SURFACE AREAS AND VOLUMES

★ INTRODUCTION

In this chapter, shall discuss problems on conversion of one of the solids like cuboid, cube, right circular cylinder, right circular cone and sphere in another.

In our day-to-day life we come across various solids which are combinations of two or more such solids, For example, a conical circus tent with cylindrical base is a combination of a right circular cylinder and a right circular cone, also an ice-cream cone is a combination of a cone and a hemi-sphere. We shall discuss problems on finding surface areas and volumes of such solids. We also come across solids which are a part of a cone. For example, a bucket , a glass tumbler, a friction clutch etc. these solids are known as frustums of a cone. In the end of the chapter, we shall discuss problems on surface area and volume of frustum of a cone.

★ UNITS OF MEASUREMENT OF AREA AND VOLUME

The inter-relationships between various units of measurement of length, area and volume are listed below for ready reference:

LENGTH

1 Centimetre (cm)	=	10 milimetre (mm)
1 Decimetre (dm)	=	10 centimetre
1 Metre (m)	=	10 dm = 100 cm = 1000mm
1 Decametre (dam)	=	10 m = 1000 cm
1 Hectometre (hm)	=	10 dam = 100 m
1 Kilometre (km)	=	1000 m = dam = 10 hm
1 Myriametre	=	10 kilocetre

AREA

$1\ cm^2 = 1\ cm \times 1\ cm = 10\ mm \times 10\ mm = 100\ mm^2$

$1\ dm^2 = 1\ dm \times 1\ dm = 10\ cm \times 10\ cm = 100\ cm^2$

$1\ m^2 = 1\ m \times 1\ m = 10\ dm \times 10\ dm = 100\ dm^2$

$1\ dam^2 = 1\ dam \times 1\ dam = 10\ m \times 10\ m = 100\ m^2$

$1\ hm^2$ 1 hectare $= 1\ hm \times 1\ hm = 100\ m \times 100\ m = 10000\ m^2 = 100\ dm^2$

$1\ km^2 = 1\ km \times 1 km = 10\ hm \times 10\ hm = 100\ hm^2$ or 100 hectare

VOLUME

$1\ cm^3 = 1\ ml = 1\ cm \times 1\ cm \times 1cm = 10\ mm \times 10\ mm \times 10\ mm = 1000\ mm^3$

1 litre $= 1000\ ml = 1000\ cm^3$

$1\ m^3 = 1\ m \times 1\ m \times 1\ m = 100\ cm \times 100\ cm \times 100\ cm = 10^6\ cm^3 = 1000$ litre = 1 kilolitre

$1 dm^3 = 1000\ cm^3$

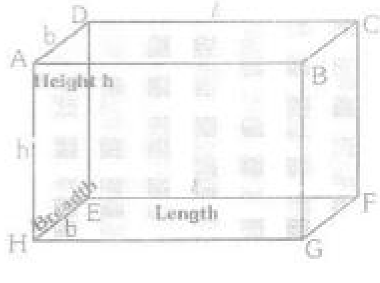
$1\ m^3 = 1000\ dm^3$

$1\ km^3 = 10^9\ m^3$

★ CUBOID

A rectangular solid bounded by six rectangular plane faces is called a cuboid. A match box, a tea-packet, a brick, a book, etc., are all examples of a cuboid.

A cuboid has 6 rectangular faces, 12 edges and 8 vertices.



The following are some definitions of terms related to a cuboid.

(i) The space enclosed by a cuboid is called its **volume.**

(ii) The line joining opposite comers of a cuboid is called its **diagonal.**
A cuboid has four diagonals.
A diagonal of a cuboid is the length of the longest rod that can be placed in the cuboid.

(iii) The sum of areas of all the six faces of a cuboid is known as its **total surface area.**

(iv) The four faces which meet the base of a cuboid are called the **lateral faces** of the cuboid.

(v) The sum of areas of the four walls of a cuboid is called its lateral **surface area.**

Formulae

For a cuboid of length = units, breadth = b units and height = h units, we have:

Sum of lengths of all edges = 4 $(\ell + b + h)$ **units.**
Diagonal of cuboid $= \sqrt{\ell^2 + b^2 + h^2}$ **units.**
Total Surface Area of cuboid = $2(\ell b + bh + \ell h)$ **sq. units.**
Lateral Surface Area of cuboid = [2 $(\ell + b) \times h$ **] sq. units.**
Area of four walls of a room = [2 $(\ell + b) \times h$ **] sq. units.**
Volume of cuboid = $(\ell \times b \times h)$ **cubic units.**

REMARK: For the calculation of surface area, volume etc. of a cuboid, the length, breadth and height must be expressed in the same units.

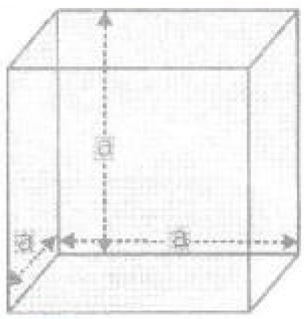
★ CUBE

A cuboid whose length, breadth and height are all equal

is called a cube .

Ice-cubes, Sugar, Dice, etc. are all examples of a cube.

Each edge of a cube is called its side.



Cube

Formulae

For a cube of edge = a units, we have;

Sum of length of all edges = 12 a nuits.
Diagonal of cube = $(a\sqrt{3})$ **nuits.**
Total Surface Area of cube = ($6a^2$) sq. nuits.
Leateral Surface Area of cube = ($4a^2$) sq. nuits.
Volume of cube = a^3 cubic units.

★ CROSS SECTION

A cut which is made through a solid perpendicular to its length is called its cross section. If the cut has the same shape and size at every point of its length, then it is called **uniform cross-section.**

Volume of a solid with uniform cross section = (Area of its cross section) x (length).
Lateral Surface Area of a solid with uniform cross section
= (Perimeter of cross section) x (length).

Ex.1 The length, breadth and height of a rectangular solid are ratio 6 : 5 : 4. If the total surface area is 5328 cm^2, find the length, breadth and height of the solid.

Sol. Let length = (6x) cm, breadth = (5x) cm and height = (4x) cm.

Then, total surface area = $[2(6x \times 5x + 5x \times 4x + 4x \times 6x)]\ cm^2 = [2(30x^2 + 20x^2)]\ cm^2$
$= (148x^2) cm^2$.

$\therefore$ 148x2 = 5328 ⇨ $x^2 = 36$ ⇨ x = 6.

Hence, length = 36 cm, breadth = 30 cm, height = 24 cm.

Ex.2 An open rectangular cistern is made of iron 2.5 cm thick. When measured from outside, it is 1 m 25 cm long. 1 m 5 cm broad and 90 cm deep.

Find: (i) the capacity of the cistern in litres:
(ii) the volume of iron used;
(iii) the total surface area of the cistern.

Sol. External dimensions of the cistern are :
Length = 125 cm, Breadth = 105 cm and Depth = 90 cm.
Internal dimensions of the cistern are :
Length = 120 cm, Breadth = 100 cm and Depth = 87.5 cm.

(i) Capacity = Internal volume = $(120 \times 100 \times 87.5)\ cm^3 = \left(\frac{120 \times 100 \times 87.5}{1000}\right)$ litres = 1050 litres.

(ii) Volume of iron = (External volume) – (Internal volume) = $[(125 \times 105 \times 90) - (120 \times 100 \times 87.5)]\ cm^3$
$= (1181250 - 1050000)\ cm^3 = 131250\ cm^3$.

