + 4 = 2x^2 + x + 2$

$\Rightarrow 2x^2 + 11x + 2 = 0$

i.e., $x = \frac{-11 \pm \sqrt{(11)^2 - 4 \times 2 \times 2}}{2 \times 2} = \frac{-11 \pm \sqrt{105}}{4}$

When y = 1 $\Rightarrow \sqrt{\dfrac{2x^2+x+2}{x^2+3x+1}} = 1$

$\Rightarrow \dfrac{2x^2+x+2}{x^2+3x+1} = 1$ [Squaring]

i.e., $2x^2 + x + 2 = x^2 + 3x + 1$

$\Rightarrow x^2 - 2x + 1 = 0$

i.e., $(x - 1)^2 = 0$

$\Rightarrow x - 1 = 0$ and $x = 1$

$\therefore$ Required solution = $\dfrac{-11 \pm \sqrt{105}}{4}, 1$

Ex.28 Solve for x :

$$\left(\frac{2x+1}{x+1}\right)^4 - 6\left(\frac{2x+1}{x+1}\right)^2 + 8 = 0;\ x \neq -1$$

Sol. Let $\left(\dfrac{2x+1}{x+1}\right)^2 = y \Rightarrow \left(\dfrac{2x+1}{x+1}\right)^4 = y^2$

$\therefore$ The given equation reduces to :

$y^2 - 6y + 8 = 0$, i.e., $y^2 - 4y - 2y + 8 = 0$

$\Rightarrow (y - 4)(y - 2) = 0$, i.e., $y = 4$ or $y = 2$

When y = 4 $\Rightarrow \left(\dfrac{2x+1}{x+1}\right)^2 = 4$, i.e.,

$\dfrac{2x+1}{x+1} = \pm 2$

$\dfrac{2x+1}{x+1} = 2$

$\Rightarrow 2x + 1 = 2x + 2$; which gives no solution.

$\dfrac{2x+1}{x+1} = -2$

$\Rightarrow 2x + 1 = -2x - 2$ i.e., $4x = -3$ and $x = -\dfrac{3}{4}$

When y = 2

$\Rightarrow \left(\dfrac{2x+1}{x+1}\right)^2 = 2$ i.e., $4x^2 + 4x + 1 = 2(x^2 + 2x + 1)$

$\Rightarrow 4x^2 + 4x + 1 = 2x^2 + 4x + 2$, i.e., $2x^2 = 1$

$\Rightarrow x^2 = \dfrac{1}{2}$ and $x = \pm \dfrac{1}{\sqrt{2}}$

$\therefore$ Required solution = $-\dfrac{3}{4}, \pm \dfrac{1}{\sqrt{2}}$

Ex.29 Solve for x : $4^{1+x} + 4^{1-x} = 10$

Sol. $4^{1+x} + 4^{1-x} = 10 \Rightarrow 4 \times 4^x + \dfrac{4}{4^x} = 10$

Substituting $4^x = y$, we get : $4y + \dfrac{4}{y} = 10$

$\Rightarrow 4y^2 + 4 = 10y$ i.e., $2y^2 - 5y + 2 = 0$

$\Rightarrow 2y^2 - 4y - y + 2 = 0$ i.e., $(y - 2)(2y - 1) = 0$

$\Rightarrow$ y = 2 or $y = \frac{1}{2}$

When y = 2 $\Rightarrow$ $4^x = 2$ i.e., $2^{2x} = 2^1$

$[\because 4 = 2^2 \Rightarrow 4^x = 2^{2x}]$

$\Rightarrow$ 2x = 1 and $x = \frac{1}{2}$

When $y = \frac{1}{2} \Rightarrow 4^x = \frac{1}{2}$

i.e.,$2^{2x} = 2^{-1}$ $\left[\because \frac{1}{2} = 2^{-1}\right]$

$\Rightarrow$ 2x = – 1 and $x = -\frac{1}{2}$

$\therefore$ Required solution, $\frac{1}{2}, -\frac{1}{2}$

Ex.30 Solve for x : $4\left(x - \frac{1}{x}\right)^2 + 8\left(x + \frac{1}{x}\right) = 29$; $x \neq 0$

Sol. Let $x + \frac{1}{x} = y$

$\therefore \left(x + \frac{1}{x}\right)^2 - \left(x - \frac{1}{x}\right)^2 = 4$

$\Rightarrow y^2 - \left(x - \frac{1}{x}\right)^2 = 4$, i.e., $y^2 - 4 = \left(x - \frac{1}{x}\right)^2$

$\therefore 4\left(x - \frac{1}{x}\right)^2 + 8\left(x + \frac{1}{x}\right) = 29$

$\Rightarrow 4(y^2 - 4) + 8y = 29$

i.e.,$4y^2 - 16 + 8y - 29 = 0$

$\Rightarrow 4y^2 + 8y - 45 = 0$

i.e.,$4y^2 + 18y - 10y - 45 = 0$

$\Rightarrow 2y(y + 9) - 5(y + 9) = 0$

$\Rightarrow (2y + 9)(2y - 5) = 0$

i.e.,$y = -\frac{9}{2}$ or $y = \frac{5}{2}$

When $y = -\frac{9}{2}$ $\Rightarrow x + \frac{1}{x} = -\frac{9}{2}$

i.e.,$2x^2 + 9x + 2 = 0$

$\Rightarrow x = \frac{-9 \pm \sqrt{(9)^2 - 4 \times 2 \times 2}}{2 \times 2} = \frac{-9 \pm \sqrt{65}}{4}$

When $y = \frac{5}{2}$ $\Rightarrow x + \frac{1}{x} = \frac{5}{2}$

i.e.,$2x^2 - 5x + 2 = 0$

$\Rightarrow 2x^2 - 4x - x + 2 = 0$ i.e., $(x - 2)(2x - 1) = 0$

$\Rightarrow$ x = 2 or $x = \frac{1}{2}$

$\therefore$ Required solution = $\frac{-9 \pm \sqrt{65}}{4}$, 2, $\frac{1}{2}$

Condition For Common Roots

◈ **Only One Root is Common :** Let α be the common root of quadratic equations $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ then

$\therefore$ $a_1\alpha^2 + b_1\alpha + c_1 = 0$

$a_2\alpha^2 + b_2\alpha + c_2 = 0$

By Cramer's rule : $\frac{\alpha^2}{\begin{vmatrix} -c_1 & b_1 \\ -c_2 & b_2 \end{vmatrix}} = \frac{\alpha}{\begin{vmatrix} a_1 & -c_1 \\ a_2 & -c_2 \end{vmatrix}} = \frac{1}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$

or $\frac{\alpha^2}{b_1c_2 - b_2c_1} = \frac{\alpha}{a_2c_1 - a_1c_2} = \frac{1}{a_1b_2 - a_2b_1}$

$\therefore \alpha = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1} = \frac{b_1c_2 - b_2c_1}{a_2c_1 - a_1c_2}$, $\alpha \neq 0$.

$\therefore$ **The condition for only one Root common is** $_1a_2 - c_2a_1)^2 = (b_1c_2 - b_2c_1)(a_1b_2 - a_2b_1)$

◈ **Both roots are common :** Then required conditions is $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Maximum & Minimum Value of Quadratic Expression

Quadratic Expression : $ax^2 + bx + c$

(i) If a > 0 Quadratic expression has least value at $x = -\frac{b}{2a}$. This least value is given by

$\frac{4ac - b^2}{4a} = -\frac{D}{4a}$

(ii) If a < 0, Quadratic expression has greatest value at $x = -\frac{b}{2a}$. This greatest value is given by

$\frac{4ac - b^2}{4a} = -\frac{D}{4a}$

Problems on Quadratic Equations

For solving problems based on quadratic equations, the following steps must be adopted :

1. Read the given statement of the problem carefully to find the required unknown quantity.
2. Take the unknown quantity as 'x' and according to the given statement, form an equation in terms of 'x'.
3. Simplify and solve the equation to get the value/values of 'x'.

❖ EXAMPLES ❖

Ex.31 Find two consecutive natural numbers, whose product is equal to 20.

Sol. Let the required two consecutive natural numbers be x and x + 1.

Given : $x(x + 1) = 20 \Rightarrow x^2 + x - 20 = 0$

$\Rightarrow (x + 5)(x - 4) = 0 \Rightarrow x = -5$, or $x = 4$

Since, x must be a natural number,

$\therefore x = 4$

And required numbers are x and x + 1 i.e., and 5.

Ex.32 The sum of the squares of two consecutive whole numbers is 61. Find the numbers.

Sol. Let the required consecutive whole numbers be x and x + 1.

$\therefore\ x^2 + (x + 1)^2 = 61$

$\Rightarrow x^2 + x^2 + 2x + 1 - 61 = 0$

$\Rightarrow 2x^2 + 2x - 60 = 0$

$\Rightarrow x^2 + x - 30 = 0$ [Dividing each term by 2]

$\Rightarrow (x + 6)(x - 5) = 0$ [On factorising]

$\Rightarrow x = -6$, or $x = 5$

$\because$ x is a whole number,

$\therefore\ x = 5$

And, required numbers are x and x + 1 = 5 and 5 + 1 i.e., 5 and 6

Ex.33 The sum of two natural numbers is 8. If the sum of their reiprocals is $\frac{8}{15}$, find the two numbers.

Sol. Let the numbers be x and 8 – x.

$\therefore\ \frac{1}{x} + \frac{1}{8-x} = \frac{8}{15} \Rightarrow \frac{8-x+x}{x(8-x)} = \frac{8}{15}$

$\Rightarrow \frac{8}{8x-x^2} = \frac{8}{15}$ i.e., $120 = 64x - 8x^2$

$\Rightarrow 8x^2 - 64x + 120 = 0$

$\Rightarrow x^2 - 8x + 15 = 0$ [Dividing by 8]

$\Rightarrow (x - 5)(x - 3) = 0$ [On factorising]

$\Rightarrow x = 5$, or $x = 3$

When x = 5, the number are x and 8 – x = 5 and 3, and when x = 3, the numbers are x and 8 – x

= 3 and 5.

$\therefore$ Required numbers are 5 and 3.

Ex.34 Divide 16 into two parts such that twice the square of the larger part exceeds the square of the smaller part by 164.

Sol. Let larger part be x, therefore the smaller part = 16 – x

Given : $2x^2 - (16 - x)^2 = 164$

$\Rightarrow 2x^2 - (256 + x^2 - 32x) - 164 = 0$

i.e., $2x^2 - 256 - x^2 + 32x - 164 = 0$

$\Rightarrow x^2 + 32x - 420 = 0$

On factorising, it gives : $(x + 42)(x - 10) = 0$

i.e., $x = -42$ or $x = 10$

$\therefore\ x = 10$

Hence the larger part = 10 and the smaller part = 16 – x = 16 – 10 = 6

Ex.35 Two positive numbers are in the ratio 2 : 5. If difference between the squares of these numbers is 189 ; find the numbers.

Sol. Let numbers be 2x and 5x

$\therefore\ (5x)^2 - (2x)^2 = 189$

$\Rightarrow 25x^2 - 4x^2 = 189$ and $21x^2 = 189$

i.e., $x^2 = \frac{189}{21} = 9$

$\Rightarrow x = \pm 3$

Since, the required numbers are positive,

$\therefore\ x = 3$

And, required numbers = 2x and 5x = 2×3 and 5×3 = 6 and 15

Ex.36 A two digit number is such that the product of the digits is 35. When 18 is added to this number the digits interchange their places. Determine the number.

Sol. Let ten's digit of the numbers = x and its unit digit = y.

$\therefore$ The two digit number is $10x + y$.

Given : $x \cdot y = 35$ and $10x + y + 18 = 10y + x$

$\Rightarrow y = \frac{35}{x}$ and $9x + 18 = 9y$ i.e., $x + 2 = y$

On substituting $y = \frac{35}{x}$ in $x + 2 = y$; we get :

$$x + 2 = \frac{35}{x} \Rightarrow x^2 + 2x = 35$$

and $x^2 + 2x - 35 = 0$

On factorising, we get : $(x + 7)(x - 5) = 0$

i.e., $x = -7$ or $x = 5$

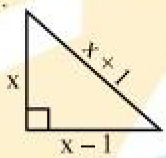
Since, x is digit, therefore x = 5 and $y = \frac{35}{x} = \frac{35}{x} = 7$

$\therefore$ The required two digit number = $10x + y$

$= 10 \times 5 + 7 = 57$

Ex.37 The sides (in cm) of a right triangle are $x - 1$, x and $x + 1$. Find the sides of triangle.

Sol. It is clear that the largest side $x + 1$ is hypotenuse of the right triangle.



According to Pythagoras Theorem, we have :

$x^2 + (x - 1)^2 = (x + 1)^2$

$\Rightarrow x^2 + x^2 - 2x + 1 = x^2 + 2x + 1$

This gives $x^2 - 4x = 0$

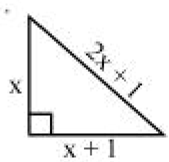
$\Rightarrow x(x - 4) = 0$ i.e., $x = 0$ or $x = 4$

Since, with x = 0 the triangle is not possible; hence x = 4.

