SIMILAR TRIANGLES

★ INTRODUCTION

In earlier classes, you have learnt about congruence of two geometric figures, and also some basic theorems and results on the congruence of triangle. Two geometric figures having same shape and size are congruent to each other but two geometric figures having same shape are called similar. Two congruent geometric figures are always similar but the converse may or may not be true.

All regular polygons of same number of sides such as equilateral triangle, squares, etc, are similar. All circles are similar.

In some cases, we can easily notice that two geometric figures are not similar. For example, a triangle and a rectangle can never be similar. In case, we are given two triangles, they may appear to be similar but actually they may not be similar. So, we need some criteria to determine the similarity of two geometric figures. In particular, we shall discuss similar triangles.

★ HISTORICAL FACTS

EUCLID was a very great Greek mathematician born about 2400 years ago. He is called the father of geometry because he was the first to establish a school of mathematics in Alexandria. He wrote a book on geometry called "The Elements" which has 13 volumes and has been used as a text book for over 2000 years. This book was further systematized by the great mathematician of Greece tike Thales, Pythagoras. Pluto and Aristotle.

Abraham Lincoln, as a young lawyer was of the view that this greek book was a splendid sharpner of human mind and improver his power of logic and language.

A king once asked Euclid, "Isn't there an easier way to understand geometry"

Euclid replied : "There is no royal-road way to geometry. Every one has to think for himself when studying."

THALES (640-546 B.C.) a Greek mathematician was the first who initiated and formulated the theoretical study of geometry to make astronomy a more exact science. He is said to have introduced geometry in Greece. He is believed to have found the heights of the pyramids in Egypt, using shadows and the principle of similar triangles. The use of similar triangles has made possible the measurements of heights and distances. He proved the well-known and very useful theorem credited after his name : Thales Theorem.

Euclid : Father of Geometry (about 300 B.C. Greece)

Thales (640-546 B.C.)

★ CONGRUENT FIGURES

Two geometrical figures are said to be congruent, provided they must have same shape and same size. Congruent figures are alike in every respect.

EX.
1. Two squares of the same length.
2. Two circle of the same radii.
3. Two rectangles of the same dimensions.
4. Two wings of a fan.
5. Two equilateral triangles of same length.

★ **SIMILAR FIGURES**

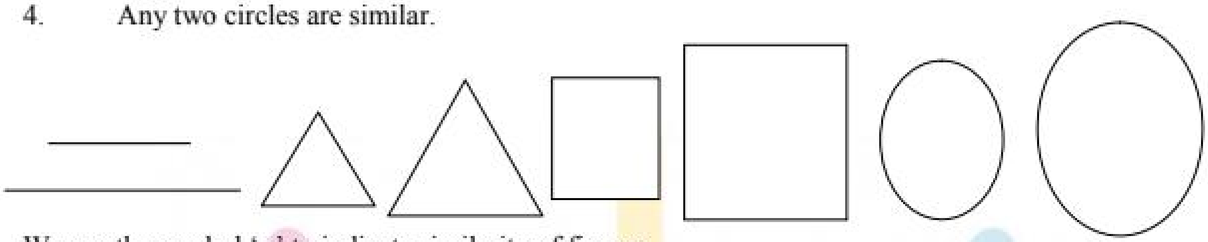
Two figures are said to be similar, if they have the same shape. Similar figures may differ in size. Thus, two congruent figures are always similar, but two similar figures need not be congruent.

EX. 1. Any two line segments are similar.

2. Any two equilateral triangles are similar

3. Any two squares are similar.

4. Any two circles are similar.



We use the symbol '~' to indicate similarity of figures.

★ **SIMILAR TRIANGLES**

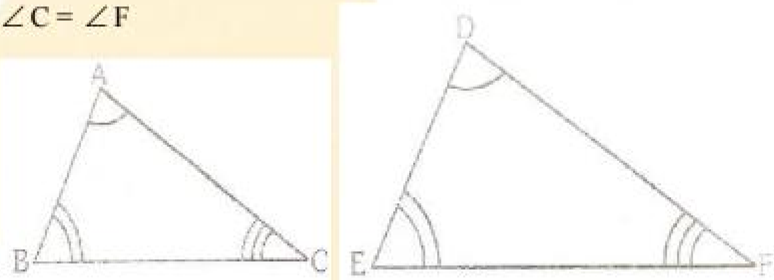
ΔABC and ΔDEF are said to be similar, if their corresponding angles are equal and the corresponding sides are proportional.

i.e., when $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$

and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$.

And, we write $\Delta ABC \sim \Delta DEF$.

The sign '~' is read as 'is similar to'.

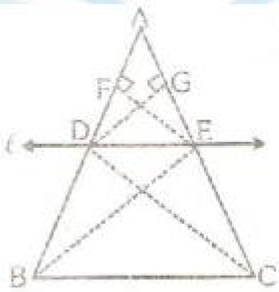


THEOREM-1 (Thales Theorem or Basic Proportionality Theorem) : If a line is drawn parallel to one side of a triangle intersecting the other two sides, then the other two sides are divided in the same ratio.

Given : A ΔABC in which line ℓ parallel to BC ($DE \parallel BC$) intersecting AB at D and AC at E.

To prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Join D to C and E to B. Through E drawn EF perpendicular to AB i.e., $EF \perp AB$ and through D draw $DG \perp AC$.



Proof :

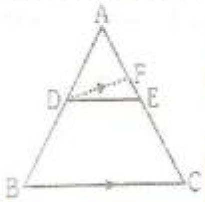
	STATEMENT	REASON
1.	Area of $(\Delta ADE) = \frac{1}{2}(AD \times EF)$ Area of $(\Delta BDE) = \frac{1}{2}(BD \times EF)$	Area of $\Delta = \frac{1}{2}$ base × altitude
2.	$\frac{Area(\Delta ADE)}{Area(\Delta BDE)} = \frac{\frac{1}{2}AD \times EF}{\frac{1}{2}BD \times EF} = \frac{AD}{DB}$	By 1.
3.	$\frac{Area(\Delta ADE)}{Area(\Delta BDE)} = \frac{\frac{1}{2}AE \times DG}{\frac{1}{2}EC \times DG} = \frac{AE}{EC}$	Similarly
4.	Area (ΔBDE) = Area (ΔCDE)	Δs BDE and CDE are on the same base BC and between the same parallel lines DE and BC.
5.	$\frac{Area(\Delta ADE)}{Area(\Delta BDE)} = \frac{AE}{EC}$	By 3. & 4.
6.	$\frac{AD}{DB} = \frac{AE}{EC}$	By 1. & 5.

Hence proved.

THEOREM-2 (Converse of Basic Proportionality Theorem) : If a line divided any two sides of a triangle proportionally, the line is parallel to the third side.

Given : A ΔABC and DE is a line meeting AB and AC at D and E respectively such that $\frac{AD}{DB} = \frac{AE}{EC}$

To prove : DE $\parallel$ BC



Proof :

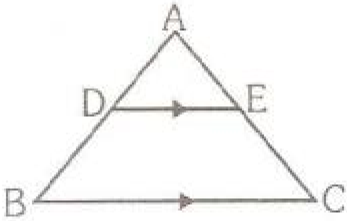
	STATEMENT	REASON
1.	If possible, let DE be not parallel to BC. Then, draw DF $\parallel$ BC	
2.	$\frac{AD}{DB} = \frac{AE}{FC}$	By Basic Proportionality Theorem.
3.	$\frac{AD}{DB} = \frac{AE}{EC}$	Given
4.	$\therefore \frac{AF}{FC} = \frac{AE}{EC}$	From 2 and 3.
	$\Rightarrow \frac{AF}{FC} + 1 = \frac{AE}{EC} + 1$	Adding 1 on both sides.
	$\Rightarrow \frac{AF + FC}{FC} = \frac{AE + EC}{EC}$	By adding.
	$\Rightarrow \frac{AC}{FC} = \frac{AC}{EC}$	AF + FC = AC and AE + EC = AC.
	$\Rightarrow FC = EC \Rightarrow$ E and F coincide. But, DF $\parallel$ BC. Hence DE $\parallel$ BC.	

Hence, proved.

Ex.1 In the adjoining figure. DE $\|$ BC.

(i) If AD = 3.4 cm, AB = 8.5 cm and AC = 13.5 cm, find AE

(ii) If $\frac{AD}{DB}=\frac{3}{5}$ and AC = 9.6 cm, find AE.



Sol. (i) Since DE $\|$ BC, we have $\frac{AD}{AB}=\frac{AE}{AC}$

$\therefore \quad \frac{3.4}{8.5}=\frac{AE}{13.5} \Rightarrow \frac{3.4\times 13.5}{8.5}=5.4$

Hence, AE = 5.4 cm.

(i) Since DE $\|$ BC, we have $\frac{AD}{DB}=\frac{AE}{EC}$

$\therefore \quad \frac{AE}{AC}=\frac{3}{5}\left[\because \frac{AD}{DB}=\frac{3}{5}(Given)\right]$

Let AE = x cm. Then, EC = (AC – AE) = (9.6 – x) cm.

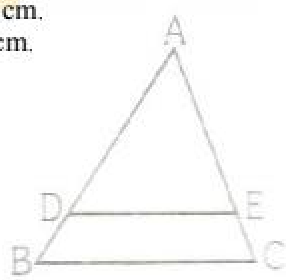
$\therefore \quad \frac{x}{9.6-x}=\frac{3}{5} \Rightarrow 5x = 3(9.6-x)$

$\Rightarrow \quad 5x = 28.8 - 3x \Rightarrow 8x = 28.8 \Rightarrow x = 3.6.$

$\therefore$ AE = 3.6 cm.

Ex.2 In the adjoining figure, AD = 5.6 cm, AB = 8.4 cm, AE = 3.8 cm and AC = 5.7 cm. Show that DE $\|$ BC.

Sol. We have, AD = 5.6 cm, DB = (AB – AD) = (8.4 – 5.6) cm = 2.8 cm.
AE = 3.8 cm, EC = (AC – AE) = (5.7 – 3.8) cm = 1.9 cm.



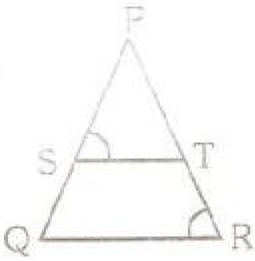
$\therefore \quad \frac{AD}{DB}=\frac{5.6}{2.8}=\frac{2}{1}$ and $\frac{AE}{EC}\frac{3.8}{1.9}=\frac{2}{1}$

Thus, $\frac{AD}{DB}=\frac{AE}{EC}$

$\therefore$ DE divides AB and AC proportionally.

Hence, DE $\|$ BC

Ex.3 In fig, $\frac{PS}{SQ}=\frac{PT}{TR}$ and $\angle$PST = $\angle$PRQ. Prove that PQR is an isosceles triangle. **[NCERT]**



Sol. It is given that $\frac{PS}{SQ}=\frac{PT}{TR}$

So, ST $\|$ QR [Theorem]

Therefore, $\angle$PST = $\angle$PQR [Corresponding angles] – (1)

Also, it is given that

$\angle$PST = $\angle$PRQ (2)

So, $\angle$PRQ = $\angle$PQR [Form 1 and 2]

Therefore PQ = PR [Sides opposite the equal angles]

i.e., PQR is an isosceles triangle.

Ex.4 Prove that any line parallel to parallel sides of a trapezium divides the non-parallel sides proportionally (i.e., In the same ratio).

OR

ABCD is a trapezium with DE $\parallel$ AB. E and F are points on AD and BC respectively such that EF $\parallel$ AB. Show that $\frac{AE}{ED}=\frac{BF}{FC}$ **[NCERT]**

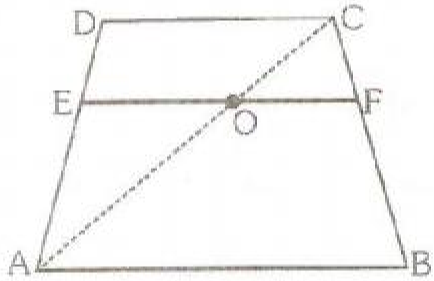
Sol. We are given trapezium ABCD.

CD $\parallel$ BA

EF $\parallel$ AB and CD both

We join AC.

It mets EF at O.



In ΔACD, OE $\parallel$ CD

$\Rightarrow \quad \frac{AO}{OC}=\frac{CF}{ED}$ ...(i)

(Basic Proportionality Theorem)

In ΔCAB, OF $\parallel$ AB

$\Rightarrow \quad \frac{CO}{OA}=\frac{CF}{FB}$ [B.P.T.]

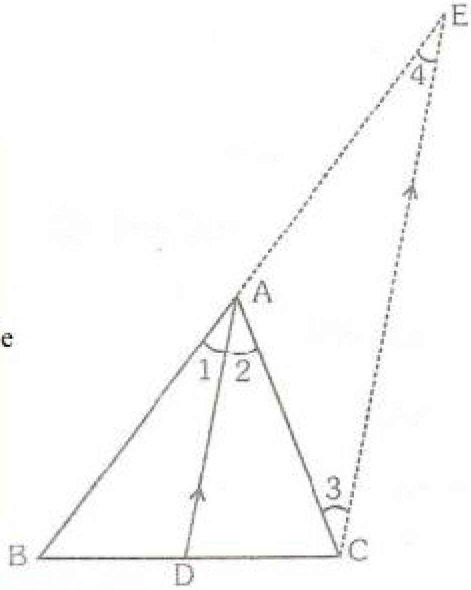
$\Rightarrow \quad \frac{AO}{OC}=\frac{BF}{FC}$ ...(ii)

From (i) and (ii)

$\frac{AE}{ED}=\frac{BF}{FC}$

Hence, proved.

Ex.5 Prove that the internal bisector of an angle of a triangle divides the opposite side in the ratio of the sides containing the angle.
(Internal Angle Bisector Theorem)



Sol. **Given :** A ΔABC in which AD is the internal bisector of $\angle$A.

To Prove : $\frac{BD}{DC}=\frac{AB}{AC}$

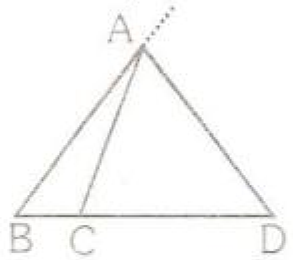
Construction : Draw CE $\parallel$ DA, meeting BA produced at E.

Proof :

	STATEMENT	REASON
1.	$\angle 1=\angle 2$	AD is the bisector of $\angle$A
2.	$\angle 2=\angle 3$	Alt. $\angle$s are equal, as CE $\parallel$ DA and AC is the transversal
3.	$\angle 1=\angle 4$	Corres. $\angle$s are equal, as CE $\parallel$ DA and BE is the transversal
4.	$\angle 3=\angle 4$	From 1, 2 and 3.
5.	AE = AC	Sides opposite to equal angles are equal
6.	In ΔBCE, DA $\parallel$ CE	
	$\Rightarrow \frac{BD}{DC}=\frac{BA}{AE}$	By B. P. T.
	$\Rightarrow \frac{BD}{DC}=\frac{AB}{AC}$	Using 5

Hence, proved.