(iii) External area = (Area of 4 faces) + (Area of the base) = $([2(125 + 105) \times 90] + (125 \times 105)\}\ cm^2$.

$= (41400 + 13125)\ cm^2 = 54525\ cm^2$.
Internal area = $([2(120 + 100) \times 87.5] + (120 \times 100)\}\ cm^2 = (38500 + 12000)\ cm^2 = 50500\ cm^2$.
Area at the top = Area between outer and inner rectangles = $[(125 \times 105) - (120 \times 100)]\ cm^2$
$= (13125 - 12000)\ cm^2 = 1125\ cm^2$.
Total surface area = $(54525 + 50500 + 1125)\ cm^2 = 106150\ cm^2$.

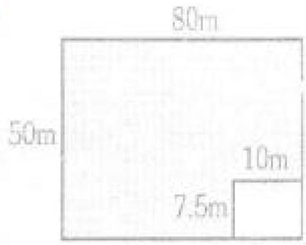
Ex.3. A field is 80 m long and 50 m broad. In one corner of the field, a pit which is 10 m long, 7.5 m broad and 8 m deep has been dug out. The earth taken out of it is evenly spread over the remaining part of the field. Find the rise in the level of the field.

Sol. Area of the field = $(80 \times 50)\ m^2 = 4000\ m^2$

Area of the pit = $(10 \times 7.5)\ m^2 = 15\ m^2$

Area over which the earth is spread out = (4000 – 75) m2 = 3925 m2

Volume of earth dug out = $(10 \times 7.5\ 8)\ m^3 = 600\ m^3$.



$\therefore$ Rise in level = $\left(\frac{volume}{Area}\right) = \left(\frac{600}{3925}\right) m = \left(\frac{600 \times 100}{3925}\right) cm = 15.3 cm$

Ex.4. A room is half as long again as it is broad. The cost of carpeting the room at Rs 18 per m^2 is Rs 972 and the cost of white-washing the four walls at Rs 6 per m^2 is Rs 1080. Find the dimensions of the room.

Sol. Let breadth = (x) in. Then, length $=(\frac{3}{2}\times)$ m.

Let height of the room = y m.

Area of the floor $=\left(\frac{colt\ of\ carpeting}{Rate}\right)=\left(\frac{972}{18}\right)=54\ m^2$

$\therefore \quad x\times\frac{3}{2}x=54\Rightarrow x^2=\left(54\times\frac{2}{3}\right)=36\Rightarrow x=6$.

So, breadth = 6 m and length $=\left(\frac{3}{2}\times 6\right)$ m = 9 m.

Now, area of four walls $=\left(\frac{\cos t\ of\ white-washing}{Rate}\right)=\left(\frac{1080}{6}\right)m^2=180\ m^2$.

$\therefore \quad 2(9+6)\times y=180\Rightarrow 30y=180\Rightarrow y=\left(\frac{180}{30}\right)=6$.

Hence, length = 9 m, breadth = 6 m, height = m.

Ex.5. The water in a rectangular reservoir having a base 80 x m 60 m, is 6.5 m deep. In what time can the water be emptied by a pipe of which the cross section is a square of side 20 cm, if water runs through the pipe at the rate of 15 km/ hr ?

Sol. Volume of water in the reservoir = (80 x 60 x 6.5) m^3 = 31200 m^3.

Area of cross section of the pipe $=\left(\frac{20}{100}\times\frac{20}{100}\right)m^3=\frac{1}{25}m^2$**.**

Volume of water emptied in 1 hr $=\left(\frac{1}{25}\times 15000\right)m^3=600\ m^3$**.**

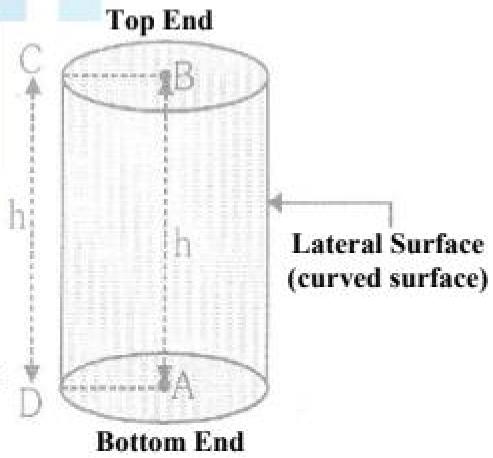
Time taken to empty the reservoir $=\left(\frac{31200}{600}\right)$ **hrs = 52 hrs.**

RIGHT CIRCULAR CYLINDER

Solids like circular pillars, circular pencils, measuring jars, road rollers and gas cylinders, etc., are said to be in cylindrical shape.

In mathematical terms, **a right circular cylinder is a solid generated by the revolution of a rectangle about its sides.**

Let the rectangle ABCD revolve about its side AB, so as to describe a right circular cylinder as shown in the figure.



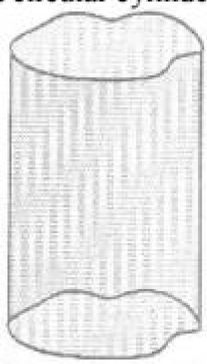
You must have observed that the cross-section of a right circular cylinder are circles congruent and parallel to each other.

Cylinders Not Right Circular

There are two cases when the cylinder is not a right circular cylinder.

Case-I : In the following figure, we see a cylinder, which is certainly, but is not at right angles to the base, So we cannot say it is a right circular cylinder,

Case-II : In the following figure, we see a cylinder, with a non-circular base as the base is not circular. So we cannot say it is a right circular cylinder,



REMARK : Unless stated otherwise, here in this chapter the word cylinder would mean a right circular cylinder.

The following are definitions of some terms related to a right circular cylinder :

(i) The radius of any circular end is called the **radius** of the right circular cylinder.
Thus, in the above figure, AD as well as BC is a radius of the cylinder .

(ii) The line joining the centres of circular ends of the cylinder, is called the axis of the right circular cylinder. In the above figure, the line AB is the axis of the cylinder. Clearly, the axis is perpendicular to the circular ends.

REMARK : If the line joining the centres of circular ends of a cylinder is not perpendicular to the circular ends, then the cylinder is not a right circular cylinder.

(iii) The length of the axis of the cylinder is called the **height or length** of the cylinder.

(iv) The curved surface joining the two bases of a right circular cylinder is called its **lateral surface.**

Formulae

For a right circular cylinder of radius = r units & height = h units, we have :

Area of each circular end = πr^2 sq. units.

Curved (Lateral) Surface Area = $(2\pi rh)$ sq. units.

Total Surface Area = Curved Surface Area + Area of two circular ends.

$= (2\pi rh + 2\pi r^2)$ sq. units.

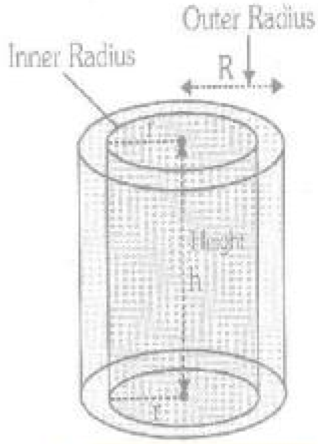
$= [2\pi r (h + r)]$ sq. units.

Volume of cylinder = $\pi r^2 h$ cubic units.

The above formulae are applicable to solid cylinders only.