$\therefore$ Sides, are $x - 1$, x and $x + 1 = 4 - 1$ i.e., 3 cm, 4 cm and 5 cm

Ex.38 The hypotenuse of a right triangle is 1 m less than twice the shortest side. If the third side is 1 m more than the shortest side, find the sides of the triangle.

Sol. Let the shortest side be x m.



$\therefore$ Hypotensuse = $(2x - 1)$ m and the third side = $(x + 1)$ m

Applying Pythagoras theorem, we get ;

$(2x - 1)^2 = x^2 + (x + 1)^2$

$\Rightarrow 4x^2 - 4x + 1 = x^2 + x^2 + 2x + 1$

i.e., $2x^2 - 6x = 0 \Rightarrow x^2 - 3x = 0$

i.e.,$x(x - 3) = 0$

$\Rightarrow$ $x = 0$ or $x = 3$

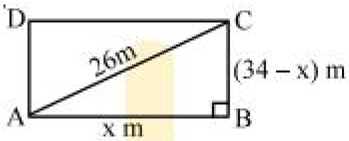
Since, $x = 0$ makes the triangle impossible. therefore, $x = 3$

And, sides of the triangle are $= x,\ 2x - 1$ and $x + 1$

$= 3,\ 2 \times 3 - 1$ and $3 + 1 = 3$m, 5m and 4m

Ex.39 If the perimeter of a rectangular plot is 68 m and its diagonal is 26 m. Find its area.

Sol. Let the length of plot = x m



$\because$ 2(length + breadth) = perimeter

$\Rightarrow$ 2(x + breadth) = 68

$\Rightarrow$ x + breadth = $\frac{68}{2}$ and breadth = (34 – x) m

Given its diagonal = 26 m and we know each angle of the rectangle = 90º.

$\therefore$ $x^2 + (34 - x)^2 = 26^2$

[Applying Pythagoras Theorem]

$\Rightarrow$ $x^2 + 1156 - 68x + x^2 - 676 = 0$

$\Rightarrow$ $2x^2 - 68x + 480 = 0$

$\Rightarrow$ $x^2 - 34x + 240 = 0$

i.e.,$x^2 - 34x + 240 = 0$

On factorising, we get : $(x - 24)(x - 10) = 0$

i.e.,$x = 24$ or $x = 10$

$x = 24$

$\Rightarrow$ length = 24 m and breadth = (34 – 24) m

= 10 m

and, $x = 10$

$\Rightarrow$ length = 10 m and breadth = (34 – 10) m

= 24 m

$\therefore$ Dimensions of the given rectangular plot are 24 m and 10 m.

Hence, its area = length × breadth = 24 m × 10m = 240 m^2

Ex.40 A train travels a distance of 300 km at a uniform speed. If the speed of the train is increased by 5 km an hour, the journey would have taken two hours less. Find the original speed of the train.

Sol. Let the original speed of the train be x km/hr.

In 1st case, Distance = 300 km and speed = x km/hr.

$\Rightarrow$ Time taken = $\frac{\text{distance}}{\text{speed}} = \frac{300}{x}$ hrs.

In 2nd case, Distance = 300 km and

speed = (x +5) km/hr.

$\therefore$ Time taken = $\frac{\text{distance}}{\text{speed}} = \frac{300}{x+5}$ hrs.

Given : $\frac{300}{x} - \frac{300}{x+5} = 2$

$\Rightarrow \frac{300(x+5)-300x}{x(x+5)} = 2$

i.e., $\frac{300x+1500-300x}{x^2+5x} = 2$

$\Rightarrow 2(x^2 + 5x) = 1500$

$\Rightarrow x^2 + 5x - 750 = 0$

On factorising, we get : $(x + 30)(x - 25) = 0$

i.e., $x = -30$ or $x = 25$

Neglecting $x = -30$; we get $x = 25$ i.e., $x = 25$ km per hour

Ex.41 A motor boat, whose speed is 15 km/hr in still water, goes 30 km downstream and comes back in a total of 4 hours 30 minutes. Determine the speed of the stream.

Sol. Let the speed of the stream = x km/hr

$\Rightarrow$ The speed of the boat downstream = (15 + x) km/hr.

and, the speed of the boat upstream = (15 – x) km/hr

Now, time taken to go 30 km downstream $= \frac{30}{15+x}$ hrs.

and, time take to come back 30 km upstream $= \frac{30}{15-x}$ hrs.

Given : the time taken for both the journeys = 4 hours 30 min. $= 4\frac{1}{2}$ hrs $= \frac{9}{2}$ hrs

$\therefore \frac{30}{15+x} + \frac{30}{15-x} = \frac{9}{2}$

$\Rightarrow \frac{30(15-x)+30(15+x)}{(15+x)(15-x)} = \frac{9}{2}$

i.e., $\frac{450-30x+450+30x}{225-x^2} = \frac{9}{2}$

$\Rightarrow 2 \times 900 = 9(225 - x^2)$

On dividing both the sides by 9, we get :

$2 \times 100 = 225 - x^2$

i.e., $x^2 = 225 - 200 \Rightarrow x^2 = 25$ and $x = \pm 5$

Rejecting the negative value of x, we get : x = 5

i.e., the speed of the steam = 5 km/hr

Ex.42 The hotel bill for a number of people for overnight stay in Rs. 4, 800. If there were 4 people more, the bill each person had to pay would have reduced by Rs. 200. Find the number of people staying overnight.

Sol. Let the number of people staying overnight be x.

$\therefore$ For x people, the hotel bill = Rs 4,800

$\Rightarrow$ For 1 person, the hotel bill = Rs $\frac{4,800}{x}$

When 4 people were more :

For (x + 4) people, the hotel bill = Rs 4,800

$\Rightarrow$ For 1 person, the hotel bill = Rs $\frac{4,800}{x+4}$

It is given that now the bill paid by each person is reduced by Rs 200.

$\therefore \quad \frac{4,800}{x} - \frac{4,800}{x+4} = 200$

$\Rightarrow \quad \frac{4,800(x+4) - 4,800x}{x(x+4)} = 200$

i.e.,$200(x^2 + 4x) = 4800x + 19200 - 4800x$

$\Rightarrow x^2 + 4x = \frac{19200}{200} = 96$

i.e.,$x^2 + 4x - 96 = 0$

On factorising, we get : $(x + 12)(x - 8) = 0$

i.e.,$x = -12$ or $x = 8$

$\because$ No. of people can not be negative

$\Rightarrow$ No. of people staying overnight = 8

Ex.43 In an auditorium, the number of rows was equal to the number of seats in each row. If the number of ros is doubled and the number of seats in each row is reduced by5, then the total number of seats is increased by 375. How many rows were there ?

Sol. Let the number of rows be x

$\Rightarrow$ No. of seats in each rows = x

$\therefore$ The total number of seats in the auditorium $= x \times x = x^2$

Now, the new no. of rows = 2x and the new no. of seats in each row = x – 5

$\therefore$ The new no. of total seats in the auditorium $= 2x(x - 5)$.

Given : $2x(x - 5) - x^2 = 375$

$\Rightarrow 2x^2 - 10x - x^2 = 375$ and $x^2 - 10x - 375 = 0$

On factorising, we get : $(x - 25)(x + 15) = 0$

i.e.,$x = 25$ or $x = -15$

Neglecting $x = -15$, we get : no. of rows = 25

Ex.44 Two years ago, a man's age was three times the square of his son's age. In three years time, his age will be four times his son's age. Find their present ages.

Sol. Let present age of son = x years

Two years ago : The age of son was (x – 2) years and so the age of the man was $3(x - 2)^2$

$\therefore$ Man's present age $= 3(x - 2)^2 + 2$

$= 3(x^2 - 4x + 4) + 2 = 3x^2 - 12x + 14$

In 3 years time : The age of son will be (x + 3) years and the age of man will be

$(3x^2 - 12x + 14) + 3 = 3x^2 - 12x + 17$ years

Given : $3x^2 - 12x + 17 = 4(x + 3)$

$\Rightarrow 3x^2 - 12x + 17 = 4x + 12$

i.e.,$3x^2 - 16x + 5 = 0$

On factorising, we get : $(x - 5)(3x - 1) = 0$

i.e.,$x = 5$ or $x = \frac{1}{3}$

Since, $x = \frac{1}{3}$ is not possible ; $x = 5$

$\therefore$ The present age of man

$= 3x^2 - 12x + 14 = 3 \times 5^2 - 12 \times 5 + 14 = 29$ years.

And, the present age of son = x = 5 years

Examples based on Solution of Quadratic Equation

Ex.45 The roots of the equation $x^2 - 2x - 8 = 0$ are -

(A) – 4, 2 (B) 4, – 2

(C) 4, 2 (D) – 4,– 2

Sol.[B] Quadratic Equation $x^2 - 2x - 8 = 0$

After factorization $(x - 4)(x + 2) = 0$

$\Rightarrow x = 4, -2$

Ex.46 The roots of the equation $x^2 - 4x + 1 = 0$ are -

(A) $2 \pm \sqrt{3}$ (B) 2, 4

(C) $-2 \pm \sqrt{3}$ (D) $\sqrt{3} \pm 2$

Sol.[A] Here a = 1, b = 4, c = 1

Using Hindu Method

$$x = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}$$

Examples based on Nature of Roots

Ex.47 The roots of the quadratic equation $7x^2 - 9x + 2 = 0$ are -

(A) Rational and different

(B) Rational and equal

(C) Irrational and different

(D) Imaginary and different

Sol.[A] $b^2 - 4ac = 81 - 56 = 25 > 0$ and a perfect square so roots are rational and different.

Ex.48 The roots of the quadratic equation $2x^2 - 7x + 4 = 0$ are -

(A) Rational and different

(B) Rational and equal

(C) Irrational and different

(D) Imaginary and different

Sol.[C] $b^2 - 4ac = 49 - 32 = 17 > 0$ (not a perfect square)

$\therefore$ Its roots are irrational and different.

Ex.49 The roots of the quadratic equation

$x^2 - 2(a + b)x + 2(a^2 + b^2) = 0$ are -

(A) Rational and different

(B) Rational and equal

(C) Irrational and different

(D) Imaginary and different

Sol.[D] $A = 1, B = -2(a + b), C = 2(a^2 + b^2)$

$B^2 - 4AC = 1[2(a + b)]^2 - 4(1)(2a^2 + 2b^2)$

$= 4a^2 + 4b^2 + 8ab - 8a^2 - 8b^2$

$= -4a^2 - 4b^2 + 8ab$

$= -4(a-b)^2 < 0$

So roots are imaginary and different.

Ex.50 The roots of the equation $x^2 - 2\sqrt{2}\,x + 1 = 0$ are -

(A) Real and different

(B) Imaginary and different

(C) Real and equal

(D) Rational and different

Sol.[A] The discriminant of the equation

$(-2\sqrt{2})^2 - 4(1)(1) = 8 - 4 = 4 > 0$ and a perfect square so roots are real and different but we can't say that roots are rational because coefficients are not rational therefore.

$\alpha, \beta = \dfrac{2\sqrt{2} - \sqrt{\left(2\sqrt{2}\right)^2 - 4}}{2} = \dfrac{2\sqrt{2} \pm 2}{2} = \sqrt{2} \pm 1$ this is irrational.

$\therefore$ the roots are real and different.

Ex.51 The roots of the equation

$(b + c)x^2 - (a + b + c)x + a = 0$ are $(a, b, c \in Q)$ -

(A) Real and different

(B) Rational and different

(C) Imaginary and different

(D) Real and equal

Sol.[B] The discriminant of the equation is

$(a + b + c)^2 - 4(b + c)(a)$

$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca - 4(b+c)a$

$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca - 4ab - 4ac$

$= a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$

$(a - b - c)^2 > 0$

So roots are rational and different.