Remark : The external bisector of an angle divides the opposite side externally in the ratio of the sides containing the angle. i.e., if in a ΔABC, AD is the bisector of the exterior of angle $\angle A$ and intersect BC produced in D, $\frac{BD}{CD} = \frac{AB}{AC}$.



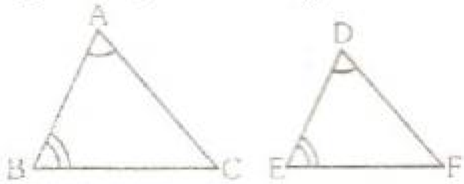
★ AXIOMS OF SIMILARITY OF TRIANGLES

1. AA (Angle-Angle) Axiom of Similarity :

If two triangles have two pairs of corresponding angles equal, then the triangles are similar. In the given figure, ΔABC and ΔDEF are such that

$\angle A = \angle D$ and $\angle B = \angle E$.

$\therefore \quad \Delta ABC \sim \Delta DEF$



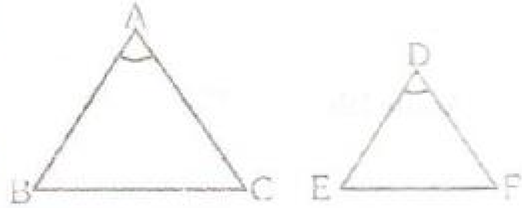
2. SAS (Side-Angle-Side) Axiom of Similarity :

If two triangles have a pair of corresponding angles equal and the sides including them proportional, then the triangles are similar.

In the given fig, ΔABC and ΔDEF are such that

$\angle A = \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$

$\therefore \quad \Delta ABC \sim \Delta DEF$

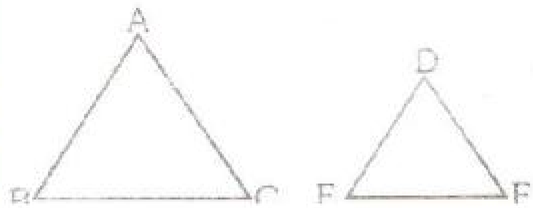


3. SSS (Side- Side- Side) Axiom of Similarity :

If two triangles have three pair of corresponding sides proportional, then the triangles are similar.

If in ΔABC and ΔDEF we have :

$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$, then $\Delta ABC \sim \Delta DEF$.



Ex.6. In figure, find $\angle L$.

Sol. In ΔABC and ΔLMN,

$$\frac{AB}{LM} = \frac{4.4}{11} = \frac{2}{5}$$

$$\frac{BC}{MN} = \frac{4}{10} = \frac{2}{5} \text{ and } \frac{CA}{NL} = \frac{3.6}{9} = \frac{2}{5}$$

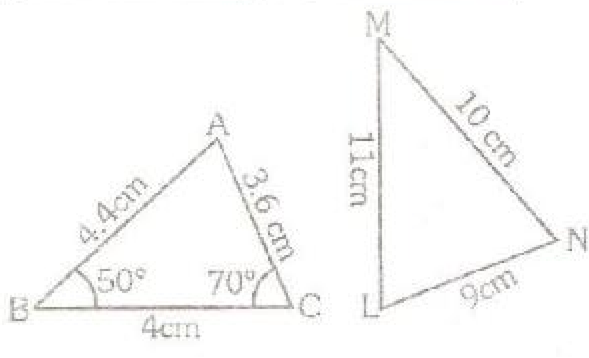
$\Rightarrow \quad \frac{AB}{LM} = \frac{BC}{MN} = \frac{CA}{NL}$

$\Rightarrow \quad \Delta ABC \sim \Delta LMN \qquad$ (SSS Similarity)

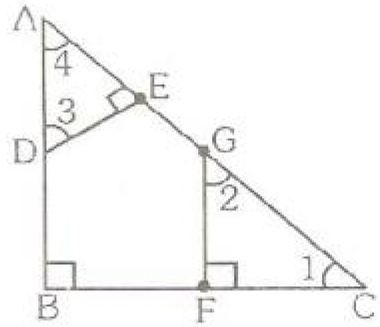
$\Rightarrow \quad \angle L = \angle A = 180^0 - \angle B - \angle C$

$\qquad = 180^0 - 50^0 - 70^0 = 60^0$

$\therefore \quad \angle L = 60^0$



Ex.7 In the figure, AB ⊥ BC, DE ⊥ AC, and GF ⊥ BC, Prove that $\Delta ADE \sim \Delta GCF$.



Sol. $\angle 1 + \angle 4 = \angle 1 + \angle 2$ (each side = 90^0)

$\Rightarrow \quad \angle 4 = \angle 2$

$\Rightarrow \quad \angle A = \angle G \quad \ldots(i)$

Also $\quad \angle E = \angle F \quad \ldots(ii)$ (each equal to 90^0)

From (i) and (ii), we get AA similarity for triangle ADE and GCF.

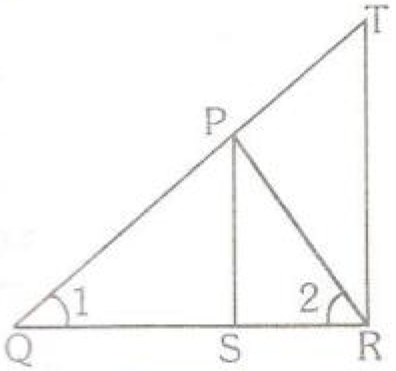
$\Rightarrow \quad \Delta ADE \sim \Delta GCF$

Ex.8 In fig, $\frac{QT}{PR} = \frac{QR}{QS}$ and $\angle 1 = \angle 2$. Prove that $\Delta PQS \sim \Delta TQR$.

Sol. $\angle 1 = \angle 2$ (Given)

$\Rightarrow \quad PR = PQ \quad \ldots(i)$

(Sides opposite to equal angles in ΔQRP)



Also $\quad \frac{QT}{PR} = \frac{QR}{QS}$ (Given) $\quad \ldots(i)$

From (i) and (ii), we have

$\frac{QT}{PR} = \frac{QR}{QS} \Rightarrow \frac{QP}{QT} = \frac{QS}{QR} \quad \ldots(iii)$

Now, in triangles PQR and TQR, we have

$\angle PQS = \angle TQR$ (each = $\angle 1$)

and $\quad \frac{PQ}{TQ} = \frac{QS}{QR}$ (from (3))

$\Rightarrow \quad \Delta PQS \sim \Delta TQR$ (SAS Similarity)

Ex.9 In fig, CD and GH are respectively, the medians of ΔABC and ΔFEG, If $\Delta ABC \sim \Delta FEG$, prove that

(i) $\Delta ADC \sim \Delta FHG$

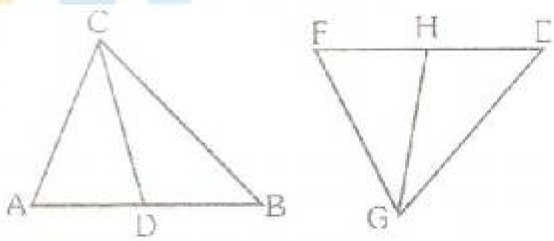
(ii) $\frac{CD}{GH} = \frac{AB}{FE}$ **(NCERT)**

Sol. $\Delta ABC \sim \Delta FEG$ (given)

$\Rightarrow \quad \angle A = \angle F, \quad \ldots(i)$ ($\because$ the corresponding angles of the similar triangles are equal)

Also, $\quad \frac{AC}{FG} = \frac{AB}{FE}$ (Corresponding sides are proportional)

$\Rightarrow \quad \frac{AC}{FG} = \frac{2AB}{2FH}$ $\begin{pmatrix} D \text{ is mid-point of } AB \\ H \text{ is mid-point of } FE \end{pmatrix}$



$\Rightarrow \quad \frac{AC}{AD} = \frac{FG}{FH} \quad \ldots(ii)$

Now, in triangles ADC and FHG, we have

$\angle A = \angle F$ and $\frac{AC}{AD} = \frac{FG}{FH}$ (By (i) and (ii))

$\Rightarrow \quad \Delta ADC \sim \Delta FHG$ (SAS similarity)

(ii) $\quad \Delta ADC \sim \Delta FHG$

$\Rightarrow \quad \frac{CD}{GH} = \frac{AD}{FH}$ (Corresponding sides proportional)

$\Rightarrow \quad \frac{CD}{GH} = \frac{2 \times AD}{2 \times FH} \quad \Rightarrow \quad \frac{CD}{GH} = \frac{AB}{FE}$

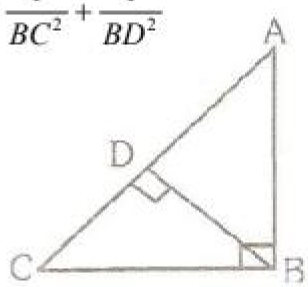
Ex,10 ABC is a right triangle, right angled at B. If BD is the length of perpendicular drawn from B to AC. Prove that :

(i) $\Delta ADB \sim \Delta ABC$ and hence $AB^2 = AD \times AC$

(ii) $\Delta BDC \sim \Delta ABC$ and hence $BC^2 = D \times AC$

(iii) $\Delta ADB \sim \Delta BDC$ and hence $BD^2 = AD \times DC$

(iv) $\frac{1}{AB^2} + \frac{1}{BC^2} + \frac{1}{BD^2}$



Sol. **Given :** ABC is right angled triangle at B and $BD \perp AC$

To prove :

(i) $\Delta ADB \sim \Delta ABC$ and hence $AB^2 = AD \times AC$

(ii) $\Delta BDC \sim \Delta ABC$ and hence $BC^2 = CD \times AC$

(iii) $\Delta ADB \sim \Delta BDC$ and hence $BD^2 = AD \times DC$

(iv) $\frac{1}{AB^2} + \frac{1}{BC^2} + \frac{1}{BD^2}$

Proof : (i) In two triangles ADB and ABC, we have :

$\angle BAD = \angle BAC$ (Common) $\angle ADB = \angle ABC$ (Each is right angle)

$\angle ABD = \angle ACB$ (Third angle) $\angle ADB = \angle ABC$ (AAA Similarity)

Triangle ADB and ABC are similar and so their corresponding sides must be proportion.

$$\frac{AD}{AB} = \frac{DB}{DC} = \frac{AB}{AC} \Rightarrow \frac{AD}{AB} = \frac{AB}{AC} \Rightarrow AB \times AB = AC \times AD \Rightarrow AB^2 = AD \times AC$$ This proves (a).

(ii) Again consider two triangles BDC and ABC, we have

$\angle BCD = \angle ACB$ (Common) $\angle BDC = \angle ABC$ (Each is right angle)

$\angle DBC = \angle BAC$ (Third angle

$\therefore$ Triangle are similar and their corresponding sides must be proportional.

i.e., $\angle BAD = \angle BAC$ $\frac{BD}{AB} = \frac{DC}{BC} = \frac{BC}{AC}$

(iii) In two triangle ADB and BDC, we have :

$\Rightarrow \frac{DC}{BC} = \frac{BC}{AC} \Rightarrow BC \times BC = DC \times AC \Rightarrow BC^2 = CD \times AC$ This proves (ii)

$\angle BDA = \angle BDC = 90^0$

$\angle 3 = \angle 2 = 90^0 \angle 1$ $[\because \angle 1 + \angle 2 = 90^0, \angle 1 + \angle 3 = 90^0]$

$\angle 1 = \angle 4 = 90^0 \angle 2$ $[\because \angle 1 + \angle 2 = 90^0, \angle 2 + \angle 4 = 90^0]$

$\Delta ADB \sim \Delta BDC$ (AAA criterion of similarity)

$\Rightarrow$ Their corresponding sides must be proportional.

$$\frac{AD}{BD} = \frac{DB}{DC} = \frac{AB}{BC} \Rightarrow \frac{AD}{BD} = \frac{DB}{DC} \Rightarrow BD \times BD = AD \times AC$$

$\therefore$ BD is the mean proportional of AD and DC

(iv) From (i), we have : $AB2 = AD \times AC$ (ii), we have : $BC2 = CD \times AC$

(iii), we have : BD2 = ADDC

Consider $$\frac{1}{AB^2} + \frac{1}{BC^2} = \frac{1}{AD \times AC} + \frac{1}{CD \times AC} + \frac{1}{AC}\left[\frac{1}{AD} + \frac{1}{DC}\right]$$

$$\frac{1}{AB^2} + \frac{1}{BC^2} = \frac{1}{AC}\left[\frac{DC}{AD} + \frac{AD}{DC}\right] = \frac{1}{AC}\left[\frac{AD + DC}{AD \times DC}\right] = \frac{1}{AC}\left[\frac{AC}{AD \times DC}\right]$$

$$= \frac{1}{AD \times DC} = \frac{1}{BD^2}$$ (from (iii))

$$\frac{1}{AB^2} + \frac{1}{BC^2} = \frac{1}{BD^2}$$

Thus we have proved the following :

If a perpendicular is drawn from the vertex containing the right angle of a right triangle to the hypotenuse then:

(a) Thu triangle on each side of the perpendicular are similar to each other and also similar to the original triangle.

i.e., $\Delta ADB \sim \Delta BDC$, $\Delta ADB \sim \Delta ABC$, $\Delta BDC \sim \Delta ABC$

(b) **The square of the perpendicular is equal to the product of the length of two parts into which the hypotenuse is divided by the perpendicular i.e., BD2 = AD** $\times$ 0DC.

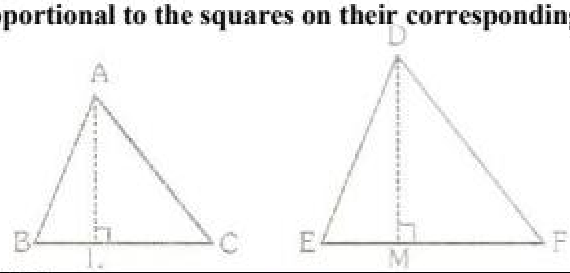
★ **RESULTS ON AREA OF SIMILAR TRIANGLES**

Theorem-3 : The areas of two similar triangles are proportional to the squares on their corresponding sides.

Given : $\Delta ABC \sim \Delta DEF$

To prove : $\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DEF} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$

Construction : Draw AL $\perp$ BC and DM $\perp$ EF.