Hollow Right Circular Cylinders

Solids like iron pipes, rubber, tubes, etc., are in the shape of hollow cylinder.
A solid bounded by two coaxial cylinders of the same height and different radii is called a hollow cylinder



Formulae

For a hollow cylinder of height h and with external and internal radii R and r respectively, we have :

Thickness of cylinder = (R – r) units.
Area of a cross-section = ($\pi R^2 - \pi r^2$) sq. units.
= $\pi(R^2 - r^2)$ sq. units.
Curved (Lateral) Surface Area = (External Curved Surface Area) + (Internal Curved Surface Area)
= ($2\pi Rh + 2\pi rh$) sq. units = $2\pi h$ (R + r) sq. units.
Total Surface Area = (Curved Surface Area) + 2 (Area of Base Ring)
= [($2\pi Rh + 2\pi rh$) + ($\pi R^2 - \pi r^2$)] sq. units
= 2π(Rh + rh +v$R^2 - r^2$) sq. units
Volume of Material = $\pi(R^2 - r^2)$ h cubic units
Volume of Hollow region = $\pi r^2 h$ cubic units

Ex. 6 **2.2 cu dm of brass is to drawn into a cylindrical wire of diameter 0.50 cm. Find the length of the wire.**

Sol. Volume of brass = 2.2 cu dm = (2.2 x 10 x 10 x 10) cm^3 = 2200 cm^3. Let the required length of wire be x cm.

Then, its volume = $(\pi r^2 x) cm^3 = \left(\frac{22}{7} \times 0.25 \times 0.25 \times x\right) cm3$

$\therefore \quad \frac{22}{7} \times 0.25 \times 0.25 \times x = 2200$

$\Rightarrow \quad x = \left(2200 \times \frac{7}{22} \times \frac{1}{0.25 \times 0.25}\right) = 11200\ cm = 112\ m$.

Hence, the length of wire is 112 m.

Ex. 7 **A well 14 m diameter is dug 8 m deep. The earth taken out of it has been evenly spread all around it to a width of 21 m to form an embankment. Find the height of the embankment.**

Sol. Volume of earth dug out from the well = $\pi r^2 h = \left(\frac{22}{7} \times 7 \times 7 \times 8\right) m^3 = 1232\ m^3$.

Area of the embankment $= \pi (R^2 - r^2) = \frac{22}{7} \times \{(28)^2\} m^2 = \left(\frac{22}{7} \times 35 \times 21\right) m^2 = 2310\ m^2$.

Height of the embankment $= \frac{Volume\ of\ earth\ dug\ out}{Area\ of\ embankment} = \left(\frac{1232}{2310} \times 100\right) cm. = 53.3\ cm$.

Ex. 8 The difference between the outside and inside surface of a cylinder metallic pipe 14 cm long is 44 cm^2. If the pipe is made of 99 cu cm of metal, find outer and inner radii of the pipe.

Sol. Let, external radius = R cm and internal radius = r cm.

Then, outside surface = π Rh = $\left(2\times\frac{22}{7}\times R\times 14\right)cm^2 = (88R)\ cm^2$.

Inside surface = 2 π rh = $\left(2\times\frac{22}{7}\times r\times 14\right)cm^2 = (88r)\ cm^2$.

$\therefore \quad (88R-88r)=44 \Rightarrow (R-r)=\frac{44}{88}=\frac{1}{2} \Rightarrow (R-r)=\frac{1}{2}$

Internal volume = π R^2h = $\left(\frac{22}{7}\times R^2\times 14\right)cm^3 = (44R^2)\ cm^3$

$\therefore \quad (44R^2-44r^2)=99 \Rightarrow (R^2-r^2)=\frac{99}{44} \Rightarrow (R^2-r^2)=\frac{9}{4}$

On dividing (ii) by (i), we get: (R + r) = $\left(\frac{9}{4}\times\frac{2}{1}\right) \Rightarrow (R+r)=\frac{9}{2}$

Solving (i) and (ii), we get, R = 2.5 and r = 2.

Hence, outer radius = 2.5 cm and inner radius = 2 cm.

Ex. 9 A solid iron rectangular block of dimensions 4.4 m, 2.6 m and 1 m is cast into a hollow cylindrical pipe of internal radius 30 cm and thickness 5 cm. Find the length of the pipe.

Sol. Volume of iron = (440 x 260 x 100) cm^3.
Internal radius of the pipe = 30 cm.
External radius of the pipe = (30 + 5) cm = 35 cm.
Let the length of the pipe be h cm.
Volume of iron in the pipe = (External volume) – (Internal volume)

$$= [\pi\times(35)^2\times h - \pi\times(30)^2\times h]cm^3 = (\pi h)\ \{(35)^2-(30)^2\}\ cm^3$$
$$= (65\times 5)\pi h\, cm^3 = (325\pi h)\, cm^3.$$

$\therefore \quad 325\pi h = 440\times 260\times 100 \qquad \Rightarrow h = \left(\frac{440\times 260\times 100}{325}\times\frac{7}{22}\right)cm$

$$\Rightarrow h = \left(\frac{11200}{100}\right)m = 112\,m.$$

Hence, the length of the pipe is 112 m.

Ex. 10 A cylindrical pipe has inner diameter of 7 cm and water flows through it at 192.5 litres per minute. Find the rate of flow in kilometers per hour.

Sol. Volume of water that flows per hour = (192.50 x 60) liters = (192.5 x 60 x 1000) cm^3.
Inner radius of the pipe = 3.5 cm.
Let the length of column of water that flows in 1 hour be h cm.

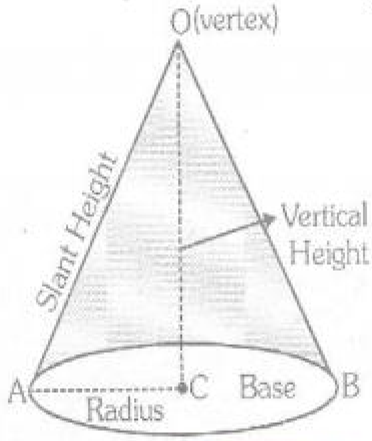
Then, $\frac{22}{7}\times 3.5\times 3.5\times h = 192.5\times 60\times 1000$

$$\Rightarrow h = \left(\frac{192.5\times 60\times 1000\times 7}{3.5\times 3.5\times 22}\right)cm = 300000\,cm = 3\text{ km}$$

Hence, the rate of flow = 3 km per hour.

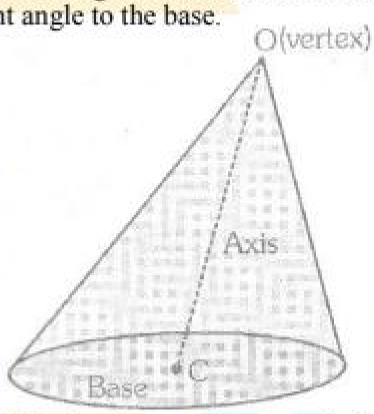
★ **RIGHT CIRCULAR CONE**

Solids like an ice-cream cone, a conical tent, a conical vessel, a clown's cap etc. are said to be in conical shape. In mathematical terms, **a right circular cone is a solid generated by revolving a right-angled triangle about one of the sides containing the right angle.**
Let a triangle AOC revolve about it's OC, so as to describe a right circular cone, as shown in the figure.