Ex.52 If the roots of the equation $x^2 + 2x + P = 0$ are real then the value of P is -

(A) $P \le 2$ (B) $P \le 1$

(C) $P \le 3$ (D) None of these

Sol.[B] Here $a = 1$, $b = 2$, $c = P$

$\therefore$ discriminant $= (2)^2 - 4(1)(P) \ge 0$

(Since roots are real)

$\Rightarrow 4 - 4P \ge 0 \quad \Rightarrow 4 \ge 4P$

$\Rightarrow P \le 1$

Examples based on **Sum and Product of Roots**

Ex.53 If the product of the roots of the quadratic equation $mx^2 - 2x + (2m-1) = 0$ is 3 then the value of m is -

(A) 1 (B) 2 (C) –1 (D) 3

Sol.[C] Product of the roots $c/a = 3 = \dfrac{2m-1}{m}$

$\therefore 3m - 2m = -1 \quad \Rightarrow m = -1$

Ex.54 If α and β are roots of the equation $x^2 - 5x + 6 = 0$ then the value of $\alpha^3 + \beta^3$ is-

(A) 35 (B) 40

(C) 45 (D) None of these

Sol.[A] Here $\alpha + \beta = 5$, $\alpha\beta = 6$

Now $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$= (5)^3 - 3.6(5) = 125 - 90 = 35$

Ex.55 If the equation $(k-2)x^2 - (k-4)x - 2 = 0$ has difference of roots as 3 then the value of k is-

(A) 1,3 (B) 3,3/2 (C) 2, 3/2 (D) 3/2, 1

Sol.[B] $(\alpha - \beta) = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

Now $\alpha + \beta = \frac{(k-4)}{(k-2)}$, $\alpha\beta = \frac{-2}{k-2}$

$\therefore\ (\alpha - \beta) = \sqrt{\left(\frac{k-4}{k-2}\right)^2 + \frac{8}{(k-2)}}$

$= \frac{\sqrt{k^2 + 16 - 8k + 8(k-2)}}{(k-2)}$

$\Rightarrow 3 = \frac{\sqrt{k^2 + 16 - 8k + 8k - 16}}{(k-2)}$

$\Rightarrow 3k - 6 = \pm k$

$\therefore\ k = 3,\ 3/2$

Ex.56 If α, β are roots of the equation $ax^2 + bx + c = 0$ then the value of $\frac{1}{(a\alpha+b)^2} + \frac{1}{(a\beta+b)^2}$ is -

(A) $\frac{b^2 - 2ac}{ac}$ (B) $\frac{2ac - b^2}{ac}$ (C) $\frac{b^2 - 2ac}{a^2c^2}$ (D) $\frac{b^2}{a^2c}$

Sol.[C] Since α, β are the root of the $ax^2 + bx + c$

then $a\alpha^2 + b\alpha + c = 0$

$\Rightarrow \alpha(a\alpha + b) + c = 0$

$\Rightarrow (a\alpha + b) = -c/\alpha$ ...(1)

Similarly

$(a\beta + b) = -c/\beta$ ...(2)

$\therefore\ \frac{1}{(a\alpha+b)^2} + \frac{1}{(a\beta+b)^2} = \frac{1}{(-c/\alpha)^2} + \frac{1}{(-c/\beta)^2}$

$\Rightarrow \frac{\alpha^2}{c^2} + \frac{\beta^2}{c^2} = \frac{\alpha^2 + \beta^2}{c^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{c^2}$

$= \frac{b^2/a^2 - 2c/a}{c^2} = \frac{b^2 - 2ac}{a^2c^2}$

Example based on **Formation of Quadratic Equation with given Roots**

Ex.57 The equation whose roots are 3 and 4 will be -

(A) $x^2 + 7x + 12 = 0$

(B) $x^2 - 7x + 12 = 0$

(C) $x^2 - x + 12 = 0$

(D) $x^2 + 7x - 12=0$

Sol.[B] The quadratic equation is given by

x^2 – (sum of the roots) x + (product of roots) = 0

∴ The required equation

$= x^2 - (3 + 4)\ x + 3.4 = 0$

$= x^2 - 7x + 12 = 0$

Ex.58 The quadratic equation with rational coefficients whose one root is $2 + \sqrt{3}$ is -

(A) $x^2 - 4x + 1= 0$ (B) $x^2 + 4x + 1= 0$

(C) $x^2 + 4x - 1= 0$ (D) $x^2 + 2x + 1=0$

Sol.[A] The required equation is

$x^2 - \{(2+\sqrt{3}) + (2-\sqrt{3})\}\ x$

$+ (2 + \sqrt{3})(2-\sqrt{3}) = 0$

or $x^2 - 4x + 1 = 0$

Ex.59 If α,β are the root of a quadratic equation $x^2 - 3x + 5 = 0$ then the equation whose roots are $(\alpha^2 - 3\alpha + 7)$ and $(\beta^2 - 3\beta + 7)$ is -

(A) $x^2 + 4x + 1= 0$ (B) $x^2 - 4x + 4 = 0$

(C) $x^2 - 4x - 1= 0$ (D) $x^2 + 2x + 3 = 0$

Sol.[B] Since α,β are the roots of equation $x^2 - 3x + 5 = 0$

So $\alpha^2 - 3\alpha + 5 = 0$ & $\beta^2 - 3\beta + 5 = 0$

∴ $\alpha^2 - 3\alpha = -5$ & $\beta^2 - 3\beta = -5$

putting in $(\alpha^2 - 3\alpha + 7)$ & $(\beta^2 - 3\beta + 7)$ ...(1)

$-5 + 7, -5 + 7$

∴ 2 and 2 are the roots

∴ the required equation is

$x^2 - 4x + 4 = 0.$

Ex.60 If α,β are roots of the equation $x^2 - 5x + 6 = 0$ then the equation whose roots are $\alpha + 3$ and $\beta + 3$ is -

(A) $x^2 - 11x + 30 = 0$

(B) $(x-3)^2 - 5(x-3) + 6 = 0$

(C) Both (1) and (2)

(D) None

Sol.[C] Let $\alpha + 3 = x$

∴ $\alpha = x - 3$ (Replace x by x – 3)

So the required equation is

$(x - 3)^2 - 5(x-3) + 6 = 0$ ...(1)

$\Rightarrow x^2 - 6x + 9 - 5x + 15 + 6 = 0$

$\Rightarrow x^2 - 11x + 30 = 0$ ...(2)

Ex.61 If α,β are roots of the equation $2x^2 + x - 1 = 0$ then the equation whose roots are $1/\alpha$, $1/\beta$ will be -

(A) $x^2 + x - 2 = 0$ (B) $x^2 + 2x - 8 = 0$

(C) $x^2 - x - 2 = 0$ (D) None of these

Sol.[C] From the given equation

$\alpha + \beta = -1/2$, $\alpha\beta = -1/2$

The required equation is -

$$x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \frac{1}{\alpha\beta} = 0$$

$$\Rightarrow x^2 - \left(\frac{\alpha+\beta}{\alpha\beta}\right)x + \frac{1}{\alpha\beta} = 0$$

$$\Rightarrow x^2 - \left(\frac{-1/2}{-1/2}\right)x + \frac{1}{-1/2} = 0$$

$$\Rightarrow x^2 - x - 2 = 0$$

Short cut : Replace x by 1/x

$\Rightarrow 2(1/x)^2 + 1/x - 1 = 0$ $\quad\Rightarrow x^2 - x - 2 = 0$

Examples based on **Roots under Particular Cases**

Ex.62 The roots of the equation $x^2 - 3x - 4 = 0$ are –

(A) Opposite and greater root in magnitude is positive

(B) Opposite and greater root in magnitude is negative

(C) Reciprocal to each other

(D) None of these

Sol.[A] The roots of the equation $x^2 - 3x - 4 = 0$ are of opposite sign and greater root is positive ($\because$ a > 0 , b < 0, c < 0)

Ex.63 The roots of the equation $2x^2 - 3x + 2 = 0$ are -

(A) Negative of each other

(B) Reciprocal to each other

(C) Both roots are zero

(D) None of these

Sol.[B] The roots of the equations $2x^2 - 3x + 2 = 0$ are reciprocal to each other because here a = c

Ex.64 If equation $\frac{x^2 - bx}{ax - c} = \frac{k-1}{k+1}$ has equal and opposite roots then the value of k is -

(A) $\frac{a+b}{a-b}$ (B) $\frac{a-b}{a+b}$

(C) $\frac{a}{b}+1$ (D) $\frac{a}{b}-1$

Sol.[B] Let the roots are α & $-\alpha$.

given equation is

$(x^2 - bx)(k + 1) = (k - 1)(ax - c)$

$\Rightarrow x^2(k + 1) - bx(k + 1) = ax(k - 1) - c(k - 1)$

$\Rightarrow x^2(k + 1) - bx(k - 1) - ax(k - 1) + c(k - 1) = 0$

Now sum of roots = 0 $\quad(\because \alpha - \alpha = 0)$

$\therefore$ b (k + 1) + a (k − 1) = 0

$\Rightarrow k = \frac{a-b}{a+b}$

Examples based on **Condition for Common Roots**

Ex.65 If one root of the equations $x^2 + 2x + 3k = 0$ and $2x^2 + 3x + 5k = 0$ is common then the values of k is -

(A) 1, 2 (B) 0, −1

(C) 1, 3 (D) None of these

Sol.[B] Since one root is common, let the root is α.

$$\frac{\alpha^2}{10k-9k} = \frac{\alpha}{6k-5k} = \frac{1}{3-4}$$

$\alpha^2 = -k$ ...(1)

$\alpha = -k$ ...(2)

$\therefore \alpha^2 = k^2$

$\Rightarrow k^2 = -k \quad \Rightarrow k^2 + k = 0$

$\Rightarrow k(k+1) = 0 \Rightarrow k = 0$ and $k = -1$

Ex.66 If the equations $2x^2 + x + k = 0$ and $x^2 + x/2 - 1 = 0$ have 2 common roots then the value of k is -

(A) 1 (B) 3 (C) −1 (D) −2

Sol.[D] Since the given equation have two roots in common so from the condition

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad \Rightarrow \frac{2}{1} = \frac{1}{1/2} = \frac{k}{-1}$$

$\therefore k = -2$

Ex.67 If $x^2 + x - 1 = 0$ and $2x^2 - x + \lambda = 0$ have a common root then –

(A) $\lambda^2 - 7\lambda + 1 = 0$ (B) $\lambda^2 + 7\lambda - 1 = 0$

(C) $\lambda^2 + 7\lambda + 1 = 0$ (D) $\lambda^2 - 7\lambda - 1 = 0$

Sol.[C] Let the common root is α then

$\alpha^2 + \alpha - 1 = 0$

$2\alpha^2 - \alpha + \lambda = 0$

By cross multiplication

$$\frac{\alpha^2}{\lambda-1} = \frac{\alpha}{-2-\lambda} = \frac{1}{-1-2}$$

$$\alpha^2 = \frac{\lambda-1}{-3} = \frac{1-\lambda}{3}, \quad \alpha = \frac{2+\lambda}{3}$$

$$\left(\frac{2+\lambda}{3}\right)^2 = \frac{1-\lambda}{3} \Rightarrow \lambda^2 + 7\lambda + 1 = 0$$

Example based on **Maximum & Minimum value of Quadratic Expression**

Ex.68 The minimum value of the expression

$4x^2 + 2x + 1$ is -

(A) 1/4 (B) 1/2 (C) 3/4 (D) 1

Sol.[C] Since a = 4 > 0 therefore its minimum value is $= \frac{4(4)(1)-(2)^2}{4(4)} = \frac{16-4}{16} = \frac{12}{16} = \frac{3}{4}$

Ex.69 The maximum value of $5 + 20x - 4x^2$ for all real value of x is -

(A) 10 (B) 20 (C) 25 (D) 30

Sol.[D] Since a = –4 < 0 therefore its maximum value is -

$$= \frac{4(-4)(5)-(20)^2}{4(-4)} = \frac{-80-400}{-16} = \frac{-480}{-16} = 30$$

❖ SOME MORE EXAMPLES ❖

Ex.70 If r and s are positive, then roots of the equation $x^2 - rx - s = 0$ are -

(A) imaginary

(B) real and both positive

(C) real and of opposite signs

(D) real and both negative

Sol.[C] Here Discriminant

$= r^2 + 4s > 0$ $(\because\ r, s > 0)$

$\Rightarrow$ roots are real

Again a = 1 > 0 and c = – s < 0

$\Rightarrow$ roots are of opposite signs.

Ex.71 Both roots of the equation

$(x - b)(x - c) + (x - c)(x - a) + (x - a)(x - b) = 0$ are -

(A) positive (B) negative

(C) real (D) imaginary

Sol.[C] The given equation can be written in the following form :

$3x^2 - 2(a + b + c)x + (ab + bc + ca) = 0$

Here discriminant

$= 4(a + b + c)^2 - 12(ab + bc + ca)$

$= 4[(a^2 + b^2 + c^2) - (ab + bc + ca)] > 0$

$[\because\ a^2 + b^2 + c^2 > ab + bc + ca]$

$\therefore$ Both roots are real.

Ex.72 For the equation $\frac{1}{x+a} - \frac{1}{x+b} = \frac{1}{x+c}$, if the product of roots is zero, then the sum of roots is -

(A) 0 (B) $\frac{2ab}{b+c}$ (C) $\frac{2bc}{b+c}$ (D) $\frac{-2bc}{b+c}$

Sol.[D] $\frac{1}{x+a} - \frac{1}{x+b} = \frac{1}{x+c}$

$\frac{b-a}{x^2+(b+a)x+ab} = \frac{1}{x+c}$

or $x^2 + (a + b)x + ab = (b - a)x + (b - a)c$

or $x^2 + 2ax + ab + ca - bc = 0$

Since product of the roots = 0

$ab + ca - bc = 0 \Rightarrow a = \frac{bc}{b+c}$

Thus sum of roots $= -2a = \frac{-2bc}{b+c}$

Ex.73 If α, β are roots of the equation

$ax^2 + 3x + 2 = 0$ $(a < 0)$, then $\alpha^2/\beta + \beta^2/\alpha$ is greater than -

(A) 0 (B) 1

(C) 2 (D) None of these

Sol.[D] Since a < 0, therefore discriminant

$D = 9 - 8a > 0$. So, α and β are real.