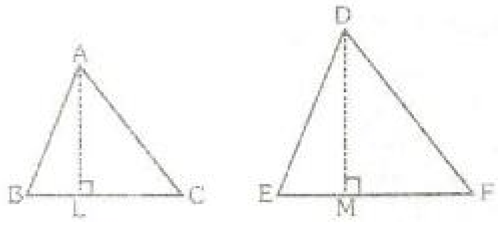


Proof:

	STATEMENT	REASON
1.	$\frac{Area\,\Delta ABC}{Area\,\Delta DEF} = \frac{\frac{1}{2}\times BC \times AL}{\frac{1}{2}\times EF \times DM}$	Area of $\Delta = \frac{1}{2} \times$ Base $\times$ Height
	$\Rightarrow \frac{Area\,\Delta ABC}{Area\,\Delta DEF} = \frac{BC}{EF}\times\frac{AL}{DM}$	
2.	In Δ ALB and Δ DME, we have	
	(i) $\angle$ ALB = $\angle$ DME	Each equal to 90^0
	(ii) $\angle$ ABL = $\angle$ DEM	$\Delta ABC \sim \Delta DEF \Rightarrow \angle B = \angle E$
	$\therefore \Delta ALB \sim \Delta DME$	AA=axiom
	$\Rightarrow \frac{AL}{DM} = \frac{AB}{DE}$	Corresponding sides of similar Δ s are proportional.
3.	Δ ABC ~ Δ DEF	Given.
	$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$	Corresponding sides of similar Δ s are proportional.
4.	$\frac{AL}{DM} = \frac{BC}{EF}$	From 2 and 3.
5.	Substituting $\frac{AL}{DM} = \frac{BC}{EF}$ in 1, we get :	
	$\frac{Area\,\Delta ABC}{Area\,\Delta DEF} = \frac{BC^2}{EF^2}$	
6.	Combining 3 and 5, we get :	
	$\frac{Area\,\Delta ABC}{Area\,\Delta DEF} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$	

Conrollary-1 : The areas of two similar triangles are proportional to the squares on their corresponding altitude.

Given : $\Delta ABC \sim \Delta DEF$, $AL \perp BC$ and $DM \perp EF$.



To prove : $\dfrac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DEF} = \dfrac{AL^2}{DM^2}$

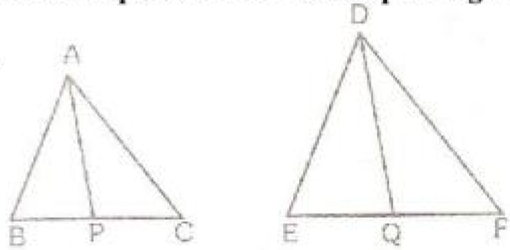
Proof :

	STATEMENT	**REASON**
1.	$\dfrac{Area\,\Delta ABC}{Area\,\Delta DEF} = \dfrac{\frac{1}{2}\times BC \times AL}{\frac{1}{2}\times EF \times DM}$ $\Rightarrow \dfrac{Area\,\Delta ABC}{Area\,\Delta DEF} = \dfrac{BC}{EF} = \dfrac{AL}{DM}$	Area of $\Delta = \dfrac{1}{2} \times$ Base $\times$ Height
2.	In Δ ALB and Δ DME, we have (i) $\angle ALB = \angle DME$ (ii) $\angle ABL = \angle DEM$ $\Rightarrow \Delta ALB \sim \Delta DME$ $\Rightarrow \dfrac{AL}{DM} = \dfrac{AB}{DE}$	Each equal to 90^0 $\Delta ABC \sim \Delta DEF \Rightarrow \angle B = \angle E$ AA=axiom Corresponding sides of similar Δ s are proportional.
3.	$\Delta ABC \sim \Delta DEF$ $\Rightarrow \dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$	Given. Corresponding sides of similar Δ s are proportional.
4.	$\dfrac{BC}{EF} = \dfrac{AL}{DM}$	From 2 and 3.
5.	Substituting $\dfrac{BC}{EF} = \dfrac{AL}{DM}$ in 1, we get : $\dfrac{Area\,\Delta ABC}{Area\,\Delta DEF} = \dfrac{AL^2}{DM^2}$	

Hence, proved

Corollary-2 : The areas of two similar triangles proportional to the squares on their corresponding medians.

Given : $\Delta ABC \sim \Delta DEF$ and AP, PQ are their medians.



To prove : $\dfrac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DEF} = \dfrac{AP^2}{DQ^2}$

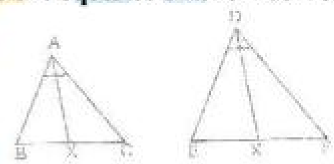
Proof :

	STATEMENT	REASON
1.	$\Delta ABC \sim \Delta DEF$ $\Rightarrow \dfrac{Area\ \Delta ABC}{Area\ \Delta DEF} = \dfrac{AB^2}{DE^2}$ ……I.	Given Area of two similar Δs are proportional to the squares on their corresponding sides.
2.	$\Delta ABC \sim \Delta DEF$ $\Rightarrow \dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{2BP}{2EQ} = \dfrac{BP}{EQ}$ ……II.	Corresponding sides of similar Δs are proportional
3.	$\dfrac{AB}{DE} = \dfrac{BP}{EQ}$ and $\angle A = \angle D$ $\Rightarrow \Delta APB \sim \Delta DQE$ $\Rightarrow \dfrac{BP}{EQ} = \dfrac{AP}{DQ}$ …….III $\Rightarrow \dfrac{AB}{DE} = \dfrac{AP}{DQ}$ $\Rightarrow \dfrac{AB^2}{DE^2} = \dfrac{AP^2}{DQ^2}$ ……..IV	From II and the fact the $\Delta ABC \sim \Delta DEF$ By SAS-similarity axiom From II and III.
4.	$\Rightarrow \dfrac{Area\ \Delta ABC}{Area\ \Delta DEF} = \dfrac{AP^2}{DQ^2}$	From I and IV.

Hence, proved

Corollary-3 : The areas of two similar triangles proportional to the squares on their corresponding angle bisector segments.

Given : $\Delta ABC \sim \Delta DEF$ and AX, DY are their

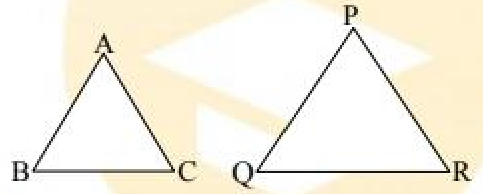


To prove : $\dfrac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DEF} = \dfrac{AX^2}{DY^2}$

Proof :

	STATEMENT	REASON
1.	$\frac{Area\ \Delta ABC}{Area\ \Delta DEF} = \frac{AB^2}{DE^2}$	Area of two similar Δs are proportional to the squares on their corresponding sides.
2.	$\Delta ABC \sim \Delta DEF$ $\Rightarrow \angle A = \angle D$ $\Rightarrow \frac{1}{2}\angle A = \frac{1}{2}\angle D$ $\Rightarrow \angle BAX = \angle EDY$	Given $\Rightarrow \angle BAX = \frac{1}{2}\angle A$ and $\angle EDY = \frac{1}{2}\angle D$
3.	**In** ΔABX and ΔEDY, we have $\angle BAX = \angle EDY$ $\angle B = \angle E$ $\therefore \Delta ABX \sim \Delta DEY$ $\Rightarrow \frac{AB}{DE} = \frac{AX}{DY} \Rightarrow \frac{AB^2}{DE^2} = \frac{AX^2}{DY^2}$	Given From 2. $\Delta ABC \sim \Delta DEF$ By AA similarity axiom
4.	$\frac{Area\ \Delta ABC}{Area\ \Delta DEF} = \frac{AX^2}{DY^2}$	From 1 and 3.

Ex.11 It is given that $\Delta ABC \sim \Delta PQR$, area ($\Delta ABC$) = 36 cm^2 and area ($\Delta PQR$) = 25 cm^2. If QR = 6 cm, find length of BC.



Sol. We know that the areas of similar triangles are proportional to the squares of their corresponding sides.

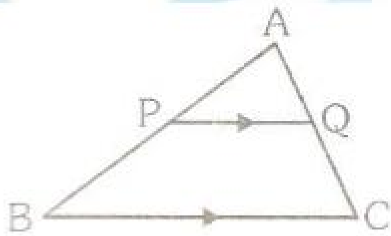
$\therefore \quad \frac{Area\ of\ (\Delta ABC)}{Area\ of\ (\Delta PQR)} = \frac{BC^2}{QR^2}$

Let BC = x cm. Then.

$\frac{36}{25} = \frac{x^2}{6^2} \Leftrightarrow \frac{36}{25} = \frac{x^2}{36} \Leftrightarrow x^2 = \frac{36 \times 36}{25} \Leftrightarrow x = \left(\frac{6 \times 6}{5}\right) = \frac{36}{5} = 7.2$

Hence BC = 7.2 cn

Ex.12 P and Q are points on the sides AB and AC respectively of ΔABC such that PQ $\|$ BC and divides ΔABC into parts, equal in area. Find PB : AB.



Sol. Area (ΔAPQ) = Area (trap. PBCQ) [Given]

$\Rightarrow$ Area (ΔAPQ) = [Area (ΔABC) – Area (ΔAPQ)

$\Rightarrow$ 2 Area (ΔAPQ) = Area (ΔABC)

$\Rightarrow \quad \dfrac{Area\ of\ (\Delta APQ)}{Area\ of\ (\Delta ABC)} = \dfrac{1}{2}$ ...(i)

Now, in ΔAPQ and ΔABC, we have

$\angle$PAQ = $\angle$BAC [Common $\angle$A]

$\angle$APQ = $\angle$ABC [PQ $\|$ BC, corresponding $\angle$s are equal]

$\therefore$ ΔAPQ ~ ΔABC.

We known that the areas of similar Δs are proportional to the squares of their corresponding sides.

$\therefore \dfrac{Area\ of\ (\Delta APQ)}{Area\ of\ (\Delta ABC)} = \dfrac{AP^2}{AB^2} \Rightarrow \dfrac{AP^2}{AB^2} = \dfrac{1}{2}$ [Using (i)]

$\Rightarrow \quad \dfrac{AP}{AB} = \dfrac{1}{\sqrt{2}}$ i.e., AB = $\sqrt{2}$. AP $\quad \Rightarrow \quad$ AB = $\sqrt{2}$ (AB – PB) $\Rightarrow \sqrt{2}$ PB = ($\sqrt{2}$ – 1) AB

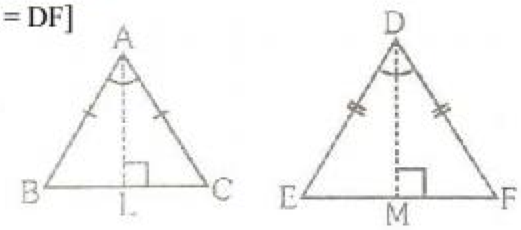
$\Rightarrow \quad \dfrac{PB}{AB} = \dfrac{(\sqrt{2}-1)}{\sqrt{2}}$. $\quad \therefore \quad$ PB : AB = ($\sqrt{2}$ – 1) : $\sqrt{2}$

Ex.13 Two isosceles triangles have equal vertical angles and there areas are in the ratio 16 : 25. Find the ratio of their corresponding heights.

Sol. Let ΔABC and ΔDEF be the given triangles in which AB = AC, DE = DF, $\angle$A = $\angle$D and

$\dfrac{Area\ of\ (\Delta ABC)}{Area\ of\ (\Delta DEF)} = \dfrac{16}{25}$ Draw AL $\perp$ BC and DM $\perp$ EF

Now, $\dfrac{AB}{AC} = 1$ and $\dfrac{DE}{DF} = 1$ [$\because$ AB = AC and DE = DF]



$\Rightarrow \quad \dfrac{AB}{DE} = \dfrac{AC}{DF}$.

$\therefore$ In ΔABC and ΔDEF, we have

$\dfrac{AB}{DE} = \dfrac{AC}{DF}$ and $\angle$A = $\angle$D

$\Rightarrow$ ΔABC ~ ΔDEF [By SAS similarity axiom]

But, the ratio of the areas of two similar Δs is the same as the ratio of the square of their corresponding heights.

$\dfrac{Area\ of\ (\Delta ABC)}{Area\ of\ (\Delta DEF)} = \dfrac{AL^2}{DM^2} \quad \Rightarrow \quad \dfrac{16}{25} = \left(\dfrac{AL^2}{DM^2}\right) \quad \Rightarrow \quad \dfrac{AL}{DM} = \dfrac{4}{5}$

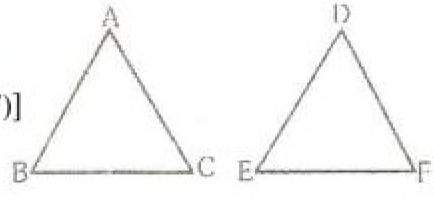
$\therefore$ AL : DM = 4 : 5, i.e., the ratio of their corresponding heights = 4 : 5,

Ex.14 If the areas of two similar triangles are equal, prove that they are congruent.

Sol. Let ΔABC ~ ΔDEF and area (ΔABC) = area (ΔDEF).

Since the ratio of the areas of two similar Δs is equal to the ratio of the squares on their corresponding sides, we have

$\dfrac{Area\ of\ (\Delta ABC)}{Area\ of\ (\Delta DEF)} = \dfrac{AB^2}{DE^2} = \dfrac{AC^2}{DF^2} = \dfrac{BC^2}{EF^2}$



$\Rightarrow \quad \dfrac{AB^2}{DE^2} = \dfrac{AC^2}{DF^2} = \dfrac{BC^2}{EF^2} = 1$ [$\because$ Area (ΔABC) = Area (ΔDEF)]

$\Rightarrow$ $AB^2 = DE^2$, $AC^2 = DF^2$ and $BC^2 = EF^2$

$\Rightarrow$ AB = DE, AC = DF and BC = EF

ΔABC $\cong$ ΔDEF [By SSS congruence]

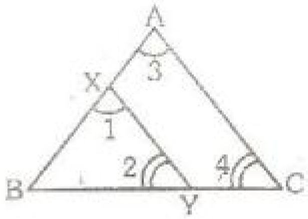
Ex.15 In fig, the line segment XY is parallel to side AC of ΔABC and it divides the triangle into two parts of equal areas. Find the ratio $\dfrac{AX}{AB}$. **[NCERT]**

Sol, We are given that XY $\|$ AC.

$\Rightarrow$ $\angle$1 = $\angle$3 and $\angle$2 = $\angle$4 [Corresponding angles]

$\Rightarrow$ ΔBXY~ ΔBAC [AA similarity]

$$\Rightarrow \quad \frac{ar(\Delta BXY)}{ar(\Delta BAC)} = \frac{(BY)^2}{(BA)^2} \qquad \text{[By theorem]} \quad \ldots(i)$$



Also, we are given that

$$ar(\Delta BXY) = \frac{1}{2} \times ar(\Delta BAC) \Rightarrow \frac{ar(\Delta BXY)}{ar(\Delta BAC)} = \frac{1}{2} \qquad \ldots(ii)$$

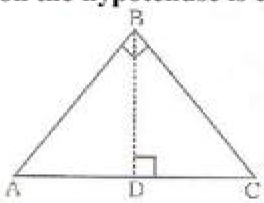
From (i) and (ii), we have $\left(\frac{BX}{BA}\right)^2 = \frac{1}{2} \Rightarrow \frac{BX}{BA} = \frac{1}{\sqrt{2}}$ ...(iii)

Now, $\frac{AX}{AB} = \frac{AB - BX}{AB} = 1 - \frac{BX}{AB} = 1 - \frac{BX}{BA} = 1 - \frac{1}{\sqrt{2}}$ [By (iii)]

$$= \frac{\sqrt{2}-1}{\sqrt{2}} = \frac{2-\sqrt{2}}{2}$$

Hence, $= \frac{AX}{AB} = \frac{2-\sqrt{2}}{2}$

Theorem-4 [Pythagoras Theorem] : In a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.