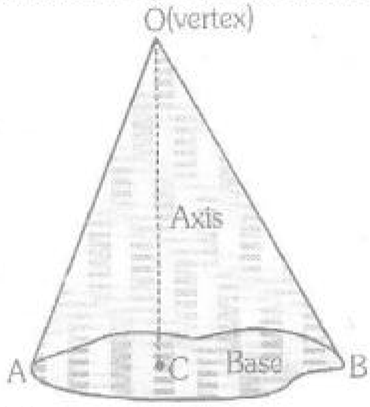


Cones Not Right circular
There are two cases when we cannot call a right circular cone.

Case-I : The figure shown blown below is not a right circular cone because the line joining its vertex to the centre of its base is not at right angle to the base.



Case-II: The figure shown below is not a right circular cone because the base is not circular.



REMARK : Unless stated otherwise, by 'cone' in this chapter, we shall mean 'a right circular cone
The following are definitions of some terms related to right circular cone :
(i) The fixed point O is called the **vertex** of the cone.
(ii) The fixed line OC is called the **axis** of the cone.
(iii) A right circular cone has a plane end, which is in circular shape. This is called the **base** of the cone.

The vertex of a right circular cone is farthest from its base.

(iv) The length of the line segment joining the vertex to the centre of the base is called the **height** of the cone. Length OC is the height of the cone.

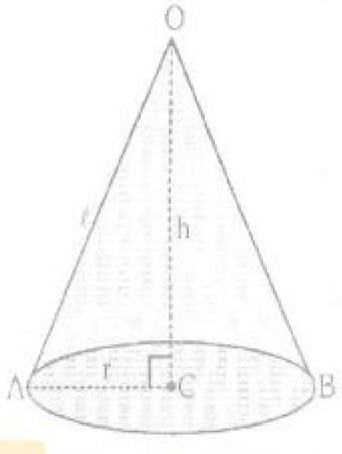
(v) The length of the line segment joining the vertex to any point on the circular edge of the base, is called the **slant height** of the cone.

(vi) The radius AC of the base circle the **radius** of the cone.

Relation Between Slant Height, Radius and Vertical Height.

Let us take a right circular cone with vertex at O, vertical height h, slant height ℓ and radius r. A is any point on the rim of the base of the cone and C is the centre of the base. Here, OC = h, AC = r and OA = ℓ. The cone is right circular and therefore, OC is at right angle to the base of the cone. So, we have OC $\perp$ CA, i.e., ΔOCA is right angled at C.

Then by Pythagoras theorem, we have :



$$\ell^2 = r^2 + h^2$$

Formulae

For a right circular cone of Radius = r, Height = h & Slant Height = ℓ, we have :

Area of the curved (lateral) surface = $(\pi r\ell)$ sq. units. = $\left(\pi r\sqrt{h^2 + r^2}\right)$ sq. units

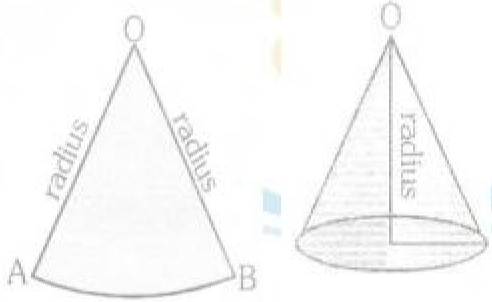
Total Surface Area of cone = (Curved surface Area + Area of Base)

= $(\pi r\ell + \pi r^2)$ sq. units = π r $(\ell + r)$ sq. units.

Volume of cone = $\left(\frac{1}{3}\pi r^2 h\right)$ cubic units.

Hollow Right Circular Cone

Suppose a sector of a circle is folded to make the radii coincide, then we get a hollow right circular cone. In such a cone;



(i) Centre of the circle is vertex of the cone.

(ii) Radius of the circle is slant height of the cone.

(iii) Length of arc AB is the circumference of the base of the cone.

(iv) Area of the sector is the curved surface area of the cone.

Ex.11 The total surface area of a right circular cone of slant height 13 cm us 90π cm^2.

Calculate : (i) its radius in cm, (ii) its volume in cm^3, in terms of π.

Sol. Given : slant height, $\ell = 13$ cm.
Let, radius = r cm and height = h cm.

(i) Total surface area = **π r** $(\pi + \text{r}) = [\pi r(13+r)]$ **cm²**.

$\therefore \quad \pi r(13+r) = 90\pi \Rightarrow r^2 + 13r - 90 = 0 \Rightarrow (r+18)(r-5) = 0$

$\Rightarrow$ r = 5 [Neglecting r = - 18, as radius cannot be negative]

$\therefore$ Radius of the cone = 5 cm.

(ii) $h = \sqrt{\ell^2 - r^2} = \sqrt{(13)^2 - (5)^2}$

= Volume of the cone $= \frac{1}{3}\pi r^2 h = \left(\frac{1}{3} \times 5 \times 5 \times 12\right) cm^3$

= 100π cm³.

Ex.12 A girl fills a cylindrical bucket 32 cm in height and 18 cm in radius with sand. She empties the bucket on the ground and makes a conical heap of the sand. If the height of the conical heap is 24 cm, find :

(i) its radius, **(ii) its slant height.**

Sol. Height of cylindrical bucket, H = 32 cm.

Radius of cylindrical bucket, R = 18 cm.

Volume of sand = $\pi R^2 H = \left(\frac{22}{7} \times 18 \times 18 \times 32\right)$ cm³.

(i) Height of conical heap, h = 24 cm.

Let the radius of the conical heap be r cm.

Then, volume of conical heap $= \frac{1}{3}\pi r^2 h \left(\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24\right)$ cm³.

Now, Volume of conical heap = Volume of sand

$\Rightarrow \quad \left(\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24\right) = \left(\frac{22}{7} \times 18 \times 18 \times 32\right)$

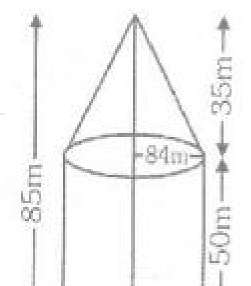
$\Rightarrow \quad r^2 = \left(\frac{18 \times 18 \times 32}{24}\right) = (18 \times 18 \times 4)$

$\Rightarrow \quad r = \sqrt{(18 \times 18 \times 4} = (18 \times 2)$ cm. = 36 cm.

$\therefore$ Radius of the heap = 36 cm.

(ii) Slant height, $\ell = \sqrt{h^2 + r^2} = \sqrt{(24)^3 + (36)^2} = \sqrt{1872} = 12\sqrt{13}$ cm.

Ex.13 An exhibition tent is in the form of a cylinder surmounted by a cone. The height of the tent above the ground is 85 m and the height of the cylindrical part is 50 m. If the diameter of the base is 168 m, find the quantity of canvas required to make the tent. Allow 20% extra for folds and stitching.



Given your answer to the nearest m^2.

Sol. Radius of the tent, $r = \left(\frac{168}{2}\right)$ m = 84 m.

Height of the tent = 85 m.
Height of the cylindrical part, H = 50 m.
Height of the conical part, h = (85 – 50) m = 35 m.

Slant height of the conical part, $\ell = \sqrt{h^2 + r^2} = \sqrt{(35)^2 + (84)^2} = \sqrt{8281}$ m = 91m.

Quantity of canvas required = Curved surface area of the tent
= Curved surface area of the cylindrical part
+ Curved surface area of the conical part
$= 2\pi rH + \pi r\ell = \pi r(2H + \ell)$

$= \left[\frac{22}{7} \times 84(2 \times 50 + 91)\right] m^2 = (22 \times 12 \times 191)\ m^2 = 50424$ m^2.