We have : $\alpha + \beta = \frac{-3}{a}$ and $\alpha\beta = \frac{2}{a}$

$$\therefore \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^3}{\alpha\beta} - 3(\alpha+\beta)$$

$$= -\frac{27}{2a^2} + \frac{9}{a} < 0 \qquad [\because a < 0]$$

Ex.74 If α, β are the roots of $x^2 - p(x + 1) - c = 0$ then $\frac{\alpha^2 + 2\alpha + 1}{\alpha^2 + 2\alpha + c} + \frac{\beta^2 + 2\beta + 1}{\beta^2 + 2\beta + c}$ is equal to -

(A) 0 (B) 1

(C) 2 (D) None of these

Sol.[B] Here the equation is $x^2 - p(x + 1) - c = 0$

$\therefore \alpha + \beta = p,\ \alpha\beta = -(p + c)$

$\Rightarrow (\alpha + 1)(\beta + 1) = 1 - c$

Now given expression

$$= \frac{(\alpha+1)^2}{(\alpha+1)^2 - (1-c)} + \frac{(\beta+1)^2}{(\beta+1)^2 - (1-c)},$$

Putting value of $1 - c = (\alpha + 1)(\beta + 1)$

$$= \frac{\alpha+1}{\alpha-\beta} + \frac{\beta+1}{\beta-\alpha} = \frac{\alpha+1-\beta-1}{\alpha-\beta} = 1$$

Note : Some times an intermediate step calculation is necessary before the main problem is attempted. As above

$(\alpha + 1)(\beta + 1) = \alpha\beta + (\alpha + \beta) + 1$

$= -p - c + p + 1 = 1 - c$

Without which the main result would be difficult to find.

Ex. 75 If α, β be the roots $x^2 + px - q = 0$ and γ, δ be the roots of $x^2 + px + r = 0$ then $\frac{(\alpha-\gamma)(\alpha-\delta)}{(\beta-\gamma)(\beta-\delta)} =$

(A) 1 (B) q (C) r (D) q + r

Sol.[A] Here $\left.\begin{matrix}\alpha+\beta=-p\\ \gamma+\delta=-p\end{matrix}\right\} \Rightarrow \alpha + \beta = \gamma + \delta$ (note)

Now $(\alpha - \gamma)(\alpha - \delta) = \alpha^2 - \alpha(\gamma + \delta) + \gamma\delta$

$= \alpha^2 - \alpha(\alpha + \beta) + r$

$= -\alpha\beta + r$

$= -(-q) + r = q + r$

By symmetry of the results

$(\beta - \gamma)(\beta - \delta) = q + r$

Hence the ratio is 1

Note : If we ignore the equality of $\alpha + \beta$ and $\gamma + \delta$, the problem may be seen to be difficult, or at least the calculations are increased unnecessarily.

Ex.76 If α, β be roots of $x^2 + px + 1 = 0$ and γ, δ are the roots of $x^2 + qx + 1 = 0$ then $(\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta) =$

(A) $p^2 + q^2$ (B) $p^2 - q^2$

(C) $q^2 - p^2$ (D) none of these

Sol.[C] Here $\left.\begin{matrix}\alpha+\beta=-p; & \alpha\beta=1 \\ \gamma+\delta=-q; & \gamma\delta=1\end{matrix}\right\} \Rightarrow \alpha\beta = \gamma\delta$

Now $(\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta)$

$= \{\alpha\beta - \gamma(\alpha + \beta) + \gamma^2\}\{\alpha\beta + \delta(\alpha + \beta) + \delta^2\}$

$= \{1 + \gamma p + \gamma^2\}\{1 - p\delta + \delta^2\}$

$= [(\gamma^2+1) + \gamma p][(\delta^2 + 1) - p\delta)]$

$= (-q\gamma + \gamma p)(-q\delta - p\delta)$

$= \gamma\delta(q^2 - p^2) = 1(q^2 - p^2)$

Note : Remember that the root always satisfy the equation and hence this fact may be used to find some values which may be occurring in the calculations.

Ex.77 If α and β are roots of the equation $x^2+px+q=0$ and α^4 and β^4 are roots of $x^2 - rx + s = 0$, then the roots of $x^2 - 4qx + 2q^2 - r = 0$ are –

(A) both real

(B) both positive

(C) both negative

(D) one negative and one positive

Sol.[A] The discriminant of the equation

$x^2 - 4qx + 2q^2 - r = 0$ is

$D = 16q^2 - 4(2q^2 - r) = 8q^2 + 4r$ ...(1)

But α, β are roots of the equation

$x^2 + px + q = 0$

$\Rightarrow \alpha + \beta = -p$ and $\alpha\beta = q$

and α^4, β^4 are roots of the equation

$x^2 - rx + s = 0$

$\Rightarrow \alpha^4 + \beta^4 = r$ and $\alpha^4\beta^4 = s$

$\therefore D = 8\alpha^2\beta^2 + (\alpha^4 + \beta^4)$

$= 4(\alpha^2 + \beta^2)^2 \geq 0$

Thus both roots are real.

Ex.78 If one root of the equation $4x^2 + 2x - 1 = 0$ is α, then other root is -

(A) 2α (B) $4\alpha^3 - 3\alpha$

(C) $4\alpha^3 + 3\alpha$ (D) None of these

Sol.[B] Let α and β are roots of the given equation, then $\alpha + \beta = -\frac{1}{2} \Rightarrow \beta = -\frac{1}{2} - \alpha$

Now $4\alpha^2 + 2\alpha - 1 = 0$

$\Rightarrow 4\alpha^2 = 1 - 2\alpha$ ...(1)

Now $4\alpha^3 = \alpha - 2\alpha^2$

$= \alpha - \frac{1}{2}(1 - 2\alpha)$ [from (1)]

$\therefore\ 4\alpha^3 - 3\alpha = -\ 2\alpha - \frac{1}{2}(1-2\alpha)$

$= -\frac{1}{2} - \alpha = \beta$

Ex.79 If α, β are roots of $\mathbf{A}x^2 + Bx + C = 0$ and α^2, β^2 are roots of $x^2 + px + q = 0$, then p is equal to -

(A) $(B^2 - 2AC)/A^2$ (B) $(2AC - B^2)/A^2$

(C) $(B^2 - 4AC)/A^2$ (D) $(4AC - B^2)A^2$

Sol.[B] $\alpha + \beta = -B/A$, $\alpha\beta = C/A$

$\alpha^2 + \beta^2 = -p$, $\alpha^2\beta^2 = q$

$\therefore\ (\alpha + \beta)^2 = B^2/A^2$

$\Rightarrow (\alpha^2 + \beta^2) + 2\alpha\beta = B^2/\mathbf{A}^2$

$\Rightarrow -p + 2C/A = B^2/A^2$

$\Rightarrow p = \frac{2CA - B^2}{A^2}$

Ex.80 The quadratic equation whose one root is $\frac{1}{2+\sqrt{5}}$ will be -

(A) $x^2 + 4x - 1 = 0$ (B) $x^2 - 4x - 1 = 0$

(C) $x^2 + 4x + 1 = 0$ (D) None of these

Sol.[A] Given root $= \frac{1}{2+\sqrt{5}} = \sqrt{5} - 2$

So the other root $= -\sqrt{5} - 2$. Then sum of the roots $= -4$, product of the roots $= -1$

Hence the equation is $x^2 + 4x - 1 = 0$

Ex.81 If the roots of equation $x^2 + bx + ac = 0$ are α, β and roots of the equation $x^2 + ax + bc = 0$ are α, γ then the value of α, β, γ respectively -

(A) a,b,c (B) b,c,a

(C) c,a,b, (D) None of thes

Sol.[C] From the given two equation

$\alpha + \beta = -b$ ...(1)

$\alpha\beta = ac$ ...(2)

$\alpha + \gamma = -a$ ...(3)

$\alpha\gamma = bc$ ...(4)

(1) – (3) $\Rightarrow \beta - \gamma = a - b$ ...(5)

(2) / (4) $\Rightarrow \beta/\gamma = a/b$

$\beta = \frac{a\gamma}{b}$ ...(6)

putting the value of β in (5)

$\frac{a\gamma}{b} - \gamma = a - b \Rightarrow \gamma\frac{(a-b)}{b} = (a-b)$

$\therefore \gamma = b$

$\therefore \beta = a \quad \& \ \alpha = c.$

Ex.82 The value of the expression $x^2 + 2bx + c$ will be positive if -

(A) $b^2 - 4c > 0$ (B) $b^2 - 4c < 0$

(C) $c^2 < b$ (D) $b^2 < c$

Sol.[D] Expression $= (x + b)^2 - b^2 + c$

$= (x + b)^2 + (c - b^2)$

$\therefore$ expression will be positive if $c - b^2 > 0$

$\Rightarrow b^2 < c$

Aliter : Here $a = 1 > 0$. Hence exp. > 0 when $B^2 - 4AC < 0$ i.e. when $4b^2 - 4c < 0 \Rightarrow b^2 < c$.

Ex.83 If roots of the equation $x^2 + ax + 25 = 0$ are in the ratio of 2: 3 then the value of a is -

(A) $\frac{\pm 5}{\sqrt{6}}$ (B) $\frac{\pm 25}{\sqrt{6}}$

(C) $\frac{\pm 5}{6}$ (D) None of these

Sol.[B] Here $k = 2/3$

so from the condition $\frac{(k+1)^2}{k} = \frac{b^2}{ac}$

$\frac{(2/3+1)^2}{2/3} = \frac{a^2}{25}$

$\Rightarrow \frac{25}{9} \times \frac{3}{2} = \frac{a^2}{25} \Rightarrow \frac{25}{6} = \frac{a^2}{25}$

$\therefore a^2 = \frac{25 \times 25}{6} \Rightarrow a = \frac{\pm 25}{\sqrt{6}}$

Ex.84 Let α, β be the roots of $ax^2 + bx + c = 0$ & γ, δ be the roots of $px^2 + qx + r = 0$; and D_1, D_2 the respective Discriminants of these equations. If $\alpha, \beta, \gamma, \delta$ are in A.P., then $D_1 : D_2$

(A) $\frac{a^2}{p^2}$ (B) $\frac{a^2}{b^2}$ (C) $\frac{b^2}{q^2}$ (D) $\frac{c^2}{r^2}$

Sol.[A] We have $\alpha + \beta = \frac{-b}{a}$, $\alpha\beta = \frac{c}{a}$

and $\gamma + \delta = \frac{-q}{p}$, $\gamma\delta = \frac{r}{p}$

Now $\alpha, \beta, \gamma, \delta$ are in A.P.

$\Rightarrow \beta - \alpha = \delta - \gamma$; $(\beta - \alpha)^2 = (\delta - \gamma)^2$

$\Rightarrow (\beta + \alpha)^2 - 4\alpha\beta = (\gamma + \delta)^2 - 4\gamma\delta$

$\Rightarrow \frac{b^2}{a^2} - \frac{4c}{a} = \frac{q^2}{p^2} - \frac{4r}{p}$

$\Rightarrow \frac{b^2 - 4ac}{a^2} = \frac{q^2 - 4pr}{p^2}$

$\Rightarrow \frac{D_1}{a^2} = \frac{D_2}{p^2} \Rightarrow \frac{D_1}{D_2} = \frac{a^2}{p^2}$

Ex.85 If $x = 2 + \sqrt{3}$ then the value of $x^3 - 7x^2 + 13x - 12$ is –

(A) 3 (B) 6 (C) –9 (D) 9

Sol.[C] $x = 2 + \sqrt{3}$

$\Rightarrow x - 2 = \sqrt{3}$ $\Rightarrow x^2 + 4 - 4x = 3$

$\Rightarrow x^2 - 4x = -1$ $\Rightarrow x^2 - 4x + 1 = 0$

Now we can write the given equation as

$x^3 - 7x^2 + 13x - 12$

$= x(x^2 - 4x + 1) - 3x^2 + 12x - 12$

$= x(x^2 - 4x + 1) - 3(x^2 - 4x + 1) - 9$

Now putting the value of $x^2 - 4x + 1 = 0$

$= x(0) - 3(0) - 9 = -9$

IMPORTANT POINTS TO BE REMEMBERED

1. An equation of the form $ax^2 + bx + c = 0$, $a \neq 0$ is known as quadratic equation.

2. An equation of the form $ax^2 + c = 0$ is known as pure quadratic equation.

3. If $x = \alpha$, is a root of this quadratic equation, then we have

$a\alpha^2 + b\alpha + c = 0$

4. Discriminant of the quadratic equation $ax^2 + bx + c = 0$ is $D = b^2 - 4ac$

5. Roots of the quadratic equation $ax^2 + bx + c = 0$ are $\alpha = \frac{-b+\sqrt{D}}{2a}$, $\beta = \frac{-b-\sqrt{D}}{2a}$

6. Nature of Roots -

(a) If $D= b^2 - 4ac > 0$ and a perfect square, roots are rational and unequal.