Given : A Δ ABC in which $\angle B = 90^0$.

To prove : $AC^2 = BA^2 + BC^2$.

Construction : From B, Draw BD $\perp$ AC

Proof :

	STEMENT	REASON
1.	In Δ ADB and Δ ABC, we have :	
	$\angle BAD = \angle CAB = \angle A$	Common
	$\angle ADB = \angle ABC$	Each = 90^0
	$\therefore \Delta ADB \sim \Delta ABC$	By AA axiom of similarity
	$\Rightarrow \frac{AD}{AB} = \frac{AB}{AC}$	Corr. sides of similar Δ s are proportional
	$\Rightarrow AB^2 = AD \times AC$ ..(i)	
2.	In Δ CDB and Δ CBA, we have : $\Delta CDB = \Delta CBA$	Each = 90^0
	$\angle BCD = \angle ACB$	Common
	$\therefore \Delta CDB \sim \Delta CBA$	By AA axiom of similarity
	$\Rightarrow \frac{DC}{BC} = \frac{BC}{AC}$	Corr. sides of similar Δ s are proportional
	$\Rightarrow BC^2 = DC \times AC$ ..(ii)	
3.	Adding (i) and (ii), we get	
	$AB^2 + BC^2 = AD \times AC + DC \times AC$	
	$= (AD + DC) \times AC = AC^2$	$\because AD + DC = AC$

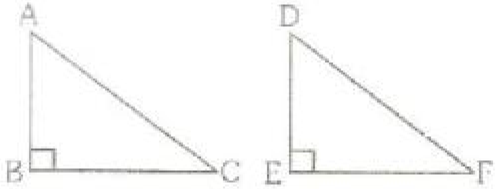
Hence, $AB^2 + BC^2 = AC^2$

Theorem-5 [Converse of pythagoras Theorem] : In a triangle if the square of one sides is equal to the sum of the squares of the squares of the other two sides, then the triangle is right angled.

Given : A Δ ABC in which $AB^2 + BC^2 = AC^2$

To prove : $\angle B = 90^0$.

Construction : Draw a Δ DEF in which CE = AB, EF = BC and $\angle E = 90^0$



Proof :

	STATEMENT	REASON
1.	In Δ DEF, we have : $\angle = 90^0$ $\therefore DE^2 + EF^2 = DF^2$ $\Rightarrow AB^2 + BC^2 = DF^2$ $\Rightarrow AC^2 = DF^2$	By Phthagoras Theorem $\because$ DE = AB and EF = BC $\because AB^2 + BC^2 = AC^2$ (Given)
2.	In Δ ABC and Δ DEF, we have : AB = DE BC = DF $\therefore \Delta ABC \cong \Delta DEF$ $\Rightarrow \angle B = \angle E$ $\Rightarrow \angle E = 90^0$	By construction By construction Proved above By SSS congruence c.p.c.t $\because \angle E = 90^0$

Hence, $\angle B = 90^0$

Ex.16 If ABC is an equilateral triangle of side a, prove that its altitude $= \frac{\sqrt{3}}{2}a$.

Sol. Δ ABC is an equilateral triangle.

We are given that AB = BC = CA = a. AD is the altitude, i.e., AD $\perp$ BC.

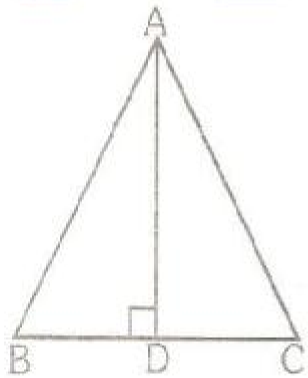
Now, in right angled triangles triangles ABD and ACD, we have

AB = AC [Given]

and AD = AD [Common side]

$\Rightarrow$ $\Delta ABD = \Delta ACD$ [By RHS congruence]

$\Rightarrow$ $BD = CD \Rightarrow BD = DC = \frac{1}{2}BC = \frac{a}{2}$



From right triangle ABD,

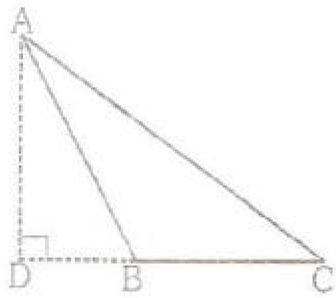
$AB^2 = AD^2 + BD^2 \quad \Rightarrow a^2 - AD^2 + \left(\frac{a}{2}\right)^2$

$\Rightarrow \quad AD^2 = a^2 - \frac{a^2}{4} = \frac{3}{4}a^2$

$\Rightarrow \quad AD = \frac{\sqrt{3}}{2}a.$

Ex.17 In a ΔABC, obtuse angled at B, if AD is perpendicular to CB produced, prove that : $AC^2 = AB^2 + BC^2 + 2BC \times BD$

Sol. In $\Delta ADB, \angle D = 90^0$.



$\therefore \quad AD^2 + DB^2 = AB^2 \quad \ldots(i)$ [By Pythagoras Theorem]

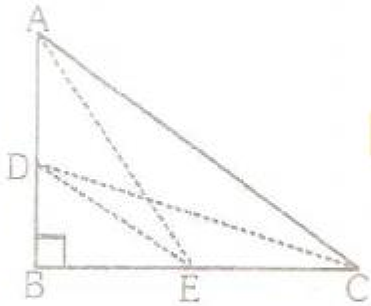
In $\Delta ADC, \angle D = 90^0$.

$\therefore \quad AC^2 = AD^2 + DC^2$ [By Pythagoras Theorem]

$= AD^2 + (DB + BC)^2$
$= AD^2 + DB^2 + BC^2 + 2DB \times BC$
$= AB^2 + BC^2 + 2BC \times BD$ [Using (i)]
Hence, $AC^2 = AB^2 + BC^2 + 2BC \times BD$.

Ex.18 In the given figure, $\angle B = 90^0$. D and E are any points on AB and BC respectively. Prove that : $AE^2 + CD^2 = AC^2 + DE^2$.

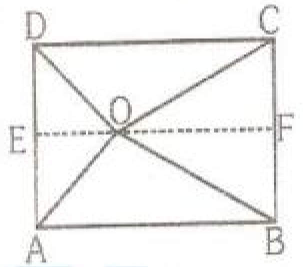
Sol. In $\Delta ABE, \angle B = 90^0$



$\therefore \quad AE^2 = AB^2 + BE^2 \ldots(i)$
In $\Delta DBC, \angle B = 90^0$.
$\therefore \quad CD^2 = BD^2 + BC^2 \ldots(ii)$
Adding (i) and (ii), we get :
$AE^2 + CD^2 = (AB^2 + BC^2) + (BE^2 + BD^2)$
$= AC^2 + DE^2$ [By Pythagoras Theorem]
Hence, $AE^2 + CD^2 = AC^2 + DE^2$.

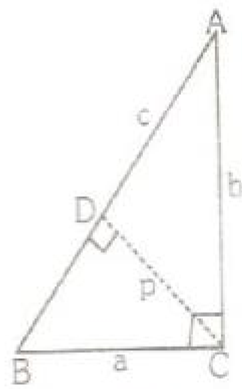
Ex.19 A point O in the interior of a rectangle ABCD is joined with each of the vertices A, B, C and D. Prove that : $OA^2 + OC^2 = OB^2 + OD^2$

Sol. Through O, draw $EOF \parallel AB$. Then, ABFE is a rectangle.



In right triangles OEA and OFC, we have :
$OA^2 = OE^2 + AE^2$
$OC^2 = OF^2 + OF^2$
$\therefore \quad OA^2 + OC^2 = OE^2 + OF^2 + AE^2 + CF^2$
Again, in right triangles OFB and OED, we have :
$OB^2 = OF^2 + BF^2$
$OD^2 = OE^2 + DE^2$
$\therefore \quad OB^2 + OD^2 = OF^2 + OE^2 + BF^2 + DE^2$
$= OE^2 + OF^2 + AE^2 + CF^2 \quad \ldots(i) \quad [\because BF = AE \ \& \ DE = CF]$
From (i) and (ii), we get
$OA^2 + OC^2 = OB^2 + OD^2$.

Ex.20 In the given figure, ΔABC is right-angled at C.
Let BC = a, CA = b, AB = c and CD = p, where $CD \perp AB$.
Prove that : (i) cp = ab (ii) $\dfrac{1}{p^2} = \dfrac{1}{a^2} + \dfrac{1}{b^2}$



Sol. (i) Area of $\Delta ABC = \dfrac{1}{2}\ BC \times CD\ \dfrac{1}{2}\ cp.$

Also, area of $\Delta ABC = \dfrac{1}{2}\ BC \times AC\ \dfrac{1}{2}\ ab.$

$\therefore\ \dfrac{1}{2}\ cp = \dfrac{1}{2}\ ab. \Rightarrow cp = ab$

(ii) $cp = ab \Rightarrow p = \dfrac{ab}{c}$

$\Rightarrow \quad p^2 = \dfrac{a^2b^2}{c^2}$

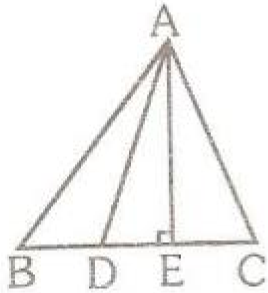
$\Rightarrow \quad \dfrac{1}{p^2} = \dfrac{c^2}{a^2b^2} = \dfrac{a^2+b^2}{a^2b^2} \qquad [\because c^2 = a^2 + b^2]$

$\Rightarrow \quad \dfrac{1}{p^2} = \dfrac{1}{a^2} = \dfrac{1}{b^2}$

Ex.21 Prove that in any triangle, the sum of the square of any two sides is equal to twice the square of half of the third side together with twice the square of the median which bisects the third side. **(Appollonius Theorem)**

Sol. Given : A ΔABC in which AD is a median.

To prove : $AB^2 + AC^2 = 2AD^2 + 2\left(\dfrac{1}{2}BC\right)^2$ or $AB^2 + AC^2 = 2(AD^2 + BD^2)$



Construction : Draw $AE \perp BC$.
Proof : $\because$ AD id median
$\therefore$ BD = DC

Now, $AB^2 + AC^2 = (AE^2 + BE^2) + (AE^2 + CE^2) = 2AE^2 + BE^2 + CE^2$
$= 2[AD^2 - DE^2] + BE^2 + CE^2$
$= 2AD^2 - 2DE^2 + (BD + DE)^2 + (DC - DE)^2$
$= 2AD^2 - 2DE^2 + (BD + DE)^2 + (DC - DE)^2$
$= 2(AD^2 + BD^2) = = 2AD^2 + 2\left(\dfrac{1}{2}BC\right)^2$

Hence, Proved.

★ SYNOPSIS

- **SIMILAR TRIANGLES.** Two triangles are said to be similar if
  (i) Their corresponding angles are equal and (ii) Their corresponding sides are proportional.

- All congruent triangles are similar but the similar triangles need not be congruent.
- Two polygons of the same numbers of sides are similar, if

(i) their corresponding angels are equal and

(ii) their corresponding sides are in the same ratio.

- **BASIC PROPORTIONALITY THEOREM.** In a triangle, a line drawn parallel to one side, to intersect the other sides in distinct points, divides the two sides in the third side.

- **CONVERSE OF BASIC PROPORTIONALITY THEOREM.** If a line divides any two sides of a triangle in the same ratio, the line must be parallel to the third side.

- **AAA-SIMILARITY.** If in two triangles, corresponding angles are equal, i.e., the two corresponding angles are equal, then the triangles are similar.

- **SSS-SIMILARITY.** If the corresponding sides of two triangles are proportional, then they are similar.

- **SSS-SIMILARITY.** If in triangles one pair of corresponding sides proportional and the included angles are equal then the two triangles are similar.

- The ratio of the areas of similar triangles is equal to the ratio of the squares of their to the sum of the squares

- **PYTHAGORAS THEOREM.** In a right triangle, if the square of one side is equal to the sum of the squares of the other two sides.

- **CONVERSE OF PYTHAGORAS THEOREM.** In a triangle, if the square of one side is equal to the sum of the squares of the other two sides then the angle opposite to the first side is a right angle.

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOOSE THE CORRECT ONE

1. Triangle ABC is such that AB = 3 cm, BC = 2 cm and CA = 2.5 cm. Triangle DEF is similar to ΔABC. If EP 4 cm, then the perimeter of ΔDEF is :
(A) 7.5 cm (B) 15 cm (C) 22.5 cm (D) 30 cm

2. In ΔABC, AB = 3 cm, AC = 4 cm and AD is the bisector of $\angle$A. Then, BD : DC is :
(A) 9 : 16 (B) 16 : 9 (C) 3 : 4 (D) 4 : 3

3. In a equilateral triangle ABC, if AD $\perp$ BC, then :
(A) $2AB^2 = 3AD^2$ (B) $4AB^2 = 3AD^2$ (C) $3AB^2 = 4AD^2$ (D) $3AB^2 = 2AD^2$

4. ABC is a triangles and DE is drawn parallel to BC cutting the other sides at D and E. If AB = 3.6 cm, AC = 2.4 cm and AD = 2.1 cm, then AE is equal to :
(A) 1.4 cm (B) 1.8 cm (C) 1.2 cm (D) 1.05 cm

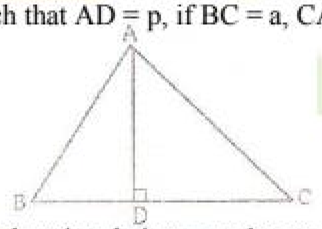
5. The line segments joining the mid points of the sides of a triangle from four triangles each of which is :
(A) similar to the original triangle (B) congruent to the original triangle.
(C) an equilateral triangle (D) an isosceles triangle.