Area of canvas required for folds and stitching = (20% of 50424) m^2 = $\left(\frac{20}{100} \times 50424\right)$ m^2 = 10084.80 m^2.

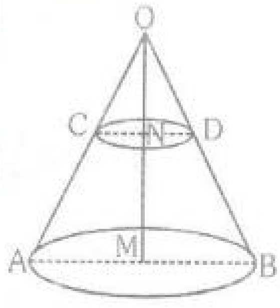
∴ Total quantity of canvas required to make the tent
= (50424 + 10084.80) m^2 = 60508.80m^2 = 60509 m^2. (to the nearest m^2)

Ex.14 The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to its base. If its volume be $\frac{1}{27}$ of the volume of the given cone, at what height, above the base is the section cut?

Sol. Let OAB be the given cone of height, H 30 cm and base radius R cm. Let this cone be cut by the plane CND to obtain the cone OCD with height h cm and base radius r cm.

Then, ΔOND ~ ΔOMB.

So, $\frac{ND}{MB} = \frac{ON}{OM} \Rightarrow \frac{r}{R} = \frac{h}{30}$ ...(i)



Volume of cone OCD $= \frac{1}{27} \times \frac{1}{3}\pi R^2 \times 30$

$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{1}{27} \times \frac{1}{3}\pi\ R^2 \times 30$

$\Rightarrow \left(\frac{r}{R}\right)^2 = \frac{10}{9h} \Rightarrow \left(\frac{h}{30}\right)^2 = \frac{10}{9h}$ [From (i)]

$\Rightarrow 9h^3 = 9000 \Rightarrow h^3 = 1000 \Rightarrow h = 10.$

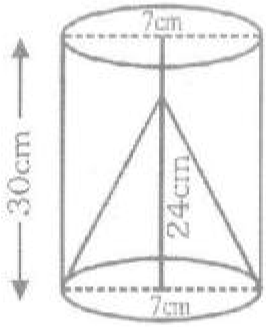
∴ Height of the cone OCD = 10 cm.

Hence, the section is cut at the height of (30 – 10) cm, i.e., 20 cm from the base.

Ex.15 From a solid cylinder of height 30 cm and radius 7 cm, a conical cavity of height 24 cm and of base radius 7 cm is drilled out. Find the volume and the total surface of the remaining solid.

Sol. Radius, r = 7 cm.
Height of the cylinder, H = 30 cm.
Height of the cone, h = 24 cm.
Slant height of the cone, $\ell = \sqrt{h^2 + r^2} = \sqrt{(24)^2 + (7)^2} = \sqrt{625} = 25\,\text{cm}$

(i) Volume of the remaining solid



= (Volume of the cylinder) – (Volume of the cone)

$= \pi r^2 h - \frac{1}{3}\pi r^2 h = \pi r^2\left(H - \frac{h}{3}\right)$

$= \left[\frac{22}{7} \times 7 \times 7 \times \left(30 - \frac{24}{3}\right)\right] cm^3 = \left[\frac{22}{7} \times 7 \times 7 \times 22\right] \text{cm}^3$

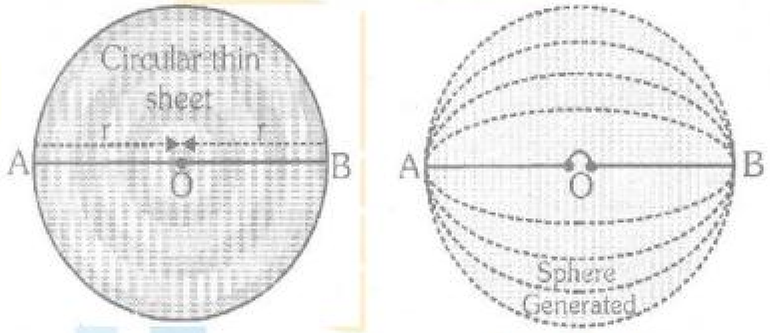
$= (22 \times 7 \times 22)\, cm^3 = 3388\, cm^3$.

(ii) Total surface area of the remaining solid
= Curved surface area of cylinder + Curved surface area of cone
+ Area of (upper) circular base of cylinder

$= 2\pi rH + \pi r\ell + \pi r^2 = \pi r(2H + \ell + r) = \left[\frac{22}{7} \times 7 \times (60 + 25 + 7\right] cm^2 = (22 \times 92)\, cm^2 = 2024\, cm^2$.

★ **SPHERE**

Objects like football, volleyball, throw-ball etc. are said to have the shape of a sphere.
In mathematical terms, **a sphere is a solid generated by revolving a circle about any of its diameters.**
Let a thin circular disc of card of card board with centre O and radius r revolve about its diameter AOB to describe a sphere as shown in figure.



Here, O is called the **centre of the sphere** and r is **radius of the sphere.** Also, the line segment AB is a **diameter of the sphere.**

Formulae

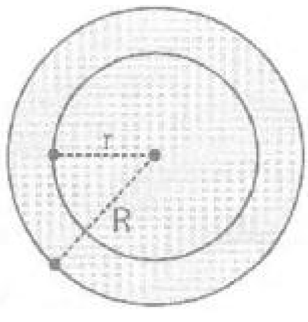
For a solid sphere of radius = r, we have :

Surface area of the sphere = ($4\pi r^2$) sq. units.

Volume of the sphere = $\left(\frac{4}{3}\pi r^3\right)$ cubic units.

SPHERICAL SHELL

The solid enclosed between two concentric spheres to called a spherical shell.

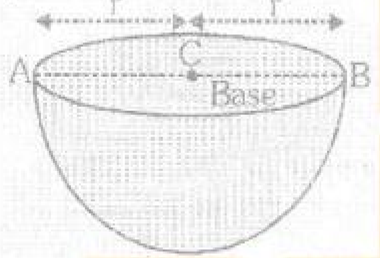


Formula
For a spherical shell with external radius = R and internal radius = r, we have :

Thickness of shell = (R – r) units.
Outer surface area = $4\pi R^2$ sq. units.
Inner surface area =$4\pi r^2$ sq. units.
Volume of material = $4/3\ \pi (R^3 - r^3)$sq. units.

HEMISPHERE

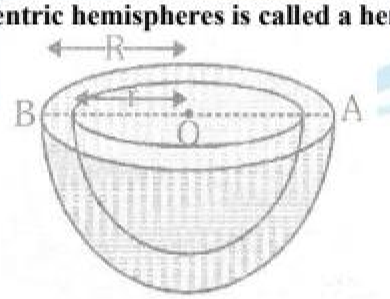
When a plane through the centre of a sphere cuts it into two equal parts, then each part is called a hemisphere.
Form a solid sphere, the obtained hemisphere is also a solid and it has a base as shown in fig.



Formula
For a hemisphere of radius r, we have :
Curved surface area = $2\pi r^2$ sq. units.
Total Surface area = $(2\pi r^2 + \pi r^2) = 3\pi r^2$ sq. units.
Volume $= \frac{2}{3}\pi r^3$ cubic units.

HEMISPHERICAL SHELL

The solid enclosed between two concentric hemispheres is called a hemispherical shell.



Formulae
For a hemispherical shell of external radius = R and internal radius = r, we have :
Thickness of the shell = (R –r) units.
Outer curved surface area = $(2\pi R^2)$ sq. units.

Inner curved surface area = $(2\pi r^2)$ sq. units.
Total surface area = $2\pi R^2 + 2\pi r^2 + \pi(R^3 - r^3) = \pi(3R^2 + r^2)$ sq. units.