(b) If $D = b^2 - 4ac > 0$ and not perfect square, roots are irrational and unequal.

(c) If $D = b^2 - 4ac = 0$, roots are real and equal.

(d) If $D = b^2 - 4ac < 0$, real roots are not possible.

7. Method to convert word problem in to quadratic.

(a) Translate the word problem to a quadratic equation with the help of given condition

(b) Solve the quadratic equation thus formed.

(c) Interpret the solution of the equation by checking.

8. Condition for Common roots :

◈ **Only One Root Common :**

The condition for only one Root common is

$(c_1a_2 - c_2a_1)^2 = (b_1c_2 - b_2c_1)(a_1b_2 - a_2b_1)$

◈ **Both Roots Common :**

Required conditions is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

◈ $(\alpha-\beta) = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \pm\frac{\sqrt{b^2-4ac}}{a} = \frac{\pm\sqrt{D}}{a}$

◈ $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = \frac{b^2-2ac}{a^2}$

◈ $\alpha^2-\beta^2 = (\alpha+\beta)\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$$= -\frac{b\sqrt{b^2-4ac}}{a^2} = \frac{\pm\sqrt{D}}{a}$$

◈ $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$.

$$\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = -\frac{b(b^2-3ac)}{a^3}$$

- $\alpha^3 - \beta^3 = (\alpha - \beta)(\alpha^2 + \beta^2 + \alpha\beta)$.

$$\alpha^3 - \beta^3 = (\alpha - \beta)^3 + 3\alpha\beta(\alpha - \beta)$$

$$= \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}\,\{(\alpha+\beta)^2 - \alpha\beta\}$$

$$= \frac{(b^2 - ac)\sqrt{b^2 - 4ac}}{a^3}$$

- $\alpha^4 + \beta^4 = \{(\alpha+\beta)^2 - 2\alpha\beta\}^2 - 2\alpha^2\beta^2 = \left(\frac{b^2 - 2ac}{a^2}\right)^2 - 2\,\frac{c^2}{a^2}$

- $\alpha^4 - \beta^4 = (\alpha^2 - \beta^2)(\alpha^2 + \beta^2)$

$$= \frac{-b(b^2 - 2ac)\sqrt{b^2 - 4ac}}{a^4}$$

- $\alpha^2 + \alpha\beta + \beta^2 = (\alpha+\beta)^2 - \alpha\beta$

- $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

- $\alpha^2\beta + \beta^2\alpha = \alpha\beta(\alpha+\beta)$

- $\left(\frac{\alpha}{\beta}\right)^2 + \left(\frac{\beta}{\alpha}\right)^2 = \frac{\alpha^4 + \beta^4}{\alpha^2\beta^2} = \frac{(\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2}{\alpha^2\beta^2}$

EXERCISE # 1

Very Short Answer Type Questions

Q.1 Which of the following are quadratic polynomials

(i) $5x^2 - 8x + 12$ (ii) $3 + 4x - 7x^2$

(iii) $8x^2 - 15$ (iv) $8x - 15$

(v) $8x^3 - 3x$ (vi) $x^2 - \sqrt{5}x + 2\sqrt{3}$

(vii) $\sqrt{3}x^2 - 10x - 5\sqrt{3}$

(viii) $\sqrt{7} - \sqrt{5}x - \sqrt{3}x^3$

(ix) $\sqrt{15}x^2 - \sqrt{5}x + 7$

Q.2 Find the value of each given polynomial at the given value of its variable :

(i) $5x^2 - 7x + 2$ at $x = 3$

(ii) $x^2 + 15x - 4$ at $x = -1$

(iii) $2y^2 - y + 2$ at $y = -2$

(iv) $3y + 8 - 2y^2$ at $y = -3$

(v) $\sqrt{2}x^2 + 3x + 1$ at $x = \sqrt{2}$

(vi) $x^3 - 3x^2 + 5x + 2$ at $x = -4$

(vii) $5\sqrt{2}x^3 + 2x^2 - \sqrt{2}x + 1$ at $x = 2\sqrt{2}$

Q.3 Find the value of constant 'm' if :

(i) $x = -2$ is a zero of quadratic polynomial $4x^2 - 3mx + 5$.

(ii) $y = -5$ is a zero of quadratic polynomial $7 + 4(m + 2)y - y^2$

Q.4 Which of the following are quadratic equations :

(i) $x^2 - 9x + 5 = 0$ (ii) $x^2 - \frac{3}{x} = 2$

Q.5 Which of the following are quadratic equations :

(i) $x - \frac{3}{x} = 2x^2$ (ii) $15x^2 + 27x - 33 = 0$

Q.6 Which of the following are quadratic equations :

(i) $\sqrt{3}x^2 + 8x = 3\sqrt{2}$ (ii) $\frac{7}{8}x^2 - \frac{3}{5}x + \frac{5}{7} = 0$

Q.7 Determine whether $x = -\frac{2}{\sqrt{3}}$ and $x = -3\sqrt{3}$ are solutions of given equation or not :

$\sqrt{3}x^2 + 11x + 6\sqrt{3} = 0$

Q.8 Determine if $x = 5$ is a root of equation given below or not :

$\sqrt{2x^2 + 4x - 5} - \sqrt{x^2 - 4x + 4} = \sqrt{1 - 12x + 3x^2}$

Q.9 In each case given below; find the value of 'm' for which the given value of the variable is a solution of the equation :

(i) $(2m + 1)x^2 + 2x - 3 = 0$; $x = 2$

(ii) $3x^2 + 2mx - 3 = 0$; $2x - 1 = 0$

(iii) $x^2 + 2ax - m = 0$; $x + a = 0$

Q.10 Determine whether $x = \frac{1}{2}$ and $x = \frac{3}{2}$ are solutions of the equation $2x^2 - 5x + 3 = 0$ or not.

Q.11-Q.27 Solve each of the following quadratic equations

Q.11 $x^2 + 5x + 6 = 0$ **Q.12** $x^2 - 8x - 33 = 0$

Q.13 $x^2 + 4x - 32 = 0$ **Q.14** $x^2 + 5x - 6 = 0$

Q.15 $x^2 - 5x - 6 = 0$ **Q.16** $x^2 - 5x + 6 = 0$

Q.17 $5x^2 - 2ax - 3a^2 = 0$ **Q.18** $x^2 + 8x = 0$

Q.19 $3x^2 + 2ax - a^2 = 0$ **Q.20** $4x^2 - 25x - 21 = 0$

Q.21 $10x^2 - 7x - 12 = 0$ **Q.22** $8x^2 - 2x - 3 = 0$

Q.23 $3x^2 - 7x - 6 = 0$ **Q.24** $x(4x - 7) = 0$

Q.25 $x(x + 1) + (x + 2)(x + 3) = 26$

Q.26 $x(x - 1) + (x - 2)(x - 3) = 42$

Q.27 $4x^2 = 25$

Q.28 Without solving, examine the nature of roots of the equations :

(i) $3x^2 + 2x - 1 = 0$ (ii) $4x^2 + 3x - 1 = 0$
(iii) $6x^2 - 5x - 6 = 0$ (iv) $x^2 - 6x + 9 = 0$
(v) $2x^2 - 5x + 5 = 0$ (vi) $3x^2 + 7x + 3 = 0$
(vii) $4x^2 - 4x + 1 = 0$ (viii) $5x^2 - 8x + 2 = 0$
(ix) $x^2 + px - q^2 = 0$

Q.29 Find the discriminant of the following quadratic equations :

(i) $x^2 - 3x + 1 = 0$ (ii) $4x^2 + 3x - 2 = 0$
(iii) $x^2 - x + 1 = 0$ (iv) $9x^2 - px + 2 = 0$
(v) $ax^2 - 3x - 5 = 0$ (vi) $4x^2 - 5x + c = 0$
(vii) $\sqrt{2}x^2 + 5\sqrt{3}x - 2\sqrt{2} = 0$
(viii) $3\sqrt{5}x^2 - 8x + 2\sqrt{5} = 0$

Q.30 Find the sum and the product of the roots of the following equations :

(i) $x^2 + 3x + 3 = 0$ (ii) $8x^2 - 3x + 4 = 0$
(iii) $4x^2 + 2x - 1 = 0$ (iv) $2x^2 - 7x + 4 = 0$
(v) $3x^2 - x + 1 = 0$ (vi) $3x^2 + x - 1 = 0$
(vii) $3x^2 + 4\sqrt{2}x + 9 = 0$ (viii) $2x^2 + 5\sqrt{3}x - 3 = 0$
(ix) $x^2 - 2\sqrt{5}x - 15 = 0$ (x) $5x^2 - 10x + 3\sqrt{5} = 0$

Short Answer Type Questions

Q.31-Q.40 Find the roots (if they exist) of the following quadratic equations by the method of completing the square :

Q.31 $x^2 - 2\sqrt{5}x + 1 = 0$ **Q.32** $4x^2 + x - 5 = 0$

Q.33 $3x^2 + 7x - 6 = 0$ **Q.34** $9x^2 + x + 15 = 0$

Q.35 $x^2 - 5x + 7 = 0$ **Q.36** $4x^2 + 3x + 5 = 0$

Q.37 $x^2 + 4x - 9 = 0$ **Q.38** $9x^2 - 15x + 6 = 0$

Q.39 $2x^2 - 5x + 3 = 0$ **Q.40** $5x^2 - 6x - 2 = 0$

Q.41-Q.53 Solve each of the following equations by using quadratic formula :

Q.41 $x^2 - 2\sqrt{2}x - 6 = 0$

Q.42 $\sqrt{6}x^2 - 4x - 2\sqrt{6} = 0$

Q.43 $\sqrt{3}x^2 + 11x + 6\sqrt{3} = 0$

Q.44 $16x^2 - 1 = 0$ **Q.45** $x^2 - 5x + 4 = 0$

Q.46 $x^2 - 7x - 18 = 0$ **Q.47** $x^2 + 2x - 15 = 0$

Q.48 $3x^2 + 2x - 8 = 0$ **Q.49** $5x^2 - x - 4 = 0$

Q.50 $4x^2 - 7x + 3 = 0$ **Q.51** $6x^2 + 7x - 5 = 0$

Q.52 $x^2 = 3x$ **Q.53** $3x^2 - 5x = 0$

Q.54 In the following, determine the set of values of p for which the quadratic equation has real roots :

(i) $px^2 + 4x + 1 = 0$ (ii) $2x^2 + 3x + p = 0$
(iii) $2x^2 + px + 3 = 0$ (iv) $3x^2 - 2px - 5 = 0$
(v) $2px^2 - 6x - 3 = 0$

Q.55 In the following, determine whether the given quadratic equations have real roots and if so find the roots :

(i) $x^2 + 6x + 6 = 0$ (ii) $x^2 - 3x + 4 = 0$
(iii) $4x^2 + x - 3 = 0$ (iv) $9x^2 + 30x + 25 = 0$
(v) $4x^2 - 12x + 9 = 0$ (vi) $3x^2 - 3x + 1 = 0$
(vii) $3x^2 - 3x - 1 = 0$ (viii) $4x^2 + 5\sqrt{3}x + 3 = 0$
(ix) $5x^2 - 2\sqrt{5}x - 3 = 0$

Q.56 Find the quadratic equation whose roots are :

(i) 5 and – 5 (ii) 8 and 3
(iii) – 8 and – 3 (iv) – 8 and 3
(v) $\sqrt{3}$ and $5\sqrt{3}$ (vi) $2\sqrt{2}$ and $-3\sqrt{2}$
(vii) $-3\sqrt{5}$ and $-4\sqrt{5}$ (viii) $1 + \sqrt{2}$ and $1 - \sqrt{2}$
(ix) $4 - \sqrt{5}$ and $4 + \sqrt{5}$ (x) $7 + \sqrt{7}$ and $7 - \sqrt{7}$
(xi) $\frac{3+\sqrt{2}}{3}$ and $\frac{3-\sqrt{2}}{3}$ (xii) $\frac{4-\sqrt{5}}{2}$ and $\frac{4+\sqrt{5}}{2}$

Long Answer Type Questions

Q.57 Find the value of 'm' so that the roots of the equation : $(4 - m)x^2 + (2m + 4)x + (8m + 1) = 0$ may be equal.

Q.58 For the quadratic equation $ax^2 + 7x + c = 0$; the sum of roots is –1 and the product of roots is 1 ; find the values of 'a' and 'c'.

Q.59 For the quadratic equation $ax^2 - 3x - b = 0$; the sum of roots is 6 and the product of roots is – 8; find the values of 'a' and 'b'.

Q.60 Find the value of p ; if one root of quadratic equation $3x^2 - px - 6 = 0$ is 3. Also, find the second (other) roots of the equation.