6. In ΔABC and ΔDEF, $\angle A = 50^0, \angle B = 70^0, \angle C = 60^0, \angle D = 60^0, \angle E = 70^0, \angle F = 50^0$, then ΔABC is similar to :
(A) ΔDEF (B) ΔEDF (C) ΔDFE (D) ΔFED

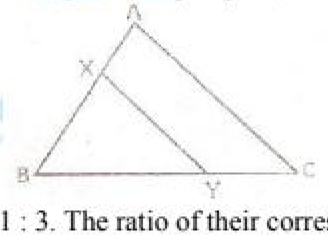
7. D, E, F are the mid points of the sides BC, CA and AB respectively of ΔABC. Then ΔDEF is congruent to triangle
(A) ABC (B) AEF (C) BFD, CDE (D) AFE, BFD, CDE

8. If in the triangles ABC and DEF, angle A is equal to angle E, both are equal to 40^0, AB : ED = AC : EF and angle F is 65^0, then angel B is :-
(A) 35^0 (B) 65^0 (C) 75^0 (D) 85^0

9. In a right angled ΔABC, right angled at A, if AD $\perp$ BC such that AD = p, if BC = a, CA = b and AB = c, then :
(A) $p^2 = b^2 + c^2$ (B) $\frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{c^2}$
(C) $\frac{p}{a} = \frac{p}{b}$ (D) $p^2 = b^2 c^2$



10. In the adjoining figure, XY is parallel to AC. If XY divides the triangle into equal parts, then the value of $\frac{AX}{AB}$ =
(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$
(C) $\frac{\sqrt{2}+1}{\sqrt{2}}$ (D) $\frac{\sqrt{2}-1}{\sqrt{2}}$

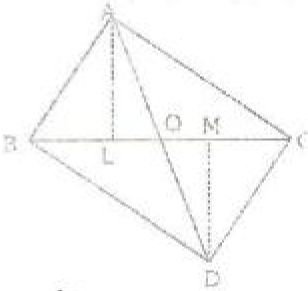


11. The ratio of the corresponding sides of two similar triangles is 1 : 3. The ratio of their corresponding heights is :
(A) 1 : 3 (B) 3 : 1 (C) 1 : 9 (D) 9 : 1

12. The areas of two similar triangles are 49 cm^2 and 64 cm^2 respectively. The ratio of their corresponding sides is :
(A) 49 : 64 (B) 7 : 8 (C) 64 : 49 (D) None of these

13. The areas of two similar triangles are 12 cm^2 and 48 cm^2. If the height of the similar one is 2.1 cm, then the corresponding height of the bigger one is :
(A) 4.41 cm (B) 8.4 cm (C) 4.2 cm (D) 0.525 cm

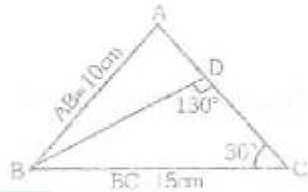
14. In the adjoining figure, ABC and DBC are two triangles on the same base BC, AL $\perp$ BC and DM $\perp$ BC. Then, $\dfrac{area(\Delta ABC)}{area(\Delta DBC)}$ is equal to ;



(A) $\dfrac{AO}{OD}$ (B) $\dfrac{AO^2}{OD^2}$

(C) $\dfrac{AO}{AD}$ (D) $\dfrac{OD^2}{AO^2}$

15. In the adjoining figure, AD : DC = 2 : 3, then $\angle$ ABC is equal to :

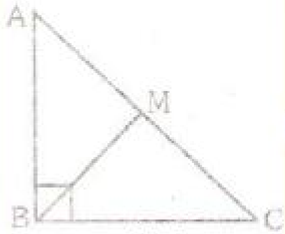


(A) 30^0 (B) 40^0
(C) 45^0 (D) 110^0

16. In ΔABC, D and E are points on AB and AC respectively such that DE $\parallel$ BC. If AE = 2 cm, EC = 3 cm and BC = 10 cm, then DE is equal to ;

(A) 5 cm (B) 4 cm (C) 15 cm (D) $\dfrac{20}{3}$ cm

17. In the given figure, $\angle$ ABC = 90^0 and BM is a median, AB = 8 cm and BC = 6 cm. Then, length BM is equal to :



(A) 3 cm (B) 4 cm (C) 5 cm (D) 7 cm

18. If D, E, F are respectively the mid points of the sides BC, CA and AB of ΔABC and the area of ΔABC is 24 sq. cm, then the area of ΔDFE is :-
(A) 24 cm^2 (B) 12 cm^2 (C) 8 cm^2 (D) 6 cm^2

19. In a right angled triangle, if the square of the hypotenuse is twice the product of the other two sides, then one of the angles of the triangle is :-
(A) 15^0 (B) 30^0 (C) 45^0 (D) 60^0

20. Consider the following statements :
1. If three sides of a triangles are equal to three sides of another triangle, then the triangles are congruent.
2. If three angles of a triangles are respectively equal to three angles of another triangle, then the two triangles are congruent.

Of these statements,
(A) 1 is correct and 2 is false (B) both 1 and 2 are false
(C) both 1 and 2 are correct (D) 1 is false and 2 is correct

(OBJECTIVE) **ANSWER KEY** EXERCISE

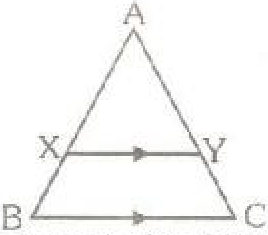
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	B	C	C	A	A	D	D	C	B	B
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	A	B	C	A	B	B	C	D	C	A

EXERCISE – 2 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

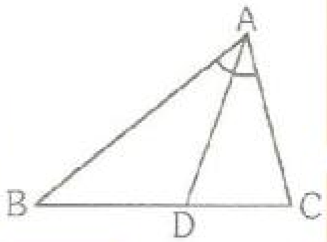
VERY SHORT ANSWER TYPE QUESTIONS

1. In the given figure, XY ∥ BC.
Given that AX = 3 cm, XB = 1.5 cm and BC = 6 cm.
Calculate :
(i) $\frac{AY}{YC}$ (ii) XY

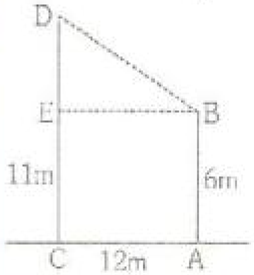


2. D and E are points on the sides AB and AC respectively of Δ ABC. For each of the following cases, state whether DE ∥ BC :
(i) AD = 5.7 cm, BD = 9.5 cm, AE = 3.6 cm, and EC = 6 cm
(ii) AB = 5.6 cm, AD = 1.4 cm, AC = 9.6 cm, and EC = 2.4 cm.
(iii) AB = 11.7 cm, BD = 5.2 cm, AE = 4.4 cm, and AC = 9.9 cm.
(iv) AB = 10.8 cm, BD = 4.5 cm, AC = 4.8 cm, and AE = 2.8 cm.

3. In Δ ABC, AD is the bisector of ∠ A. If BC = 10 cm, BD = 6 cm and AC = 6 cm, find AB.



4. AB and CD are two vertical poles height 6 m and 11 m respectively. If the distance between their feet is 12 m, find the distance between their tops.



5. Δ ABC and Δ PQR are similar triangles such that are (Δ ABC) = 49 cm^2 and area (Δ PQR) = 25 cm^2. If AB = 5.6 cm, find the length of PQ.

6. Δ ABC and Δ PQR are similar triangles such that are (Δ ABC) = 28 cm^2 and area (Δ PQR) = 63 cm^2. If PR = 8.4 cm, find the length of AC.

7. Δ ABC ~ Δ DEF. If BC = 4 cm, EF = 5 cm and area (Δ ABC) = 32 cm^2, determine the area of Δ DEF.

8. The areas of two similar triangles are 48 cm^2 and 75 cm^2 respectively. If the altitude of the first triangle be 3.6 cm, find the corresponding altitude of the other.

9. A rectangular field is 40 m long and 30 m broad. Find the length of its diagonal.

10. A man goes 15 m due west and then 8 m due north. How far is he from the starting point?

11. A ladder 17 m long reaches the window of a building 15 m above the ground. Find the distance of the foot of the ladder from the building.

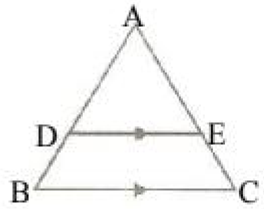
SHORT ANSWER TYPE QUESTIONS

1. In the given fig, $DE \parallel BC$.

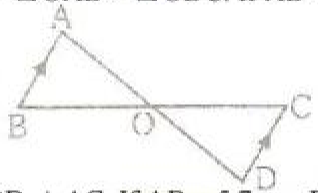
(i) If AD = 3.6 cm, AB = 9 cm and AE = 2.4 cm, find EC.

(ii) If $\frac{AD}{DB} = \frac{3}{5}$ and AC = 5.6 cm, find AE.

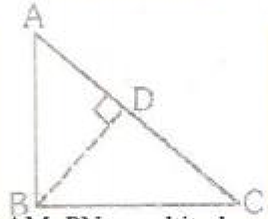
(iii) If AD = x cm, DB = (x – 2) cm, AE = (x + 2) cm and EC = (x – 1) cm, find the value of x.



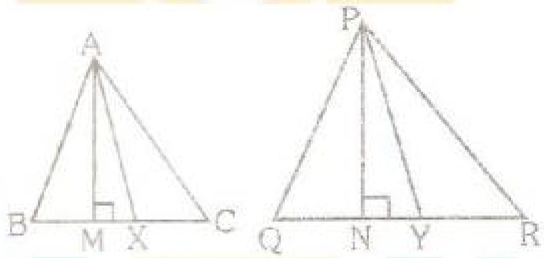
2. In the given figure, $BA \parallel DC$. Show that $\Delta OAB \sim \Delta ODC$. If AB = 4 cm, CD = 3 cm, OC = 5.7 cm and OD = 3.6 cm, find OA and OB.



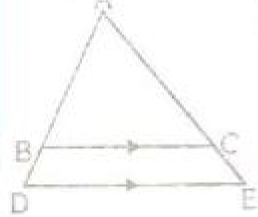
3. In the given figure, $\angle ABC = 90^0$ and $BD \perp AC$. If AB = 5.7 cm, BD = 3.8 cm and CD = 5.4 cm, find BC.



4. In the given figure, $\Delta ABC \sim \Delta PQR$ and AM, PN are altitude, whereas AX and PY are medians. Prove that $\frac{AM}{PN} = \frac{AX}{PY}$



5. In the given figure, $BC \parallel DE$, area (ΔABC) = 25 cm^2, area (trap. BCED) = 24 cm^2 and DE = 1 4 cm. Calculate the length of BC.



6. In ΔABC, $\angle C = 90^0$. If BC = a, AC = b and AB = c, find :

(i) c when a = 8 cm and b = 6 cm.

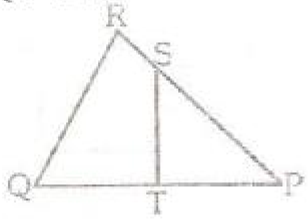
(ii) a when c = 25 cm and b = 7 cm.

(iii) b when c = 13 cm and a = 5 cm.

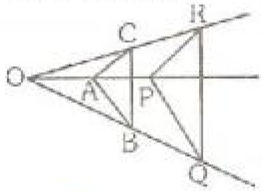
7. The sides of a right triangle containing the right angle are (5x) cm and (3x – 1) cm. If the area of triangle be 60 cm^2, calculate the length of the sides of the triangle.

8. Find the altitude of an equilateral triangle of side $5\sqrt{3}$ cm.

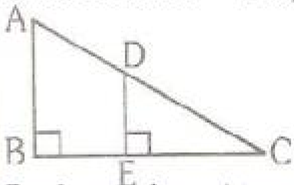
9. In the adjoining figure (not drawn to scale), PS – 4 cm, SR = 2 cm, PT = 3 cm and QT = 5 cm.
(i) Show that $\Delta PQR \sim \Delta PST$. (ii) Calculate ST, if QR = 5.8 cm.



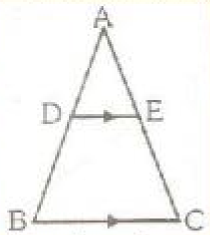
10. In the given figure, $AB \parallel PQ$ and $AC \parallel PR$. Prove that $BC \parallel QR$.



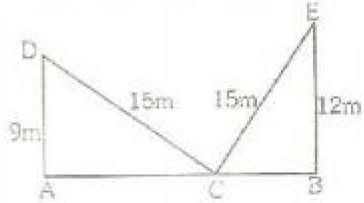
11. In the given figure, AB and DE are perpendicular to BC. If AB = 9 cm, DE = 3 cm and AC = 24 cm, calculate AD.



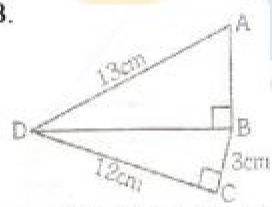
12. In the given figure, $DE \parallel BC$. If DE = 4 cm, BC = 6 cm and area (ΔADE) = 20 cm^2, find the area of ΔABC.



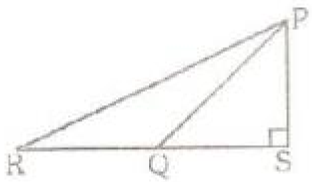
13. A ladder 15 m long reaches a window which is 9 m above the ground on one side of the street. Keeping its foot at the same point, the ladder is turned is turned to the other side of the street to reach a window 12 m high. Find the width of the street.



14. In the given figure, ABCD is a quadrilateral in which BC = 3 cm, AD = 13 cm, DC = 12 cm and $\angle ABD = \angle BCD = 90^0$. Calculate the length of AB.

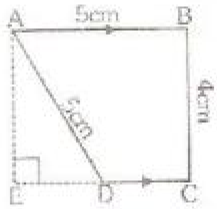


15. In the given figure, $\angle PSR = 90^0$, PQ = 10 cm, QS = 6 cm and RQ = 9 cm, calculate the length of PR.



16. In a rhombus PQRS, side PQ = 17 cm and diagonal PR = 16 cm. Calculate the area of the rhombus.

17. From the given figure, find the area of trapezium ABCD.

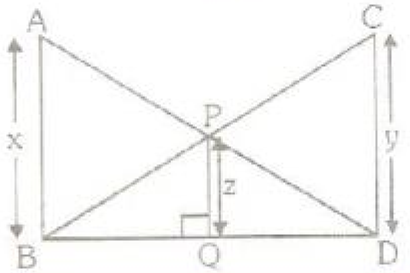


18. In a rhombus ABCD, prove that $AC^2 + BD^2 = 4AB^2$.

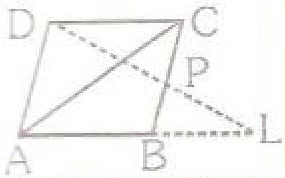
19. A ladder 13 m long rests against a vertical wall. If the foot of the ladder is 5 m from the foot of the wall, find the distance of the other end of the ladder from the ground.