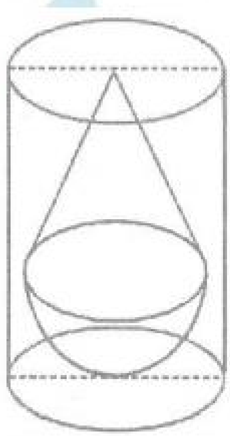
Ex.16 A solid consisting of a right circular cone, standing on a hemisphere, is placed upright, in a right circular cylinder, full of water, and touches the bottom. Find the volume of water left in the cylinder., having given that the radius of the cylinder is 3 cm and its height is 6 cm: the radius of the hemisphere is 2 cm and the height of the cone is 4 cm. Give your answer to the nearest cm^3 (Take π = 22/7)

Sol. Radius of the cylinder = 3 cm and its height = 6 cm.

Volume of water in the cylinder, when full $= \left[\pi \times (3)^2 \times 6\right] cm^3 = (54\pi)\ cm^3$.

Volume of solid consisting of cone hemisphere = (Volume of hemi-sphere) + (Volume of cone)

$$= \left[\frac{2}{3}\pi \times (2)^3 + \frac{1}{3}\pi \times (2)^2 \times 4\right] cm^3 = \left(\frac{32\pi}{3}\right) cm^3.$$



Volume of water displaced from cylinder

= Volume of solid consisting of cone and hemisphere

$$= \left(\frac{32\pi}{3}\right) cm^3$$

Volume of water left in the cylinder after placing the solid into it

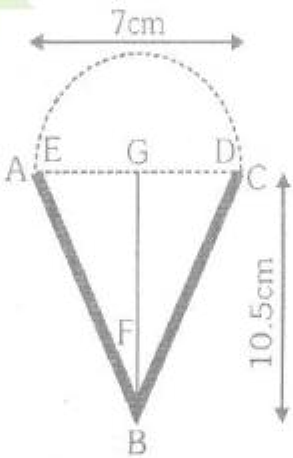
$$\left(54\pi - \frac{32}{3}\right) cm^3 = \left(\frac{130\pi}{3}\right) cm^3 = \left(\frac{130}{3} \times \frac{22}{7}\right) cm^3 = 136.19\, cm^3$$

Hence, the volume of water left in the cylinder to the nearest cm^3 is 136 cm^3.

Ex.17 The given figure shows the cross-section of an ice-cream cone consisting of a cone surmounted by a hemisphere. The radius of the hemisphere is 3.5 cm and the height of the cone is 10.5 cm. The outer shell ABCDEF is shaded and is not filled with ice-cream. AE = DC = 0.5 cm, AB ‖ EF and BC ‖ FD. Caclculate:

(i) the volume of the ice-cream in the cone (the unshaded poration including the hemisphere) in cm^3;

(ii) the volume of the outer shell (the shaded portion) in cm^3. Give your answer to the nearest cm^3.



Sol. Radius of hemisphere, R= AG = 3.5 cm.
External radius of conical shell, R = AG = 3.5 cm.
Internal radius of conical shell, r = EG = (AG – AE) = (3.5 – 0.5) cm = 3 cm
Now, ΔΔ BG ~ ΔEFG.

$$\therefore \quad \frac{FG}{BG} = \frac{EG}{AG} \Rightarrow \frac{FG}{1.05} = \frac{3}{3.5} \Rightarrow FG = 9\text{cm}.$$

So, internal height of conical shell, h = FG = 9 cm.

(i) Volume of ice-cream
= Volume of hemisphere + Internal volume of conical shell

$$= \frac{2}{3}\pi R^3 + \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(2R^3 + r^2 h)$$

= $\left[\frac{1}{3}\times\frac{22}{7}\times\{2\times(3.5)^3+(3)^2\times 9\}\right]cm^3 = \left[\frac{1}{3}\times\frac{22}{7}\times\left(\frac{343}{4}+81\right)\right]cm^3$

= $\left(\frac{1}{3}\times\frac{22}{7}\times\frac{667}{4}\right)cm^3 = \left(\frac{7337}{42}\right)cm^3 = 174.69cm^3 = cm^3$. (to the nearest cm^3)

(i) Volume of the shell = External volume – Internal volume

= $\frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(R^2H - r^2h)$

= $\frac{1}{3}\pi\left[(3.5)^2\times 10.5-(3)^2\times 9\right]cm^3 = \frac{1}{3}\pi\left[\left(\frac{7}{2}\right)^2\times\left(\frac{21}{2}\right)-(9\times 9)\right]cm^3$

= $\left[\frac{1}{3}\times\frac{22}{7}\times\left(\frac{1029}{8}-81\right)\right]cm^3 = \left(\frac{1}{3}\times\frac{22}{7}\times\frac{381}{8}\right)cm^3\left(\frac{1397}{28}\right)cm^3 = 49.89cm^3$

= $50\ cm^3$ (to the nearest cm^3)

Ex.18 A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The height and radius of the cylindrical part are 13 cm and 5 cm respectively. The radii of the hemispherical and conical part are the same as that of the cylindrical part. Calculate the surface area of the height of the conical part is 12 cm.

Sol. The toy is in the shape shown below :

Radius of the hemispherical part = 5 cm,

∴ Curved surface area of the Hemispherical part

$2\pi r^2 = [2\pi\times(5)^2\ cm^2 = (50\pi)\ cm^2$.

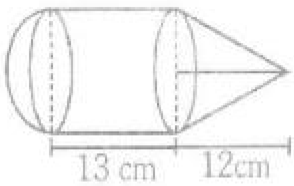
Cylindrical part has radius = 5 cm and height = 13 cm.

∴ Curved surface area of the cylindrical part = $\pi rh = (2\pi\times 5\times 13)cm^2 = (130\pi)cm^2$.

Conical part has radius = 5 cm and height = 12 cm.

∴ Its slant height $\sqrt{5^2+(12)^2} = \sqrt{25+144} = \sqrt{169} = 13cm.$

∴ Curved surface area of the conical part = $\pi r\ell$



$= (\pi\times 5\times 13)cm^2 = (65\pi)cm^2$

Hence, the surface area of the toy = $(50\pi + 130\pi + 65\pi)\ cm^2 = (245\pi)\ cm^2$.

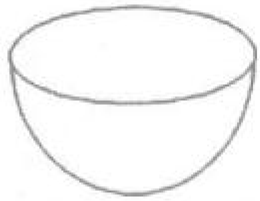
$=\left(245\times\frac{22}{7}\right)cm^2 = 770cm^2$.

Also, volume of the toy $=\left(\frac{2}{3}\pi r^3+\pi r^2h+\frac{1}{3}\pi r^2H\right)cm^3 = \left(\frac{250\pi}{3}+325\pi+100\pi\right)cm^3 = \left(\frac{1525}{3}\right)cm^3$

Ex.19 The outer and inner diameters of a hemispherical bowl are 17 cm and 15 cm respectively. Find cost of polishing it all over at 25 paise per cm^2. (Take π = 22/7).

Sol. Outer radius $=\frac{17}{2}$ cm, Inner radius $=\frac{15}{2}$ cm.

Area of outer surface = 2 $\pi R^2 = \left[2\pi \times \left(\frac{17}{2}\right)^2\right] cm^2 = \left(\frac{289\pi}{2}\right) cm^2$.



Area of inner surface = 2 $\pi r^2 = \left[2\pi \times \left(\frac{15}{2}\right)^2\right] cm^2 = \left(\frac{225\pi}{2}\right) cm^2$.