Q.61 If α and β are the roots of the equation $2x^2 + 5x - 4 = 0$; find the value of :

(i) $\alpha^2 + \beta^2$ (ii) $\alpha^2 + \beta^2 - 3\alpha - 3\beta$
(iii) $\alpha^2 + \beta^2 - 4\alpha\beta$ (iv) $\alpha^3 + \beta^3$
(v) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

Q.62 If α and β are the roots of the equation,

$x^2 - 6x + 1 = 0$; find the value of :

(i) $\alpha^2 + \beta^2$ (ii) $\alpha^4 + \beta^4$

(iii) $\alpha^3 + \beta^3$ (iv) $\alpha^2 + \beta^2 - 2\alpha\beta$

Q.63 For each equation, given below, find the value (s) of p so that the equation has equal roots :

(i) $2x^2 - 7x + p = 0$

(ii) $6x^2 + 12x - p = 0$

(iii) $px^2 + 4x + p = 0$

(iv) $2px^2 - 20x + (13p - 1) = 0$

(v) $3px^2 + 18x + p = 0$

Q.64-Q.72 Solve the following equations by reducing them to quadratic equation :

Q.64 $\sqrt{x^2-4x+3} + \sqrt{x^2-9} = \sqrt{4x^2-14x+6}$, $x \in R$

Q.65 $\sqrt{x^2+4x-21} + \sqrt{x^2-x-6} = \sqrt{6x^2-5x-39}$

Q.66 $\sqrt{2x^2-3x-9} + \sqrt{x^2-2x-3} = 5\sqrt{x-3}$

Q.67 $\sqrt{x^2-25} - (x - 5) = \sqrt{x^2-16x+55}$

Q.68 $\sqrt{x^2-16} - \sqrt{x^2-8x+16} = \sqrt{x^2-5x+4}$

Q.69 $\sqrt{2x^2+x+3} + \sqrt{2x^2+x-6} = 3$

Q.70 $\sqrt{3x^2+x+5} = x - 3$

Q.71 $9\left(x^2+\frac{1}{x^2}\right) - 9\left(x+\frac{1}{x}\right) - 52 = 0$

Q.72 $3\left(x^2+\frac{1}{x^2}\right) - \left(x-\frac{1}{x}\right) - 30 = 0$

Q.73-Q.88 Solve each of the following equations :

Q.73 $2\left(\frac{x}{x+1}\right)^2 - 5\left(\frac{x}{x+1}\right) + 2 = 0$; $x \neq -1$

Q.74 $\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = \frac{13}{6}$; $x \neq 1$; $x \neq 0$

Q.75 $\left(\frac{2x+1}{x-1}\right)^4 - 10\left(\frac{2x+1}{x-1}\right)^2 + 9 = 0$

Q.76 $9^{x+2} - 6 . 3^{x+1} + 1 = 0$

Q.77 $6\sqrt{\frac{x}{x+4}} - 2\sqrt{\frac{x+4}{x}} = 11$

Q.78 $\sqrt{x^2-16} - (x + 4) = \sqrt{3x^2+8x-16}$

Q.79 $3 \times 4^{x+1} - 13 \times 2^x = 140$

Q.80 $5 \times 9^{x+1} - 15 \times 3^{x+2} = 2430$

Q.81 $\left(x-\frac{1}{x}\right)^2 + 8\left(x+\frac{1}{x}\right) = 29$; $x \neq 0$

Q.82 $(x^2 + 3x)^2 - (x^2 + 3x) - 6 = 0$; $x \in R$

Q.83 $(x^2 - 5x)^2 - 7(x^2 - 5x) + 6 = 0$; $x \in R$

Q.84 $4\left(\frac{x}{x+1}\right)^4 - 25\left(\frac{x}{x+1}\right)^2 + 36 = 0$

Q.85 $\left(x+\frac{1}{x}\right)^2 - \frac{3}{2}\left(x-\frac{1}{x}\right) - 4 = 0$

Q.86 (i) $\sqrt{7x+9} - \sqrt{x+8} = 1$

(ii) $\sqrt{2x+4} - \sqrt{x-5} = 3$

Q.87 $3\left(x-\frac{1}{x}\right)^2 - 16\left(x+\frac{1}{x}\right) + 32 = 0;\ x \neq 0$

Q.88 $x^4 + 2x^3 - 13x^2 + 2x + 1 = 0$

ANSWER KEY

A. VERY SHORT ANSWER TYPE :

1. (i), (ii), (iii), (vi), (vii), (ix) **2.** (i) 26 (ii) –18 (iii) 12 (iv) –19 (v) $5\sqrt{2}+1$ (vi) –130 (vii) 173

3. (i) $-\frac{7}{2}$ (ii) $-\frac{29}{10}$ **4.** (i) **5.** (ii) **6.** (i), (ii) **7.** yes

8. no **9.** (i) $-\frac{5}{8}$ (ii) $\frac{9}{4}$ (iii) $-a^2$ **10.** no[only x = 3/2 is a solution of the given equation]

11. –3, –2 **12.** 11, –3 **13.** –8, 4 **14.** –6, 1 **15.** 6, – 1 **16.** 3, 2 **17.** $a, -\frac{3a}{5}$

18. 0, –8 **19.** $\frac{a}{3}, -a$ **20.** $7, -\frac{3}{4}$ **21.** $\frac{3}{2}, -\frac{4}{5}$ **22.** $\frac{3}{4}, -\frac{1}{2}$ **23.** $3, -\frac{2}{3}$ **24.** $0, \frac{7}{4}$

25. 2, – 5 **26.** 6, –3 **27.** $\frac{5}{2}, -\frac{5}{2}$

28. (i) Rational (real) and unequal. (ii) Rational (real) and unequal. (iii) Rational and unequal. (iv) Real and equal. (v) Imaginary (vi) Irrational and unequal. (vii) Real and equal (viii) Irrational and unequal. (ix) Irrational and unequal.

29. (i) 5 (ii) 41 (iii) – 3 (iv) $p^2 - 72$ (v) $9 + 20a$ (vi) $25 - 16c$ (vii) 91 (viii) –56

30. (i) –3, 3 (ii) $\frac{3}{8}, \frac{1}{2}$ (iii) $-\frac{1}{2}, -\frac{1}{4}$ (iv) $\frac{7}{2}, 2$ (v) $\frac{1}{3}, \frac{1}{3}$ (vi) $-\frac{1}{3}, -\frac{1}{3}$

(vii) $\frac{-4\sqrt{2}}{3}, 3$ (viii) $\frac{-5\sqrt{3}}{2}, -\frac{3}{2}$ (ix) $2\sqrt{5}, -15$ (x) $2, \frac{3\sqrt{5}}{5}$

B. SHORT ANSWER TYPE :

31. $\sqrt{5}+2, \sqrt{5}-2$ **32.** $1, -\frac{5}{4}$ **33.** $\frac{2}{3}, -3$ **34.** no real root. **35.** $\frac{5+\sqrt{7}}{2}, \frac{5-\sqrt{7}}{2}$

36. no real roots. **37.** 1, 5 **38.** $1, \frac{2}{3}$ **39.** $1, \frac{3}{2}$ **40.** $\frac{3+\sqrt{19}}{5}, \frac{3-\sqrt{19}}{5}$

41. $3\sqrt{2}, -\sqrt{2}$ **42.** $\sqrt{6}, -\frac{2}{\sqrt{6}}$ **43.** $-3\sqrt{3}, -\frac{-2}{\sqrt{3}}$ **44.** $\frac{1}{4}, -\frac{1}{4}$

45. 4, 1 **46.** 9, – 2 **47.** 3, – 5 **48.** $\frac{4}{3}, -2$ **49.** $1, -\frac{4}{5}$

50. $1, \frac{3}{4}$ **51.** $\frac{1}{2}, -\frac{5}{3}$ **52.** 0, 3 **53.** $0, \frac{5}{3}$

54. (i) $p \le 4$ (ii) $p \le \frac{9}{8}$ (iii) $p^2 \ge 24$ (iv) $p^2 + 15 \ge 0$ (v) $p \ge -\frac{3}{2}$

Hint :

(i) Comparing $px^2 + 4x + 1 = 0$ with
$ax^2 + bx + c = 0$; we get :
$a = p$, $b = 4$ and $c = 1$
Now the roots will be real, if $b^2 - 4ac \ge 0$
$\Rightarrow (4)^2 - 4 \times p \times 1 \ge 0$
$\Rightarrow -4p \ge p \times 1 \ge 0$
$\Rightarrow 4p \le 16$ and $p \le 4$

55. (i) yes, $-3 \pm \sqrt{3}$ (ii) no (iii) yes, $-1, \frac{3}{4}$ (iv) yes, $-\frac{5}{3}, -\frac{5}{3}$ (v) yes, $\frac{3}{2}, \frac{3}{2}$ (vi) no (vii) yes, $\frac{3 \pm \sqrt{21}}{6}$

(viii) yes, $-\sqrt{3}, -\frac{\sqrt{3}}{4}$ (ix) yes, $\frac{3\sqrt{5}}{5}$

56. (i) $x^2 - 25 = 0$ (ii) $x^2 - 11x + 24 = 0$ (iii) $x^2 + 11x + 24 = 0$

(iv) $x^2 + 5x - 24 = 0$ (v) $x^2 - 6\sqrt{3}x + 15 = 0$ (vi) $x^2 + \sqrt{2}x - 12 = 0$

(vii) $x^2 + 7\sqrt{5}x + 60 = 0$ (viii) $x^2 - 2x - 1 = 0$ (ix) $x^2 - 8x - 11 = 0$

(x) $x^2 - 14x + 42 = 0$ (xi) $9x^2 - 18x + 7 = 0$ (xii) $4x^2 - 16x + 11 = 0$

C. LONG ANSWER TYPE :

57. 0, 3 **58.** a = 7, c = 7 **59.** $a = \frac{1}{2}$, b = 4 **60.** $p = 7, -\frac{2}{3}$

61. (i) $\frac{41}{4}$ (ii) $\frac{71}{4}$ (iii) $\frac{73}{4}$ (iv) $-\frac{245}{8}$ (v) $-\frac{41}{8}$ **62.** (i) 34 (ii) 1154 (iii) 198 (iv) 32

63. (i) $6\frac{1}{8}$ (ii) –6 (iii) ±2 (iv) $2, -\frac{25}{13}$ (v) $\pm 3\sqrt{3}$ **64.** 3 **65.** 3 **66.** 3 **67.** 5. 13

68. 4, 5 **69.** – 2 **70.** no solution **71.** $3, \frac{1}{2}, -\frac{-7 \pm \sqrt{13}}{6}$

72. $-3, \frac{1}{3}, \frac{3 \pm \sqrt{13}}{2}$ **73.** 1, –2 **74.** $\frac{9}{13}, \frac{4}{13}$ **75.** $4, \frac{2}{5}, -2, 0$ **76.** –2

77. $-\frac{16}{3}$ **78.** $-4, -\frac{20}{3}$ **79.** 2 **80.** 2 **81.** $\frac{3 \pm \sqrt{5}}{2}, \frac{-11 \pm \sqrt{117}}{2}$

82. $-1, -2, \frac{-3 \pm \sqrt{21}}{2}$ **83.** $6, -1, \frac{5 \pm \sqrt{29}}{2}$ **84.** $-2, -\frac{2}{3}, -3, -\frac{3}{5}$

85. $1, -1, -\frac{1}{2}, 2$ **86.** (i) 1 (ii) 30, 6 **87.** $\frac{1}{3}, 1, 3$

88. $\frac{-5 \pm \sqrt{21}}{2}, \frac{3 \pm \sqrt{5}}{2}$

Hint : Divide both the sides by x^2 to get :

$$x^2 + 2x - 13 + \frac{2}{x} + \frac{1}{x^2} = 0; \Rightarrow \left(x^2 + \frac{1}{x^2}\right) + 2\left(x + \frac{1}{x}\right) - 13 = 0$$

EXERCISE # 2

Q.1 The value of the expression $16x^2 + 24x + 9$ for $x = -\frac{3}{4}$ is -

(A) 2 (B) 1 (C) 0 (D) –1

Q.2 x = 3 is a solution of the equation $3x^2 + (k - 1)x + 9 = 0$ if k has value -

(A) 13 (B) –13 (C) 11 (D) –11

Q.3 The roots of the equation $x^2 + 2x - 35 = 0$ are -

(A) 5, 7 (B) –5, –7

(C) –5, 7 (D) 5, – 7

Q.4 Match List I with List II :

List I	List II
a. Roots of $2x^2 - 9x + 7 = 0$	1. $\frac{7}{2}$ and 7
b. Roots of $2x^2 - 21x + 49 = 0$	2. $\frac{7}{2}$ and 1
c. Roots of $x^2 - 6x + 9 = 0$	3. $\frac{7}{2}$ and 3
d. Roots of $2x^2 - 13x + 21 = 0$	4. 3 and 3

	a b c d		a b c d
(A)	2 1 4 3	(B)	1 2 3 4
(C)	2 1 3 4	(D)	3 1 4 2

Q.5 The quadratic polynomial in x whose zeros are a, 2a is -

(A) (x + a) (x – 2a) (B) (x – 2a) (x + 2a)

(C) (x + a) (x + 2a) (D) (x – a) (x – 2a)