LONG ANSWER TYPE QUESTIONS

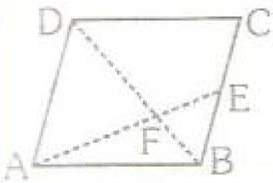
1. In the given figure, it is given that $\angle ABD = \angle CDB = \angle PQB = 90^0$. If AB = x units, CD = y units and PQ = z units, prove that $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$



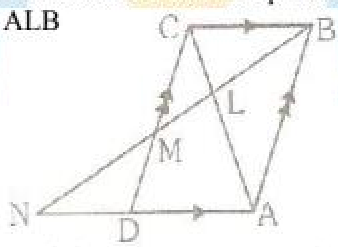
2. In the adjoining figures, ABCD is a parallelogram, P is a point on side BC and DP when produced meets AB produced at L. Prove that : (i) DP : PL = DC : BL (ii) DL : DP = AL : DC.



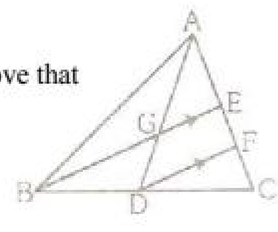
3. In the given figure, ABCD is a parallelogram, E is a point on BC and the diagonal BD intersects AE at F. Prove that DF $\times$ FE = FB $\times$ FA.



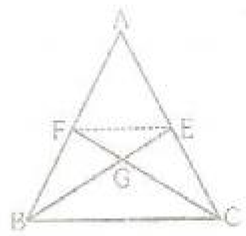
4. In the adjoining figure, ABCD is a parallelogram in which AB = 16 cm BC = 10 cm and L is a point on AC such that CL : LA = 2 : 3. If BL produced meets CD at M and AD produced at N, prove that :
(i) $\Delta CLB \sim \Delta ALN$ (ii) $\Delta CLM \sim \Delta ALB$



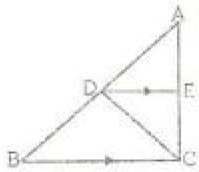
5. In the given figure, medians AD and BE of ΔABC meet at G and DF$\parallel$BE. Prove that
(i) EF = FC (ii) AG : GD = 2 : 1.



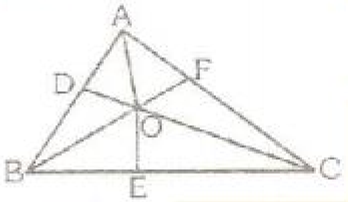
6. In the given figure, the medians BE and CF of ΔABC meet at G. Prove that :
(i) $\Delta GEF \sim \Delta GBC$ and therefore, BG = 2 GE. (ii) $AB \times AF = AE \times AC$.



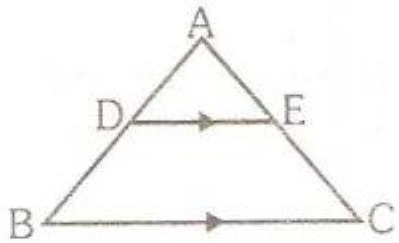
7. In the given figure, DE $\|$ BC and BD – DC.
(i) Prove that DE bisects $\angle ADC$.
(ii) If AD = 4.5 cm, AE = 3.9 cm and DC = 7.5 cm, find CE.
(iii) Find the ratio AD : DB.



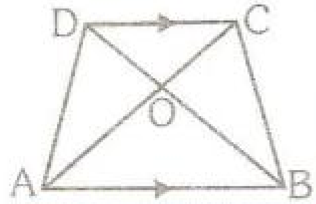
8. O is point inside a ΔABC. The bisectors of $\angle AOB$. $\angle BOC$ and $\angle COA$ meet the sides AB, BC and in points D, E and F respectively. Prove that AD-BE-CF = DB.EC.FA



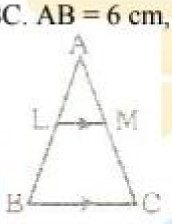
9. In the figure, DE $\|$ BC.
(i) Prove that ΔADE and ΔABC are similar.
(ii) Given that AD = $\frac{1}{2}$ BD. Calculate DE, if BC = 4.5 cm.



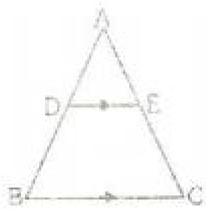
10. In the adjoining figure, ABCD is a trapezium in which AB $\|$ DC and AB = 2 DC. Determine the ratio of areas of ΔAOB and ΔCOD



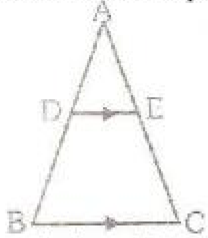
11. In the adjoining figure, LM is parallel to BC. AB = 6 cm, AL = 2 cm and AC = 9 cm. Calculate :
(i) the length of CM.
(ii) the value of $\frac{Area(\Delta ALM)}{Area(trap.LBCM)}$



12. In the given figure, DE $\|$ BC. and DE : BC = 3 : 5. Calculate the ratio of the areas of ΔADE and the trapezium BCED.



13. In ΔABC, D and E are mid-points of AB and AC respectively. Find the ratio of the areas of ΔADE and ΔABC.



14. In a ΔPQR, L and M are two points on the base QR, such that $\angle LPQ = \angle QRP$ and $\angle RPM = \angle RQP$. Prove that

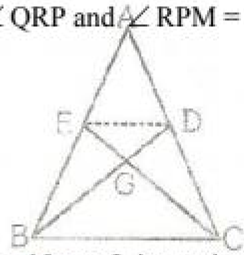
(i) $\Delta PQL - \Delta RPM$ (ii) QL.RM = PL.PM (iii) $PQ^2 = QL.QR$

15. In the adjoining figures, the medians BD and CE of a ΔABC meet at G.

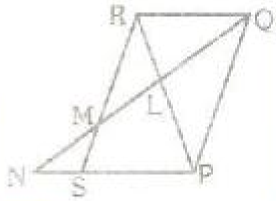
Prove that:

(i) $\Delta EGD \sim \Delta CGB$

(ii) BG = 2 GD from (i) above.



16. In the adjoining figure, PQRS is a rarallelog'am with PQ = 15 cm and RQ = 10 cm. L is a point on RP such that RL : LP = 2 : 3. QL produced meets RS at M and PS produced at N. Find the length of PN and RM.

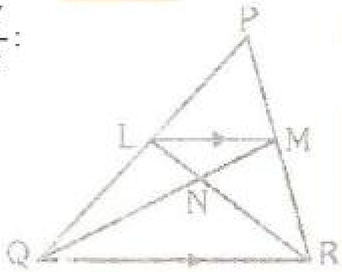


17. In ΔPQR, $LM \parallel QR$ and PM : MR = 3 : 4. Calculate:

(i) $\dfrac{PL}{PQ}$ and then $\dfrac{LM}{QR}$:

(ii) $\dfrac{Area\,(\Delta ALM)}{Area\,(\Delta MNR)}$

(iii) $\dfrac{Area\,(\Delta LQM)}{Area\,(\Delta LQN)}$

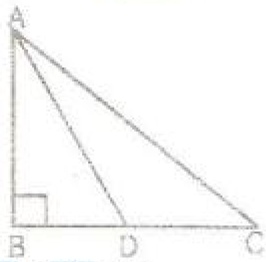


18. In ΔABC, $\angle B = 90^0$ and D is the mid point of BC.

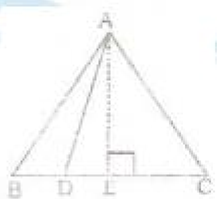
Prove that :

(i) $AC^2 = AD^2 + 3CD^2$

(ii) $BC^2 = 4(AD^2 - AB^2)$



19. In ΔABC, if AB = AC and D is a point on BC. Prove that $BC^2 - AD^2 = BD \times CD$.



VELY SHORT ANSWER TYPE QUESTIONS

1. (i) $\frac{1}{2}$ (ii) 4 cm **2.** (i) Yes, (ii) No, (iii) No, (iv) Yes **3.** 9 cm **4.** 13 m **5.** PQ = 4 cm **6.** AC = 5.6 cm **7.** 50 cm^2 **8.** 4.5 cm **9.** 50 m **10.** 17 m **11.** 8m

SHORT ANSWER TYPE QUESTIONS

1. (i) 3, 6 cm, (ii) 2.1 cm, (iii) x = 4 **2.** OA = 4.8 cm, OB = 7.6 cm **3.** 8.1 cm **5.** 10 cm **6.** (i) 10 cm, (ii) 24 cm, (iii) 12 cm **7.** 15 cm,8 cm, 17cm **8.** 7.5 cm **9.** 2.9 cm **11.** 16 cm **12.** 45 cm^2 **13.** 21m **14.** 4 cm **15.** 17 cm **16.** 240 cm^2 **17.** 14 cm^2 **19.** 12 m

LONG ANSWER TYPE QUESTIONS

7. (ii) 6.5 cm, (ii) 3 : 8 **9.** DE = 1.5 cm **10.** 4 : 1 **11.** (i) 6 cm, (ii) $\frac{1}{8}$ **12.** 9 : 16 **13.** 1 : 4

16. PN = 15 cm, RM = 10 cm **17.** (i) $\frac{PL}{PQ} = \frac{LM}{QR} = \frac{3}{7}$ (ii) 3 : 7 (iii) 10 : 7

EXERCISE – 3 (FOR SCHOOL/BOARD EXAMS)

PERVIOUS YEARS BOARD QUESTIONS

VERY SHORT ANSWER TYPE QUESTIONS

1. ΔABC and ΔDEF are similar, BC = 3 cm, EF = 4 cm and area of $\Delta ABC = 54\ cm^2$. Determine the area of ΔDEF. **Delhi-1996**

2. In ΔABC, CE $\perp$ AB, BD $\perp$ AC and & BD intersect at P, considering triangles BEP and CPD. Prove that BP $\times$ PD = EP $\times$ PC. **Delhi-1996C**

3. A right triangle has hypotenuse of length q cm and one side of length p cm. if (q – p) = 2, express the length of third side of the right triangle in terms of q. **Al-1996C**

4. In the given figure, ABC is a triangle in which AB = AC. D and E are points on the sides AB and AC respectively, such that AD = AE. Show that the points B, C, E and D are concyclic. **Al-1996C**

5. In a ΔABC, AB = AC and D is a point on side AC, such that BC2 = AC $\times$ CD. Prove that BD = BC. **Al-1997**

6. ΔABC is right angled at B. On side AC, a point D is taken such that AD = DC and AB = BD. Find the measure of $\angle CAB$ **Delhi-1998**

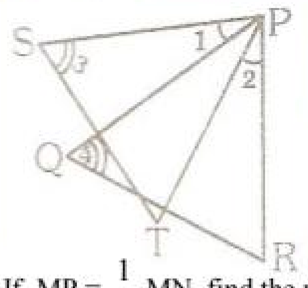
7. In a ΔABC, P and Q are points on the sides AB and AC respectively such that PQ is parallel to BC. Prove that median AD, drawn from A to BC, bisects PQ. **Al-1998**

8. Two poles of height 7 m and 12 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tips. **Al-1998C**

9. In a ΔABC, D and E are points on AB & AC respectively such that DE is parallel to BC and AD : DB = 2 : 3. Determine Area (ΔADE) : Area (ΔABC). **Foreign-1999**

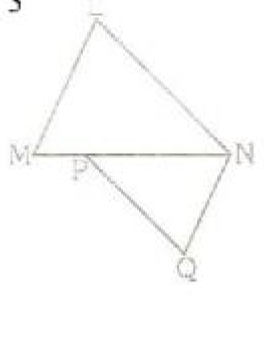
10. In the given figure, $\angle A = \angle B$ and D & E are points on AC and BC respectively such that AD = BE, show that DE $\|$ AB. **Delhi-1999**

11. In figure, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$. Show that PT. QR = PR. ST. **Foreign-2000**



12. In figure, LM $\|$ NQ and LN $\|$ PQ. If $MP = \frac{1}{3}$ MN, find the ratio of the areas of ΔLMN and ΔQNP.

Foreign-2000



13. ABC is an isosceles triangle right angled at B. Two equilateral triangles BDC and AEC are constructed with side BC and AC. Prove that area of $\Delta BCD = \frac{1}{2}$ area of ΔACE. **Delhi-2001**

14. The areas of two similar triangles are 81 cm^2 and 49 cm^2 respectively. If the altitude of the first triangle 6.3 cm, find the corresponding altitude of the other. **Al-2001**

15. L and M are the mid-points of AB and BC respectively of ΔABC, right-angled at B. prove that $4LC^2 = AB^2 + 4BC^2$. **Al-2001;Foreign-2001**

16. The areas of two similar triangles are 121 cm^2 and 64 cm^2 respectively. If the median of the first triangle is 12.1 cm. find the corresponding median of the other. **Al-2001**

17. In an equilateral triangle ABC, AD is the altitude drawn from A on side BC. Prove that $3AB^2 = 4AD^2$. **Delhi-2002**

18. (i) Prove that the equilateral triangle described on the two sides of a right angled triangle are together equal to the equilateral triangle on the hypotenuse in terms of their areas. **Al-2002**

(ii) P is a point in the interior of ΔABC, X, Y and Z are point on lines PA.PB and PC respectively such that $XY \parallel AB$ and $XZ \parallel BC$. Prove that $YZ \parallel BC$. **Al-2002 : Delhi-2003 [NCERT]**

(iii) D and E are points on the sides AB and AC respectively of ΔABC such that DE is parallel to BC and AD : DB = 4 : 5. CD and BE intersect each other at F. Find the ratio of the areas of ΔDEF and ΔBCE **Al-2000 : Al-2003**

(iv) P. Q are respectively points on sides AB and AC of triangle ABC. If AP = 2 cm. PB = 4 cm. AQ = 3 cm and QC = 6 cm. prove that BC = 3PQ. **Foreign-2003**

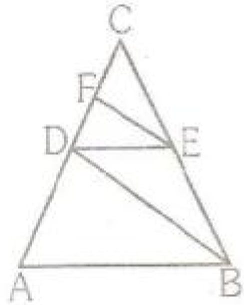
19. D is a point on the side BC of ΔABC such that $\angle ADC = \angle BAC$. Prove that $\frac{CA}{CD} = \frac{CB}{CA}$. **Delhi-2002:[NCERT]**

20. ABCD is a trapezium in which $AB \parallel DC$. The diagonals AC and BD intersect at O. Prove that $\frac{AO}{OC} = \frac{BO}{DO}$ **Al-2004:[NCERT]**

21. In a ΔABC, $AD \perp BC$ and $\frac{BD}{AD} = \frac{AD}{DC}$. Prove that ABC is a right triangle, right angled at A. **Foreign-2004**

22. In a right angled triangle ABC, $\angle A = 90^0$ and $AD \perp BC$. Prove that $AD^2 = BD \times CD$. **Delhi-2004C, 2006**

23. In fig., $AB \parallel DE$ and $BD \parallel EF$. Prove that DC2 = CF × AC. **Al-2004C : Delhi-2007**

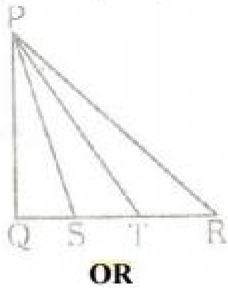


24. If one diagonal of a trapezium divides the other diagonal in the ratio of 1 : 2. prove that one of the parallel sides is double the other. **Foreign-2005**

25. In ΔABC, $AD \perp BC$, prove that $AB^2 + CD^2 = AC^2 + DB^2$. **Delhi-2005C, Al-2006 [NCERT]**

26. Prove that the sum of the squares of the sides of a rhombus is equal to sum of the squares of its diagonals. **Al-2005C [NCERT]**

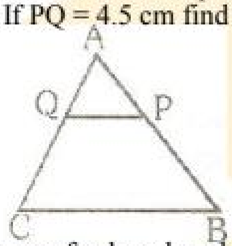
27. In figure, S and T trisect the side QR of a right triangle PQR. Prove that $8PT^2 = 3PR^2 + 5PS^2$.