Area of the ring at the top = $\pi (R^2 - r^2) = \pi [(8.5)^2 - (7.5)^2]$ cm^2 = (16π) cm^2.

∴ Total area to be polished $= \left(\frac{289\pi}{2} + \frac{225\pi}{2} + 16\pi\right) cm^2$.

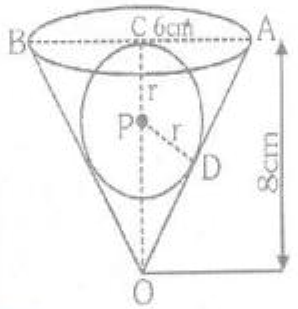
$= (273\pi) cm^2 = \left(273 \times \frac{22}{7}\right) cm^2 = 858 cm^2$.

∴ Cost of polishing the bowl = Rs $\left(\frac{858 \times 25}{100}\right)$ = Rs. 214. 50.

Ex.20 A conical vessel of radius 6 cm and height 8 cm is completely filled with water. A sphere is lowered into the water and its size is such that when it touches the sides, it is just immersed. What fraction of water overflows?

Sol. Radius of the conical vessel, R = AC = 6 cm.
Height of the conical vessel, H = OC = 8 cm.
Let the radius of the sphere be r.
Then, PC = PD = 6 cm.
[∵ lengths of two tangents from an external point to a circle are equal]

OA = $\sqrt{OC^2 + AC^2} = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$ cm.



OD = (OA – AD) = (10 – 6)
OP = (OC – PC) = (8 – R).
In right angled ΔODP, we have :
$OP^2 = OD^2 + PD^2$

⇒ $(8 - R)^2 = 4^2 + r^2 \Rightarrow 64 - 16r + r^2 = 16 + r^2$

⇒ $16r = 48 \Rightarrow r = \frac{48}{16} = 3$.

Volume of water overflown = volume of sphere $= \frac{4}{3}\pi r^3 = \left[\frac{4}{3}\pi \times (3)^3\right] cm^3 = (36\pi) cm^3$.

Volume of water in the cone before immersing the sphere

= Volume of cone $= \frac{1}{3}\pi r^2 h = \left(\frac{1}{3}\pi \times (6)^2 \times 8\right) cm^3 = (96\pi) cm^3$.

∴ Fraction of water overflown $= \frac{\textit{Volume of water overflown}}{\textit{Original volume of water}} = \frac{(36\pi)}{96\pi} = \frac{3}{8}$

★ **FRUSTUM**

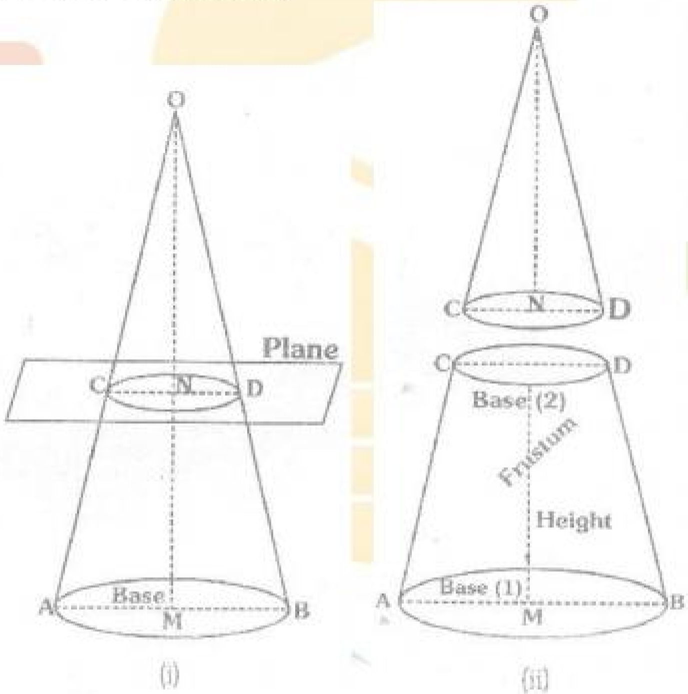
FRUSTUM OF A RIGHT CIRCULAR CONE

In our day-to-day life we come across a number of solids of the shape as shown in the figure. For example, a bucket or a glass tumbler. We observe that this type of solid is a part of a right circular cone and is obtained when the cone is cut by, a plane parallel to the base of the cone.

If right circular cone is cut off by a plane parallel to its base, the portion of the cone between the plane and the base of the cone is called a frustum of the cone.

We can see this process from the figures given below:

The lower portion in figure is the frustum of the cone. It has two parallel flat circular bases, mark as Base (1) and Base (2). A curved surface joins the two bases.



The line segment MN joining the centres of the two bases is called the height of the frustum. Diameter CD of Base (2) is parallel to diameter AB of base (1). Each of the line segments AC and BD is called the slant height of the frustum. We observe from the figures (i) and (ii) that,

1. **Height of the frustum = (the height of the cone OAB) – (the height of the cone OCD)**
2. **Slant height of the frustum = (the height of the cone OAB) – (the height of the cone)**

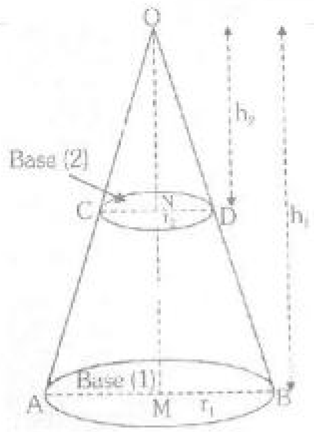
⏩ **Volume of a Frustum of a Right Circular Cone**

Let h be the height ; r_1 and r_2 be the radii of the two bases ($r_1 > r_2$) of frustrum of a right circular cone.

The frustum is made from the complete cone OAB by cutting off the conical part OCD. Let h_1 be the height of the cone OAB and h_2 be the height of the cone OCD.

Here, $h_2 = h_1 - h$.

Since right angled triangles OND and OMB are similar, therefore, we have:



$$\frac{h_2}{h_1} = \frac{r_2}{r_1}$$

$$\Rightarrow \quad \frac{h_1 - h}{h_1} = \frac{r_2}{r_1} \qquad \Rightarrow \quad 1 - \frac{h}{h_1} = \frac{r_2}{r_1}$$

$$\Rightarrow \quad \frac{h}{h_1} = 1 - \frac{r_2}{r_1} = \frac{r_1 - r_2}{r_1} \qquad \Rightarrow \quad h_1 = \frac{hr_1}{r_1 - r_2}$$

and $\quad h_2 = h_1 - h = \frac{hr_1}{r_1 - r_2} - h \Rightarrow \quad h_2 = \frac{hr_2}{r_1 - r_2}$

Volume V of the frustum of cone = Volume of the cone OAB – volume of the cone OCD

$$= \frac{1}{3}\pi r_1^2 h_1 - \frac{1}{3}\pi r_2^2 h_2 = \frac{1}{3}\pi r_1^2 \times \frac{hr_1}{(r_1 - r_2)} - \frac{1}{3}\pi r_2^2 \times \frac{hr_2}{(r_1 - r_2)}$$

$$= \frac{1}{3}\pi h \left\{\frac{r_1^3 - r_1^3}{r_1 - r_2}\right\} = \frac{1}{3}\pi h \{r_1^2 + r_1 r_2 + r_2^2\}$$