Q.6 The expresson $x^4 + 7x^2 + 16$ can be factorized as-

(A) $(x^2 + x + 1)(x^2 + x + 16)$

(B) $(x^2 + x + 1)(x^2 - x + 16)$

(C) $(x^2 + x - 4)(x^2 - x + 4)$

(D) $(x^2 + x - 4)(x^2 - x - 4)$

Q.7 The solution of $2 - x = \frac{x-2}{x}$ would include -

(A) –2, –1 (B) 2, – 1

(C) –4, 2 (D) 4, –2

Q.8 The common roots of the equations $x^2 - 7x + 10 = 0$ and $x^2 - 10x + 16 = 0$ is -

(A) –2 (B) 3 (C) 5 (D) 2

Q.9 Let $f(x) = ax^2 + bx + c$. Then, match the followig :

a. Sum of roots of $f(x) = 0$ — 1. $\frac{c}{a}$

b. Product of roots of $f(x) = 0$ — 2. $\frac{-b}{a}$

c. Roots of $f(x) = 0$ are real & distinct — 3. $b^2 - 4ac = 0$

d. Roots of $f(x) = 0$ are real and identical. — 4. $b^2 - 4ac < 0$

The correct matching is -

	a	b	c	d
(A)	2	1	4	3
(B)	1	2	3	4
(C)	4	3	1	2
(D)	3	4	1	2

Q.10 Let $f(x) = x^2 - 2qx - 1$. Match the following -

a. Sum of roots of $f(x) = 0$ — 1. $2q$

b. Product of roots of $f(x) = 0$ — 2. -1

c. The roots of $f(x) = 0$ are both equal — 3. $q = -1$

4. Never

	a	b	c
(A)	2	1	4
(B)	2	1	3
(C)	1	2	4
(D)	1	2	3

Q.11 The sum of the roots of the equation $x^2 - 6x + 2 = 0$ is -

(A) -6 (B) -2 (C) 2 (D) 6

Q.12 If the product of the roots of $x^2 - 3x + k = 10$ is -2 the value of k is -

(A) -2 (B) 8 (C) 12 (D) -8

Q.13 If one root of the equation $2x^2 + ax + 6 = 0$ is 2, then a equals -

(A) 7 (B) $\frac{7}{2}$ (C) -7 (D) $\frac{-7}{2}$

Q.14 The ratio of the sum and the product of the roots of $7x^2 - 12x + 18 = 0$ is -

(A) 7 : 12 (B) 2 : 3
(C) 3 : 2 (D) 7 : 18

Q.15 The roots of $2x^2 - 6x + 7 = 0$ are -

(A) real, unequal and rational
(B) real, unequal and irrational
(C) real and equal
(D) imaginary

Q.16 The roots of $2x^2 - 6x + 3 = 0$ are -

(A) real, unequal and rational
(B) real, unequal and irrational
(C) real and equal
(D) imaginary

Q.17 With respect to the roots of $x^2 - x - 2 = 0$, we can say that -

(A) both of them are natural numbers

(B) both of them are integers

(C) the latter of the two is negative

(D) None of these

Q.18 The equation $x^2 + 4x + k = 0$ has real roots, then-

(A) $k \geq 4$ (B) $k \leq 4$

(C) $k \leq 0$ (D) $k \geq 0$

Q.19 The value of k for which $x^2 - 4x + k = 0$ has coincident roots is -

(A) 4 (B) –4 (C) 0 (D) –2

Q.20 The equation (m being real), $mx^2 + 2x + m = 0$ has two distinct roots if -

(A) $m \neq 0$ (B) $m \neq 0, 1$

(C) $m \neq 1, -1$ (D) $m \neq 0, 1, -1$

Q.21 If the equation $x^2 + 2(k + 2)x + 9k = 0$ has equal roots, the values of k are -

(A) 1, 4 (B) –1, 4 (C) 1, –4 (D) –1, –4

Q.22 If the roots of $ax^2 + bx + c = 0$ be equal, then the value of c is -

(A) $\frac{-b}{2a}$ (B) $\frac{b}{2a}$ (C) $\frac{-b^2}{4a}$ (D) $\frac{b^2}{4a}$

Q.23 If the roots of $x^2 + 4mx + 4m^2 + m + 1 = 0$ are real, then -

(A) $m = -1$ (B) $m \leq -1$

(C) $m \geq -1$ (D) $m \geq 0$

Q.24 The roots of the equation

$(q - r)x^2 + (r - p)x + (p - q) = 0$ are -

(A) $\frac{r-p}{q-r}$, 1 (B) $\frac{p-q}{q-r}$, 1

(C) $\frac{q-r}{p-q}$, 1 (D) $\frac{r-p}{p-q}$, 1

Q.25 Vidhya and Vandana solved a quadratic equation. In solving it, Vidhya made a mistake in the constant term and got the roots as 6 and 2, while Vandana made a mistake in the coefficient of x only and obtained the roots as –7 and –1. The correct roots of the equation are -

(A) 6, – 1 (B) –7, 2

(C) –6, –2 (D) 7, 1

Q.26 A and B solved a quadratic equation. In solving it, A made a mistake in the constant term and obtained the roots as 5, – 3 while B made a mistake in the coefficient of x and obtained the roots as 1, –3. The correct roots of the equation are -

(A) 1, 3 (B) –1, 3

(C) –1, –3 (D) 1, –1

Q.27 If the equation $9x^2 + 6kx + 4 = 0$ has equal roots, then the value of k must be -

(A) zero (B) either 2 or zero

(C) either –2 or zero (D) either 2 or –2

Q.28 The roots of $\frac{x+4}{x-4}+\frac{x-4}{x+4}=\frac{10}{3}$ are -

(A) ± 4 (B) ± 6 (C) ± 8 (D) $2 \pm \sqrt{3}$

Q.29 The relationship between x and y as shown in the given table is :

x	0	1	2	3	4
y	100	90	70	40	0

(A) $y = 100 - 10x$ (B) $y = 100 - 5x - 5x^2$

(C) $y = 100 - 5x^2$ (D) $y = 20 - x - x^2$

Q.30 The roots of a quadratic equation are 5 and –2. The equation is -

(A) $x^2 - 3x + 10 = 0$ (B) $x^2 - 3x - 10 = 0$

(C) $x^2 + 3x + 10 = 0$ (D) $x^2 + 3x - 10 = 0$

Q.31 If the sum of the roots of a quadratic equation is 6 and the product of the roots is also 6, then the equation is -

(A) $x^2 - 6x + 6 = 0$ (B) $x^2 + 6x - 6 = 0$

(C) $x^2 - 6x - 6 = 0$ (D) $x^2 + 6x + 6 = 0$

Q.32 If one root of the equation $3x^2 - 10x + 3 = 0$ is $\frac{1}{3}$, the other root is -

(A) $-\frac{1}{3}$ (B) –3 (C) 3 (D) $\frac{1}{3}$

Q.33 If the equation $ax^2 - 5x + c = 0$ has 10 as the sum of rhe roots and also as the product of the roots, which of the follwoing is true ?

(A) $a = \frac{1}{2}, c = 5$ (B) $a = 2, c = 3$

(C) $a = 5, c = \frac{1}{2}$ (D) $a = 3, c = 2$

Q.34 If the sum of the roots of the equation $kx^2 + 2x + 3k = 0$ is equal to their product, then the value of k is -

(A) $\frac{1}{3}$ (B) $-\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $-\frac{2}{3}$

Q.35 If α, β be the roots of $ax^2 + bx + c = 0$, the value of $\alpha^2 + \beta^2$ is -

(A) $\frac{b^2-4ac}{2a}$ (B) $\frac{b^2-2ac}{2a}$

(C) $\frac{b^2-2ac}{a^2}$ (D) $\frac{b^2-4ac}{2ac}$

Q.36 If α, β are the the roots $x^2 - px + q = 0$, then the value of $\alpha^2 + \beta^2$ is -

(A) $p^2 + 2q$ (B) $p^2 - 2q$

(C) $p(p^2 - 3q)$ (D) $p^2 - 4q$

Q.37 If α, β are the roots of the quadratic equation $x^2 - 6x + 6 = 0$, the value of $\alpha^2 + \beta^2$ is -

(A) 36 (B) 24 (C) 12 (D) 6

Q.38 If α, β are the roots of the equation $x^2 - 8x + p = 0$ and $\alpha^2 + \beta^2 = 40$, p is equal to -
(A) 8 (B) 10 (C) 12 (D) 14

Q.39 If α, β are the roots of the equation $x^2 + x + 1 = 0$, the value of $\alpha^4 + \beta^4$ is -
(A) 0 (B) 1
(C) –1 (D) None of these

Q.40 If α, β are the roots of the equations $2x^2 - 4x + 3 = 0$, then the value of $\alpha^3 + \beta^3$ is -
(A) –1 (B) 1 (C) 2 (D) 0

Q.41 If α, β are the roots of the equation $x^2 - 5x + 6 = 0$, the value of $\alpha^2 - \beta^2$ is -
(A) ± 4 (B) ± 5 (C) ± 6 (D) 0

Q.42 If α, β are the roots of $x^2 + px + q = 0$, the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ is -
(A) $\frac{p^2 - 2q}{q}$ (B) $\frac{p^2 + 2q}{q}$
(C) $\frac{-p^2 + 2q}{q}$ (D) $\frac{-p^2 - 2q}{q}$

Q.43 If α, β are the roots of the equation $ax^2 + bx + c = 0$, the value of $\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2}$ is -
(A) $\frac{b^2 - 2ac}{ac}$ (B) $\frac{-b^3 + 3abc}{a^3}$
(C) $\frac{-b^3 + 3abc}{ac^2}$ (D) $\frac{b^2 - 2ac}{a^2}$

Q.44 If one root of $5x^2 + 13x + k = 0$ be the reciprocal of the other root, the value of k is -
(A) 0 (B) 1
(C) 2 (D) 5

Q.45 The roots of the equation $ax^2 + bx + c = 0$ will be reciprocals if -
(A) a = b (B) b = c
(C) c = a (D) None of these

Q.46 The value of k for which the roots α, β of the equation : $x^2 - 6x + k = 0$ satisfy the relation $3\alpha + 2\beta = 20$, is -
(A) 8 (B) –8 (C) 16 (D) –16

Q.47 If α, β are the roots of the equation $2x^2 - 3x + 1 = 0$, then the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is -
(A) $2x^2 + 5x + 2 = 0$ (B) $2x^2 - 5x - 2 = 0$
(C) $2x^2 + 5x - 2 = 0$ (D) $2x^2 - 5x + 2 = 0$

Q.48 If α, β are the roots of the equation $x^2 - 3x + 2 = 0$, then the equation whose roots are $(\alpha + 1)$ and $(\beta + 1)$ is -
(A) $x^2 + 5x + 6 = 0$ (B) $x^2 - 5x - 6 = 0$
(C) $x^2 + 5x - 6 = 0$ (D) $x^2 - 5x + 6 = 0$

Q.49 If α, β are the roots of the equation $2x^2 - 5x + 7 = 0$, then the equation whose roots are $(2\alpha + 3\beta)$ and $(3\alpha + 2\beta)$ is -

(A) $2x^2 + 25x + 82 = 0$

(B) $2x^2 - 25x - 82 = 0$

(C) $2x^2 - 25x + 82 = 0$

(D) $2x^2 + 25x - 82 = 0$

Q.50 The quadratic equation whose roots are the reciprocals of the roots of the equation $3x^2 - 20x + 17 = 0$, is -

(A) $20x^2 - 17x + 3 = 0$

(B) $17x^2 - 20x + 3 = 0$

(C) $20x^2 + 17x - 3 = 0$

(D) $17x^2 + 20x - 3 = 0$

Q.51 If α, β are the roots of the equation $x^2 + kx + 12 = 0$ such that $\alpha - \beta = 1$, the value of k is -

(A) 0 (B) ± 5

(C) ± 1 (D) ± 7

Q.52 The roots of the equation $x^2 + px + q = 0$ are 1 and 2. The roots of the equation $qx^2 - px + 1 = 0$ must be -

(A) $1, \frac{1}{2}$ (B) $\frac{-1}{2}, -1$

(C) $\frac{-1}{2}, 1$ (D) $-1, \frac{1}{2}$

Q.53 If the equations $x^2 + 2x - 3 = 0$ and $x^2 + 3x - k = 0$ have a common root, then the non-zero value of k is -

(A) 1 (B) 2 (C) 3 (D) 4

Q.54 If the equations $2x^2 - 7x + 3 = 0$ and $4x^2 + ax - 3 = 0$ have common root, then the values of a are -

(A) -11 or 4 (B) -11 or -4

(C) 11 or -4 (D) 11 or 4

Q.55 Consider the following statements :

I. If the roots of the equation $ax^2 + bx + c = 0$ are negative reciprocal of each other, then $a + c = 0$.

II. A quadratic equation can have maximum two roots.

III. If α, β are the roots of a quadratic equation such that $\alpha + \beta = 22$ and $\alpha - \beta = 8$, then the equation $x^2 - 22x + 112 = 0$, has α and β as its roots.