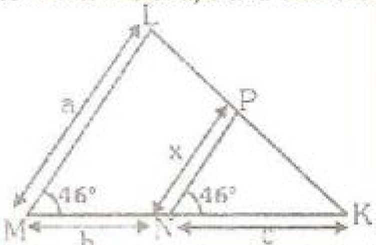
OR

If BL and CM are medians of a triangle ABC right-angled at A, then prove that $4(BL^2 + CM^2) = 5BC^2$. **Al-2006 C; Foreign-2009**

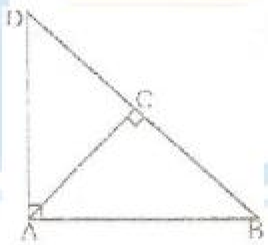
28. In the fig, P and Q are points on the sides AB and AC respectively of ΔABC such that AP = 3.5 cm, PB = 7 cm, AQ = 3 cm and QC = 6 cm. If PQ = 4.5 cm find BC. **Delhi-2008**



29. In fig. $\angle M = \angle N = 46^0$ Express x in terms of a, b and c where a, b and c are lengths of LM, MN and NK respectively. **Delhi-2009**



30. In figure, ΔABC is a right triangle, right-angled at A and $AC \perp BD$. Prove that $AB^2 = BC.\ BD$. **Al-2009**



31. In a ΔABC, $DE \| BC$. If $DE = \frac{2}{3}BC$ and area of $\Delta ABC = 81\ cm^2$, find the area of ΔADE. **Foregin-2009**

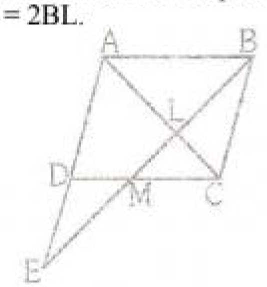
SHORT ANSWER TYPE QUESTIONS

1. P and Q are points on the sides CA and CB respectively of a ΔABC right-angled at C. prove that $AQ^2 + BP^2 = AB^2 + PQ^2$. **Delhi-1996, 2007**

2. ABC is a right triangle, right angled at B. AD and CE are the two medians drawn from A and C respectively. If AC = 5 cm and AD = $\frac{3\sqrt{5}}{2}$ cm, find the length of CE. **Al -1997**

3. In ΔABC, if AD is the median, show that $AB^2 + AC^2 = 2[AD^2 + BD^2]$. **Delhi-1997, 98**

4. In the given figure, M is the mid-point of the side CD of parallelogram ABCD. BM. When joined meets AC is L and AD produced in E. Prove that EL = 2BL. **Al-1998; Delhi-1999, Al-2009**



5. ABC is a right triangle, right-angled at C. if p is the length of the perpendicular from C to AB and a, b, c have the usual meaning, then prove that (i) pc = ab (ii) $\frac{1}{p^2} = \frac{1}{a^2} = \frac{1}{b^2}$ **Delhi-1998, 98 C**

6. In an equilateral triangle PQR, the side QR is trisected at S. Prove that $9PS^2 = 7PQ^2$. **Al-1998, 98C [NCERT]**

7. If the diagonals of a quadrilateral divide each other proportionally, prove that it is trapezium. **Foreign-1999**

8. In an isosceles triangle ABC with AB = AC, BD is a perpendicular from B to the side AC. Prove that $BD^2 - CD^2 = 2CD.AD$. **Foreign-1999**

9. ABC and DBC are two triangles on the same base BC. If AD intersect BC at O. Prove that $\frac{ar.\Delta ABC}{ar.\Delta DBC} = \frac{AO}{DO}$ **Al-1999C; Delhi-2005**

10. In ΔABC, $\angle A$ is acute. BD and CE are perpendiculars on AC and AB respectively. Prove that $AB \times AE = AC \times AD$. **Al-2003**

11. Points P and Q are on sides AB and AC of a triangle ABC in such a way that PQ is parallel to side BC. Prove that the median AD drawn from vertex A to side BC bisects the segment PQ. **Foreign -2003**

12. If the diagonals of a quadrilateral divide each other proportionally, prove that it is a trapezium.

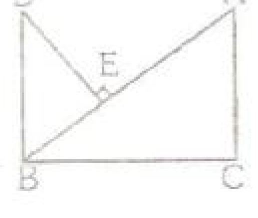
OR

Two Δs ABC and DBC are on the same base BC and on the same side of BC in which $\angle A = \angle D = 90^0$. If CA and BD meet each other at E, show that AE.ED. **Delhi-2008**

13. D and E are points on the sides CA and CB respectively of ΔABC right-angled at C. prove that $AE^2 + BD^2 = AB^2 + DE^2$.

OR

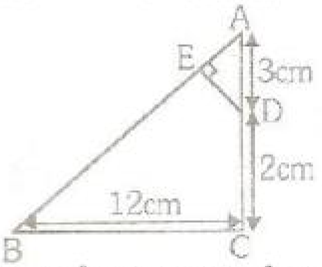
In fig. $DB \perp BC$, $DE \perp AB$ and $AC \perp BC$. Prove that $\frac{BE}{DE} = \frac{AC}{BC}$. **Al-2008**



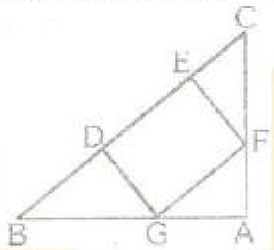
14. E is a point on the side AD produced of a $\parallel^{gm}$ ABCD and BE intersects CD at F. Show that $\Delta ABC \sim \Delta CFB$.

Foreign-2008

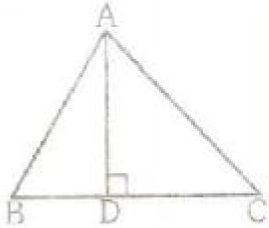
15. In fig, ΔABC is right angled at C and $DE \perp AB$. Prove that $\Delta ABC \sim \Delta ADE$ and hence find the lengths of AE and DE.



In fig, DEFG is a square and $\angle BAC = 90^0$. Show that $DE^2 = BD \times EC$ **Delhi-2009**

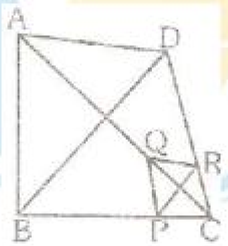


16. In fig, $AD \perp BC$ and $BD = \frac{1}{3}$ CD. Prove that $2CA^2 = 2AB^2 + BC^2$. **Al-2009**



17. In fig, two triangles ABC and DBC lie on the same side of base BC. P is a point on BC such that $PQ \parallel BA$ and $PR \parallel BD$. Prove that $QR \parallel AD$.

Foreign-2009



LONG ANSWER TYPE QUESTIONS

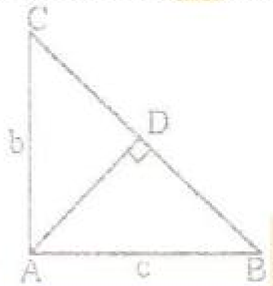
1. In a right triangle ABC, right-angled at C, P and Q are points on the sides CA and CB respectively which divide these sides in the ratio 1 : 2. Prove that **Al-1996C**

(i) $9AQ^2 = 9AC^2 + 4BC^2$ (ii) $0BP^2 = 9BC^2 + 4AC^2$ (iii) $9(AQ^2 + BP^2) = 13AB^2$.

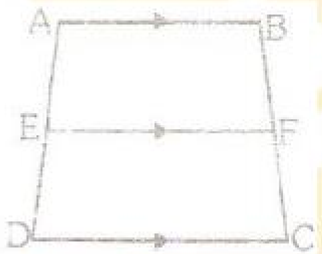
2. The ratio of the areas of similar triangles is equal to the ratio of the square on the corresponding sides, prove. Using the above theorem, prove that the area of the equilateral triangle described on the side of a square is half the area of the equilateral triangle described on its diagonal. **Delhi-1997C; 2005C; Foreign-2003**

3. Perpendiculars OD, OE and OF are drawn to sides BC, CA and AB respectively from a point O in the interior of a ΔABC. Prove that :
   (i) $AF^2 + BD^2 + CE^2 = OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$.
   (ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$. **Delhi-1997C, [NCERT]**

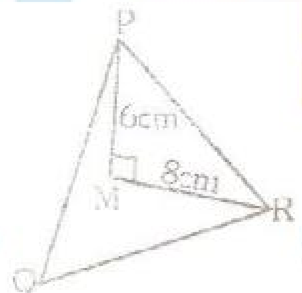
4. In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares on the other two sides. Prove. Using the above theorem, determine the length of AD in terms of b and C. **Al-1997 C**



5. If a line is drawn parallel to one side of a triangle, other two sides are divided in the same ratio, Prove. Using this result to prove the following : In the given figure, if ABCD is a trapezium in which $AB \parallel DC \parallel EF$, then $\frac{AE}{ED} = \frac{BF}{FC}$. **Foreign-1998**



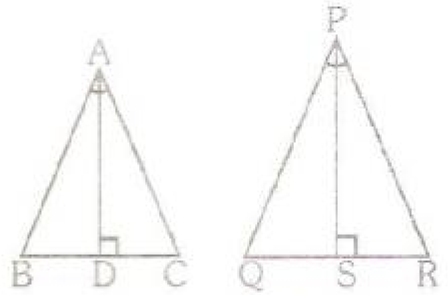
6. State and prove Pythagoras. Use the theorem and calculate are (ΔPMR) from the given figure. **Delhi-1998C, 2006**



7. In a right-angled triangle, the square of hypotenuse is equal to the sum of the squares of the two sides. Given that $\angle B$ of ΔABC is an acute angle and $AD \perp BC$. Prove that $AC^2 = AB^2 + BC^2 - 2BC.\ BD$. **Delhi-1999**

8. In a right triangle, prove that the square on the hypotenuse is equal to the sum of the squares on the other two sides. Using above, solve the following : In quadrilateral ABCD, find the length of CA, if $CD \perp DB$, CD = 6 m, DB = 12 m and AB = 11 m. **Delhi-2000**

9. Prove that the ratio of the areas of two similar triangles is equal to the squares of their corresponding sides. Using the above, do the following



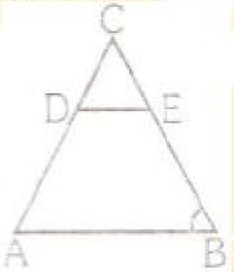
In fig, ΔABC and ΔPQR are isosceles triangles in which $\angle A = \angle P$. If $\frac{area(\Delta ABC)}{area(\Delta PQR)} = \frac{9}{16}$, find $\frac{AD}{PS}$. **AI-2000**

10. In a right-angled triangle, prove that the square on the hypotenuse is equal to the sum of the squares on the other two sides. Using the above result, find the length of the second diagonal of a rhombus whose side is 5 cm and one of the diagonals is 6 cm. **AI-2001**

11. In a triangle, if the square on one side is equal to the sum of the squares on the other two sides prove that the angle opposite the first side is a right angel.
Using the above theorem and prove that following : In triangle ABC, AD $\perp$ BC and BD = 3CD. Prove that $2AB^2 = 2AC^2 + BC^2$. **AI-2003**

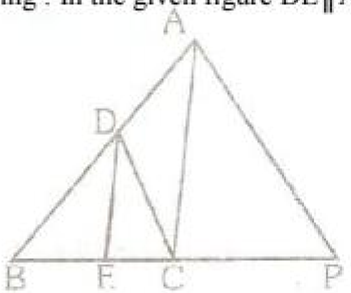
12. In a right triangle, prove that the square on hypotenuse is equal to sum of the squares on the other two sides.Using the above result, prove that following : PQR is a right triangle right angled at Q. If S bisects QR, show that $PR^2 = 4PS^2 - 3PQ^2$. **Delhi-2004C**

13. If a line is drawn parallel to one side of a trial prove that the other two sides are divided in the same ratio. Using the above result, prove from fig. that AD = BE if $\angle A = \angle B$ and DE $\parallel$ AB. **AI-2004C**



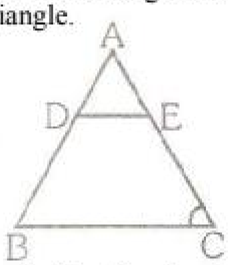
14. Prove that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides. Apply the above theorem on the following : ABC is a triangle and PQ is a straight line meeting AB in P and AC in Q. If AP = 1 cm, PB = 3 cm, AQ = 1.5 cm, QC = 4.5 cm, prove that area of ΔAPQ is one-sixteenth of the area of ΔABC. **Delhi-2005**

15. If a line is drawn parallel to one side of a triangle, prove that the other two sides are divided in the same ratio. Use the above to prove the following : In the given figure DE $\parallel$ AC and DC $\parallel$ AP. Prove that $\frac{BE}{EC} = \frac{BC}{CP}$. **AI-2005**



16. In a triangle if the square on one side is equal to the sum of squares on the other two sides, prove that the angle opposite to the first side is a right angle. Using the above theorem to prove the following :
In a quadrilateral ABCD, $\angle B = 90^0$. If $AD^2 = AB^2 + BC^2 + CD^2$, prove that $\angle ACD = 90^0$. **AI-2205**

17. If a line is drawn parallel to one side of a triangle, to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio. Using the above, prove the following : In figure, DE $\parallel$ AC and BD = CE. Prove that ABC is an isosceles triangle. **Delhi-2007, 2009**



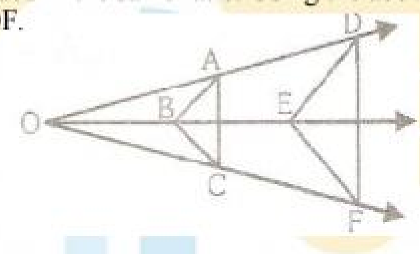
18. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides. Using the above for the following : If the areas of two similar triangles are equal, prove that they are congruent. **AI-2007**

19. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides. Using the above result, prove the following :

In a ΔABC, XY is parallel to BC and it divides ΔABC into two parts of equal area. Prove that $\frac{BX}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$ **Delhi-2008**

20. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides. Using the above, do the following :
The diagonals of a trapezium ABCD, with AB $\parallel$ DC, intersect each other at the point O. If AB = 2 CD, find the ratio of the area of ΔAOB to the area of ΔCOD. **AI -2008**

21. If a line is drawn parallel to one side of a triangle, to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio. Using the above, prove the following : In the fig, AB $\parallel$ DE and BC $\parallel$ EF. Prove that AC $\parallel$ DF. **Foreign-2008**



2. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides. Using the above, do the following : In a trapezium ABCD, AC and BD are intersecting at O, AB $\parallel$ DC and AB = 2 CD. If area of $\Delta AOB = 84\ cm^2$, find the area of ΔCOD. **Delhi-2009**

VERY SHORT ANSWER TYPE QUESTIONS

2. 96 cm^2 **3.** $2\sqrt{(q-1)}$ **6.** 60^0 **8.** 13 m **9.** 4.25 **12.** 9 : 4 **14.** 4.9 cm **16.** 8.8 cm **18.** (iii) 16 : 81

28. 13.5 cm **19.** $\left(\frac{ac}{b+c}\right)$ **31.** 36 cm^2

SHORT ANSWER TYPE QUESTIONS

2. $2\sqrt{5}$ cm **15.** AE = $\frac{15}{13}$, DE = $\frac{36}{13}$

LONG ANSWER TYPE QUESTIONS

4. $\frac{bc}{\sqrt{b^2+c^2}}$ **6.** 24 cm^2 **8.** 13 cm **9.** 3 : 4 **10.** 8 cm **21.** 4 : 1 **23.** 21 cm^2

EXERCISE – 1 (FOR OLYMPIADS)

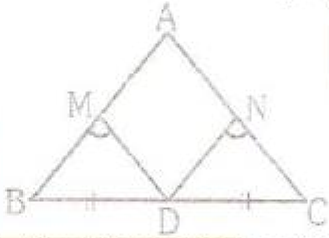
CHOOSE THE CORRECT ONE

1. In a triangle ABC, if AB, BC and AC are the three sides of the triangle, then which of the statements is necessarily true?
   (A) AB + BC < AC (B) AB + BC > AC (C) AB + BC = AC (D) $AB^2 + BC^2 = AC^2$

2. The sides of a triangle are 12 cm, 8 cm and 6 cm respectively, the triangle is :
   (A) acute (B) obtuse (C) right (D) can't be determined

3. In an equilateral triangle, the incentre, circumcentre, orthocenter and centroid are:
   (A) concylic (B) coincident (C) collinear (D) none of these

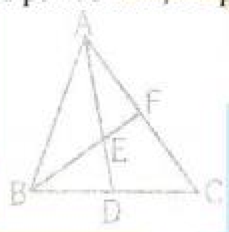
4. In the adjoining figure D is the midpoint of a ΔABC. DM and DN are the perpendiculars on AB and AC respectively and DM = DN, then the ΔABC is :
   (A) right angled
   (B) isosceles
   (C) equilateral
   (D) scalene



5. Triangle ABC is such that AB = 9 cm, BC = 6 cm, AC = 7.5 cm, Triangle DEF is similar to ΔABC, If EF = 12 cm then DE is :
   (A) 6 cm (B) 16 cm (C) 18 cm (D) 15 cm

6. In ΔABC, AB = 5 cm, AC = 7 cm. If AD is the angle bisector of < A. Then BD : CD is :
   (A) 25 : 49 (B) 49 : 25 (C) 6 : 1 (D) 5 : 7

7. In a ΔABC, D is the mid-point of BC and E is mid-point of AD, BF passes through E. What is the ratio of AF : FC
   (A) 1 : 1
   (B) 1 : 2
   (C) 1 : 3
   (D) 2 : 3

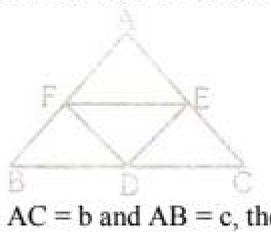


8. In a ΔABC, AB = AC and AD $\perp$ BC, then :
   (A) AB < AD (B) AB > AD (C) AB = AD (D) AB $\leq$ AD

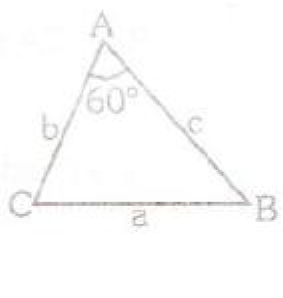
9. The difference between altitude and base of a right angled triangle is 17 cm and its hypotenuse is 25 cm. What is the sum of the base and altitude of the triangle is ?
   (A) 24 cm (B) 31 cm (C) 36 cm (D) can't be determined

10. If AB, BC and AC be the three sides of a triangle ABC, which one of the following is true ?
    (A) AB – BC = AC (B) (AB – BC) > AC (C) (AB – BA) < AC (D) $AB^2 - CB^2 = AC^2$

11. In the adjoining figure D, E and F are the mid-points of the sides BC, AC and AB respectively. ΔDEF is congruent to triangle :
    (A) ABC
    (B) AEF
    (C) CDE , BFD
    (D) AFE , BFD and CDE



12. In the adjoining figure $\angle BAC = 60^0$ and BC = a, AC = b and AB = c, then :



(A) $a^2 = b^2 + c^2$
(B) $a^2 = b^2 + c^2 - bc$
(C) $a^2 = b^2 + c^2 + bc$
(D) $a^2 = b^2 + 2bc$

13. If the medians of a triangle are equal, then the triangle is:
(A) right angled (B) isosceles (C) equilateral (D) scalene

14. The incentre of a triangle is determined by the:
(A) Medians (B) angle bisectors
(C) perpendicular bisectors (D) altitudes

15. The point of intersection of the angle bisectors of a triangle is :
(A) orthocenter (B) centroid (C) incentre (D) circumcentre

16. A triangle PQR is formed by joining the mid-points of the sides of a triangle ABC, 'O' is the circumcentre of ΔABC, then for ΔPQR, the point 'O' is :
(A) incentre (B) circumcentre (C) orthocenter (D) centroid

17. If AD, BE, CF are the altitudes of ΔABC whose orthocenter is H, then C is the orthocenter of :
(A) ΔABH (B) ΔBDH (C) ΔABD (D) ΔBEA

18. In an equilateral ΔABC, if a, b and c denote the lengths of perpendiculars from A, B and C respectively on the opposite sides, then:
(A) $a > b > c$ (B) $a > b < c$ (C) $a = b = c$ (D) $a = c \neq b$

19. Any two of the four triangles formed by joining the midpoints of the sides of a given triangle are:
(A) congruent (B) equal in area but not congruentAB > AD
(C) unequal in area and not congruent (D) none of these

20. The internal bisectors of $\angle B$ and $\angle C$ of ΔABC meet at O. If $\angle A = 80^0$ then $\angle BOC$ is :
(A) 50^0 (B) 160^0 (C) 100^0 (D) 130^0

21. The point in the plane of a triangle which is at equal perpendicular distance from the sides of the triangle is :
(A) centroid (B) incentre (C) circumcentre (D) orthocenter

22. Incentre of a triangle lies in the interior of :
(A) an isosceles triangle only (B) a right angled triangle only
(C) any equilateral triangle only (D) any triangle

23. In a triangle PQR, PQ = 20 cm and PR = 6 cm, the side QR is :
(A) equal to 14 cm (B) less than 14 cm (C) greater than 14 cm (D) none of these

24. If ABC is a right angled triangle at B and M, N are the mid-points of AB and BC, than $4\,(AN^2 + CM^2)$ is equal to-
(A) $4AC^2$ (B) $6\,AC^2$ (C) $5\,AC^2$ (D) $\frac{5}{4}\,AC^2$

25. ABC is a right angle triangle at A and AD is perpendicular to the hypotence. Then $\frac{BD}{CD}$ is equal to :

(A) $\left(\frac{AB}{AC}\right)^2$ (B) $\left(\frac{AB}{AD}\right)^2$ (C) $\frac{AB}{AC}$ (D) $\frac{AB}{AD}$

26. Let ABC be an equilateral triangle. Let BE $\perp$ CA meeting CA at E, then $(AB^2 + BC^2 + CA^2)$ is equal to :

(A) $2BE^2$ (B) $3\ BE^2$ (C) $4\ BE^2$ (D) $6\ BE^2$

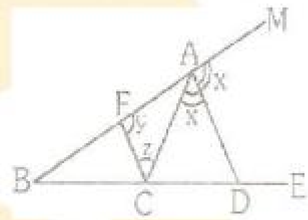
27. If D, E and F are respectively the mid-points of sides of BC, CA and AB of a ΔABC. If EF = 3 cm, FD = 4 cm, and AB = 10 cm, then DE, BC and CA respectively will be equal to :

(A) 6, 8 and 20 cm (B) 4, 6 and 8 cm (C) 5, 6 and 8 cm (D) $\frac{10}{3}$, 9 and 12 cm

28. In the right angle triangle $\angle C = 90^0$. AE and BD are two medians of a triangle ABC meeting at F. The ratio of the area of ΔABF and the quadrilateral FDCE is :

(A) 1 : 1 (B) 1 : 2 (C) 2 : 1 (D) 2 : 3

29. The bisector of the exterior $\angle$A of ΔABC intersects the side BC produced to D. Here CF is parallel to AD.



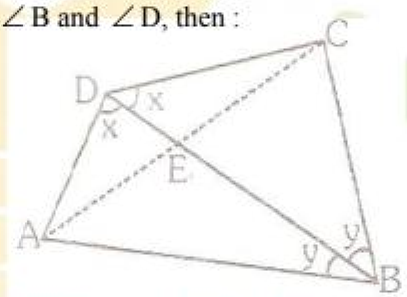
(A) $\frac{AB}{AC} = \frac{BD}{CD}$

(B) $\frac{AB}{AC} = \frac{CD}{BD}$

(C) $\frac{AB}{AC} = \frac{BC}{CD}$

(D) None of these

30. The diagonal BD of a quadrilateral ABCD bisects $\angle$B and $\angle$D, then :



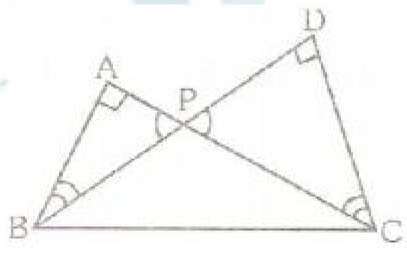
(A) $\frac{AB}{CD} = \frac{AD}{BC}$

(B) $\frac{AB}{BC} = \frac{AD}{CD}$

(C) $AB = AD \times BC$

(D) None of these

31. Two right triangles ABC and DBC are drawn on the same hypotenuse BC on the same side of BC. If AC and DB intersects at P, then



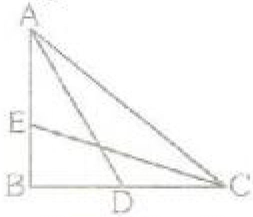
(A) $\frac{AP}{PC} = \frac{BP}{DP}$

(B) $AP \times DP = PC \times BP$

(C) $AP \times PC \times = BP \times DP$

(D) $AP \times BP \times = PC \times PD$

32. In figure, ABC is a right triangle, right angled at B. AD and CE are the two medians drawn from A and C respectively. If AC = 5 cm and AD = $\frac{3\sqrt{5}}{2}$ cm, find the length of CE:



(A) $2\sqrt{5}$ cm
(B) 2.5 cm
(C) 5 cm
(D) $4\sqrt{2}$ cm

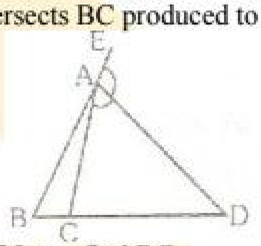
33. In a ΔABC, AB = 10 cm, BC = 12 cm and AC = 14 cm. Find the length of median AD. If G is the centroid, find length of GA :

(A) $\frac{5}{3}\sqrt{7}, \frac{5}{9}\sqrt{7}$ (B) $5\sqrt{7}, 4\sqrt{7}$ (C) $\frac{10}{\sqrt{3}}, \frac{8}{3}\sqrt{7}$ (D) $4\sqrt{7}, \frac{8}{3}\sqrt{7}$

34. The three sides of a triangles are given. Which one of the following is not a right triangle ?

(A) 20, 21, 29 (B) 16, 63, 65
(C) 56, 90, 106 (D) 36, 35, 74

35. In the figure AD is the external bisector of $\angle EAC$, intersects BC produced to D. If AB = 12 cm, AC = 8 cm and BC = 4 cm, find CD.

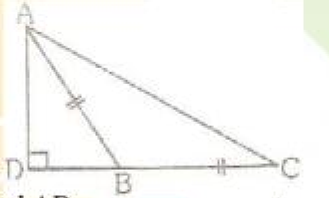


(A) 10 cm
(B) 6 cm
(C) 8 cm
(D)9 cm

36. In ΔABC, AB2 + AC2 = 2500 cm2 and median AD = 25 cm, find BC.

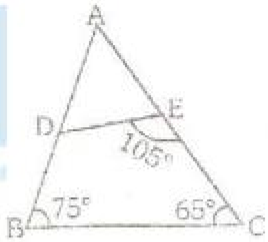
(A) 25 cm (B) 40 cm (C) 50 cm (D) 48 cm

37. In the given figure, AB = BC and $\angle BAC$ = 150. AB = 10 cm. Find the area of ΔABC.



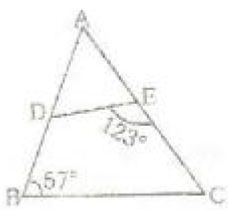
(A) 50 cm^2
(B) 40 cm^2
(C) 25 cm^2
(D) 32 cm^2

38. In the given figure, if $\frac{DE}{BC} = \frac{2}{3}$ and if AE = 10 cm. Find AB



(A) 16 cm
(B) 12 cm
(C) 15 cm
(D) 18 cm

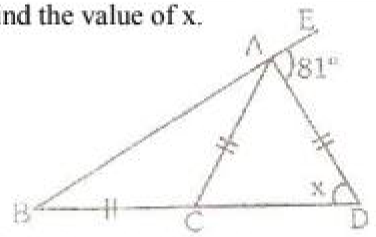
39. In the figure AD = 12 cm. AB = 20 cm and AE = 10 cm. Find EC.



(A) 14 cm
(B) 10 cm
(C) 8 cm
(D) 15 cm

40. In the given fig, BC = AC = AD, $\angle$EAD = 81^0. Find the value of x.

(A) 45^0
(B) 54^0
(C) 63^0
(D) 36^0



41. What is the ratio of inradius to the circumradius of a right angled triangle?

(A) 1 : 2 (B) $1 : \sqrt{2}$ (C) 2 : 5 (D) Can't be determined

ANSWER KEY															
Ans.	B	B	B	B	C	D	B	B	B	C	D	B	C	B	C
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	C	A	C	A	D	B	D	C	C	A	C	C	A	A	B
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**				
Ans.	C	A	D	D	C	C	C	C	A	B	D				