IV. If α, β are the roots of the equation $2x^2 - 4x + 1 = 0$, the value of $\frac{1}{\alpha + 2\beta} + \frac{1}{\beta + 2\alpha}$ is $\frac{12}{17}$.

Of these statements.

(A) I, II and IV are correct

(B) I, III & IV are correct

(C) none is correct

(D) all are correct

Q.56 The positive value of m for which the roots of the equation $12x^2 + mx + 5 = 0$ are in the ratio 3 : 2 is -

(A) $5\sqrt{10}$ (B) $\frac{5}{2}\sqrt{10}$

(C) $\frac{5}{12}$ (D) $\frac{12}{5}$

Q.57 Match List I with List II. List I contains quadratic polynomials and List II contains the conditions for these polynomials to be fractorizable into a product of real linear factors.

List I	List II
a. $4x^2 + kx + 1$	1. $k \le \frac{1}{2}$
b. $kx^2 - 4x + k$	2. $k \ge 4$ or $k \le -4$
c. $kx^2 - 2x + 2$	3. $k \ge 8$ or $k \le 0$
d. $2x^2 - kx + k$	4. $-2 \le k \le 2$

a b c d | a b c d

(A) 3 2 1 4 (B) 2 4 1 3

(C) 4 1 3 2 (D) 1 3 4 2

Q.58 The solution set of the equation $x^{2/3} + x^{1/3} - 2 = 0$ is -

(A) {8, 1} (B) {8, –1}

(C) {–8, –1} (D) {–8, 1}

Q.59 The value of x in the equation

$\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = 2\frac{1}{6}$ is -

(A) $\frac{5}{13}$ (B) $\frac{7}{13}$

(C) $\frac{9}{13}$ (D) None of these

Q.60 The value of x in the equation

$8\left(x^2 + \frac{1}{x^2}\right) - 42\left(x - \frac{1}{x}\right) + 29 = 0$ is -

(A) 4 (B) –2

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Q.61 The value of x in the equation

$\sqrt{4x-3} + \sqrt{2x+3} = 6$ is -

(A) 3 (B) 1

(C) 100 (D) 111

Q.62 The two parts into which 57 should be divided so that their product is 782, are -

(A) 43, 14 (B) 33, 24

(C) 34, 23 (D) 44, 13

Q.63 The sum of a number and its reciprocal is $2\frac{1}{20}$. the number is -

(A) $\frac{5}{4}$ (B) $\frac{3}{4}$ (C) $\frac{4}{3}$ (D) $\frac{1}{6}$

Q.64 A two digit number is such that the product of the digits is 8. When 18 is added to the number, the digits are reversed. The number is -

(A) 18 (B) 24

(C) 81 (D) 42

Q.65 The perimter of a rectangle is 82 m and its area is 400 m^2. The breadth of the rectangle is -

(A) 25 m (B) 16 m

(C) 9 m (D) 20 m

Q.66 Out of a group of swans, $\left(\frac{7}{2}\right)$ times the square root of the number are swimming in water while two remaining are playing on the shore. The total numebr of swans is -

(A) 4 (B) 8 (C) 12 (D) 16

ANSWER KEY

Q.No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Ans.	C	D	D	A	D	C	B	D	A	C	D	B	C	B	D	B	B	B
Q.No	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Ans.	A	C	A	D	B	B	D	B	D	C	B	B	A	C	A	D	C	B
Q.No	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
Ans.	B	C	C	A	B	A	C	D	C	D	D	D	C	B	D	B	D	A
Q.No	55	56	57	58	59	60	61	62	63	64	65	66						
Ans.	A	A	B	D	C	A	D	C	A	B	B	D						

HINTS & SOLUTION

8. **Hint :** $x^2 - 7x + 10 = 0 \Leftrightarrow (x - 3)(x - 2) = 0$ $\Leftrightarrow x = 5, 2$ $x^2 - 10x + 16 = 0 \Leftrightarrow (x - 8)(x - 2) = 0 \Leftrightarrow x = 8, 2$

$\therefore$ Common root is 2.

19. For coincident roots, D = 0 i.e. $16 - 4k \geq = 0$

$\Rightarrow k = 4.$

23. For real roots, $D \geq 0$

i.e., $16m^2 - 4(4m^2 + m + 1) \geq 0$

$\therefore -4m - 4 \geq 0$ or $4m \leq -4$ or $m \leq -1$

24. The given equation may be written as :

$(q - r)x^2 - [(q - r) + (p - q)]x + (p - q) = 0$

$\Rightarrow (q - r)x^2 - (q - r)x - (p - q)x + (p - q) = 0$

$\Rightarrow (q - r)x(x - 1) - (p - q)(x - 1) = 0.$

$\Rightarrow (x - 1)[(q - r)x - (p - q)] = 0.$

So, $x = 1$ or $x = \frac{p-q}{q-r}$.

25. When there is no mistake in a and b, the sum of roots must be correct. When there is no mistake in a and c, product of the roots must be correct.

$\therefore$ Sum of roots = (6 + 2) = 8 & Product of roots = (–7) (–1) = 7. So, the correct equation is :

$x^2 - 8x + 7 = 0 \Rightarrow (x - 7)(x - 1) = 0.$

$\therefore$ The roots are 7, 1.

26. Sum of roots = 5 + (–3) = 2,

Product of roots = $1 \times (-3) = -3$.

$\therefore$ Eqn. is $x^2 - 2x - 3 = 0$

$\Leftrightarrow (x - 3)(x + 1) = 0 \Leftrightarrow x = 3$ or -1.

46. $\alpha + \beta = 6$ and $3\alpha + 2\beta = 20 \Rightarrow \alpha = 4, \beta = 2.$

Product of roots = k. So, $k = (4 \times 2) = 8$

49. $\alpha + \beta = \frac{5}{2}$ and $\alpha\beta = \frac{7}{2}$.

$\therefore$ Sum of new roots $= (2\alpha + 3\beta) + (3\alpha + 2\beta) = 5(\alpha + \beta) = 5 \times \frac{5}{2} = \frac{25}{2}$.

Product of new roots $= (2\alpha + 3\beta)(3\alpha + 2\beta)$

$= 6(\alpha^2 + \beta^2) + 13\alpha\beta = 6[(\alpha + \beta)^2 - 2\alpha\beta] + 13\alpha\beta = 6(\alpha + \beta)^2 + \alpha\beta = 6 \times \frac{25}{4} + \frac{7}{2}$ $= 41.$

$\therefore$ Required equation is : $x^2 - \frac{25}{2}x + 41 = 0$

i.e., $2x^2 - 25x + 82 = 0.$

50. Putting $x = \frac{1}{y}$ in the given equation, we get :

$\frac{3}{y^2} - \frac{20}{y} + 17 = 0$ or $3 - 20y + 17y^2 = 0$

So, the required equation is : $17x^2 - 20x + 3 = 0$.

51. $\alpha + \beta = -k$ and $\alpha\beta = 12$

$\Rightarrow (\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = k^2 - 48.$

$\therefore\ k^2 - 48 = 1 \Rightarrow k^2 = 49 \Rightarrow k = \pm 7.$

52. 1 and 2 must satisfy $x^2 + px + q = 0$.

So, $p + q = -1$ and $2p + q = -4$

$\Leftrightarrow p = -3$ and $q = 2$.

$\therefore\ qx^2 - px + 1 = 0 \Leftrightarrow 2x^2 + 3x + 1 = 0$

$\Leftrightarrow (x + 1)(2x + 1) = 0 \Leftrightarrow x = -1$ or $-\frac{1}{2}$

53. Let α be a common root of the given equations.

Then, $\alpha^2 + 2\alpha - 3 = 0$ and $\alpha^2 + 3\alpha - k = 0$

$\therefore\ \frac{\alpha^2}{-2k+9} = \frac{\alpha}{-3+k} = \frac{1}{3-2},$

So, $\alpha^2 = \frac{9-2k}{1}$ & $\alpha = \frac{k-3}{1}$

So, $(9 - 2k) = (k - 3)^2$ or $k^2 - 4k = 0$

or $k(k - 4) = 0$. So, $k = 4$.

54. Let α be a common root of the given equations.

Then, $2\alpha^2 - 7\alpha + 3 = 0$ and $4\alpha^2 + a\alpha - 3 = 0$

$\therefore\ \frac{\alpha^2}{21-3a} = \frac{\alpha}{12+6} = \frac{1}{2a+28}$

$\therefore\ \alpha^2 = \frac{21-3a}{2a+28}$ and $\alpha = \frac{18}{2a+28} = \frac{9}{a+14}$.

$\therefore\ \frac{21-3a}{2a+28} = \left(\frac{9}{a+14}\right)^2 = \frac{81}{(a+14)^2}$

$\Leftrightarrow \frac{21-3a}{2} = \frac{81}{a+14}$.

$\therefore\ -3a^2 - 21a + 294 = 162 \Leftrightarrow 3a^2 + 21a - 132 = 0$

$\Leftrightarrow 3a^2 + 33a - 12a - 132 = 0$

$\Leftrightarrow (a + 11)(3a - 12) = 0$

$\therefore\ a = -11$ or 4.

55. Let the roots be α and $(-1/\alpha)$. Then, Product of roots $= -1$.

$\therefore\ \frac{c}{a} = -1$ or $a + c = 0$. So, I is true.

II is clearly true.

$\alpha + \beta = 22,\ \alpha - \beta = 8 \Rightarrow \alpha = 15,\ \beta = 7.$

So, $\alpha + \beta = 22$ & $\alpha\beta = 105$.

An equation with α, β as roots is

$$x^2 - 22x + 105 = 0$$

So, III is incorrect.

Further in IV we have : $\alpha + \beta = 2$ and $\alpha\beta = \frac{1}{2}$

$$\frac{1}{\alpha+2\beta} + \frac{1}{\beta+2\alpha} = \frac{(\beta+2\alpha)+(\alpha+2\beta)}{(\alpha+2\beta)(\beta+2\alpha)}$$

$$= \frac{3(\alpha+\beta)}{2(\alpha^2+\beta^2)+5\alpha\beta}$$

$$= \frac{3(\alpha+\beta)}{2[(\alpha+\beta)^2-2\alpha\beta]+5\alpha\beta} = \frac{3(\alpha+\beta)}{2(\alpha+\beta)^2+\alpha\beta}$$

$$= \frac{3\times 2}{2\times 4+\frac{1}{2}} = 6 \times \frac{2}{17} = \frac{12}{17}.$$

So, IV is correct. Hence I, II, IV are correct.

58. The given eqn. is : $y^2 + y - 2 = 0$, where $y = x^{1/3}$

Now, $y^2 + y - 2 = 0 \Rightarrow (y + 2)(y - 1) = 0$

$\Rightarrow y = -2$ or $y = 1$

$x^{1/3} = -2$ or $x^{1/3} = 1$

$\Rightarrow x = (-2)^3 = -8$ or $x = 1$

62. Let the parts be x and (57 – x).

$\therefore\ x(57 - x) = 782 \quad \Rightarrow x^2 - 57x + 782 = 0$

$\Rightarrow (x - 34)(x - 23) = 0$

$\Rightarrow x = 34$ or $x = 23$.

So, the required parts are 34 and 23.

63. Let the number be x. Then ,

$x + \frac{1}{x} = \frac{41}{20} \Rightarrow 20x^2 - 41x + 20 = 0$

$\Rightarrow (4x - 5)(5x - 4) = 0$

$\Rightarrow x = \frac{5}{4}$ or $x = \frac{4}{5}$ So, $x = \frac{5}{4}$

64. Let tens digit = x. Then unit digit = $\frac{8}{x}$

$\therefore \left(10x + \frac{8}{x}\right) + 18 = 10 \times \frac{8}{x} + x$

or $9x - \frac{72}{x} + 18 = 0$

or $x - \frac{8}{x} + 2 = 0$ or $x^2 + 2x - 8 = 0$

or $(x + 4)(x - 2) = 0$.

$\therefore\ x = 2$.

$\therefore$ Ten's digit = 2 & Unit digit = 4.

so, number = 24.

65. Semi-perimeter = 41.

Let length = x & breadth = (41 – x).

$\therefore$ x (41 – x) = 400

$\Rightarrow x^2 - 41x + 400 = 0$

$\Rightarrow (x - 16)(x - 25) = 0$

$\Rightarrow x = 16$ or 25.

$\therefore$ Breadth = 16 m

66. Let the number of swans be x.

Then, $\frac{7}{2}\sqrt{x} + 2 = x$

$\Rightarrow 2x - 7\sqrt{x} - 4 = 0$

$\Rightarrow 2y^2 - 7y - 4 = 0$, where $y = \sqrt{x}$

$\Rightarrow (2y + 1)(y - 4) = 0$

$\Rightarrow y = -\frac{1}{2}$ or $y = 4$

$\Rightarrow \sqrt{x} = -\frac{1}{2}$ or $\sqrt{x} = 4$

$\Rightarrow x = \frac{1}{4}$ or $x = 16$

$\therefore$ x = 16 (Number of swans can not be a fraction)