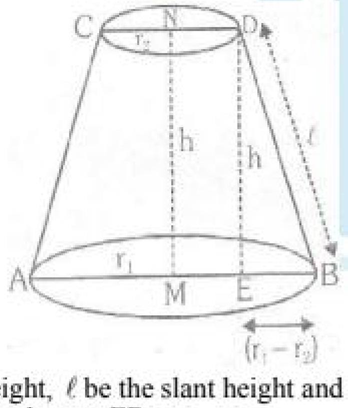
$$\therefore \quad V = \frac{1}{3}\pi h \{r_1^2 + r_1 r_2 + r_2^2\}$$

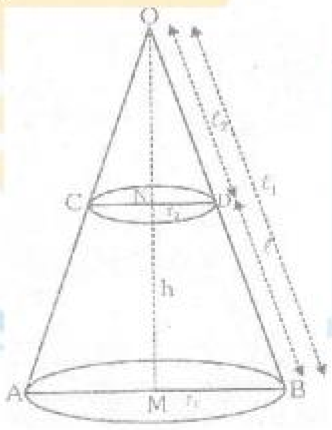
Note : Volume $V = \frac{1}{3}\pi h \{r_1^2 + r_1 r_2 + r_2^2\}$

$$= \frac{h}{3}(\pi r_1^2 + \pi r_2^2 + \pi r_1 r_2) = \frac{h}{3}\{\pi r_1^2 + \pi r_2^2 + \sqrt{(\pi r_1^2)(\pi r_2^2)}\}$$

$$= \frac{h}{3}\{(area\ of\ base) + (area\ of\ base\ 2) + \sqrt{(area\ of\ base)(area\ of\ base\ 2)}\}$$

⏩ Curbed Surface Area of a Frustum of a Right Circular Cone





Let h be the height, ℓ be the slant height and r_1, r_2 be the radii of the bases where $r_1 > r_2$.

In figure (i), we observe $EB = r_1 - r_2$

Aad $\quad \ell^2 = h^2 + (r_1 - r_2)^2$

$\therefore \quad \ell = \sqrt{h^2 + (r_1 - r_2)^2}$

In figure (ii, we have OAB as the complete cone from which cone OCD is cut off to make the frustum ABDC.

Let ℓ be the slant height of the cone OAB and ℓ_2 be the slant height of the cone OCD.

Since, ΔOMB are similar,

$$\frac{\ell_2}{\ell_1}=\frac{r_2}{r_1} \quad \Rightarrow \quad \frac{\ell_1-\ell}{\ell_1}=\frac{r_2}{r_1} \quad \Rightarrow \quad \ell_1=\frac{\ell r_1}{r_1-r_2}$$

Now, $\ell_2=\ell_1-\ell=\frac{\ell r_1}{r_1-r_2}-\ell \quad \Rightarrow \quad \ell_2=\frac{\ell r_2}{r_1-r_2}$

Curved surface area of frustum ABCD

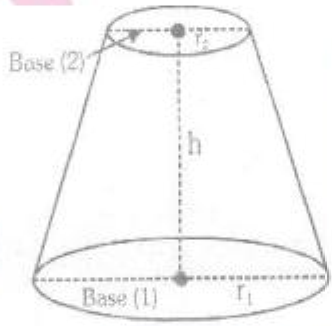
= (Curved surface area of cone OAB) – (Curved surface area of cone OCD)

$$= \pi r_1\ell_1-\pi r_2\ \ell_2=\pi r_1\times\frac{\ell r_1}{(r_1-r_2)}-\pi r_2\times\frac{\ell r_2}{(r-r_2)}=\pi\ell\left\{\frac{r_1^2-r_2^2}{r_1-r_2}\right\}$$

Therefore, curved surface area of frustum = $\pi\ell(r_1+r_2)$.

Total surface Area of a Frustum of a solid Right Circular Cone

Let h be the height, ℓ be the slant height and r_1, r_2 the radii of the bases where $r_1 > r_2$ as shown in figure.



Total surface area of this frustum

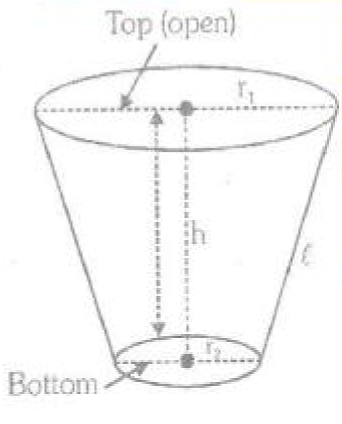
= Curved surface area + Area of Base 1 + Area of Base 2

$= \pi\ \ell(r_1+r_2)+\pi\ r_2^2$

Area of the Metal Sheet Used To Make a Bucket

A bucket is in the shape of a frustum of a right circular hollow cone.

Let h be the depth, ℓ be the slant height, r_1 be the radius of the top and r_2 be the radius of the bottom as shown in figure



The area of the metal sheet used for making the bucket

= Outer (or inner) curved surface area + Area of bottom

= $\pi\ \ell(r_1+r_2)+\pi r_2^2$

Ex.21 A bucket of height 16 cm and made up of metal sheet is in the form of frustum of a right circular cone with radii of its lower and upper ends as 3 cm and 15 cm respectively. Calculate:

(i) the height of the cone of which the bucket is a part.

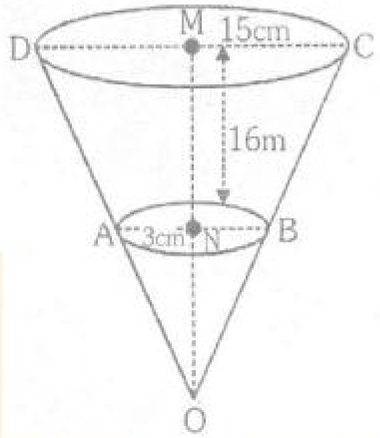
(ii) the volume of water which can be filled in the bucket.

(iii) the slant height of the bucket.

(iv) the area of the metal sheet required to make the bucket.

Sol. Let ABCD be the bucket which is frustum of a cone with vertex 0 (as shown in figure). Let ON = x cm

$\Delta OAB - \Delta OMC$



$\therefore \quad \frac{x}{16+x} = \frac{3}{15} \qquad \left\{\because \frac{ON}{OM} = \frac{NB}{MC}\right\}$

$\Rightarrow \quad \frac{x}{16+x} = \frac{1}{5} \qquad \Rightarrow \quad 5x = 16 + x1$

$\Rightarrow \quad 4x = 16 \qquad \Rightarrow \quad x = 4$

$\therefore$ ON = 4 cm and OM = 4 + 16 = 20 cm

$\therefore$ the height of the cone = 20 cm

volume of the bucket $= \frac{1}{3}\pi(15)^2 \times 20 - \frac{1}{3}\pi(3)^2 \times 4cm^3$

{i.e., Volume of the large cone – Volume of the small cone}

$= \frac{1}{3}\pi[225 \times 20 - 36]cm^3$

$= \pi[75 \times 20 - 12]cm^3$

$= 1488\pi\ cm^3$

Slant height of cone of radius 15 cm

$= \sqrt{(15)^2 + (20)^2}cm = \sqrt{625}\ cm = 25cm$

Slant height of cone of radius 3 cm

$= \sqrt{(4)^2 + (3)^2}cm = 5cm$

$\therefore$ Slant height of bucket $= (25-5)cm = 20cm, i.e, \ell = 20cm$

$\therefore$ The area of the metal sheet $= \pi\,\ell(R+r) + \pi\,r^2$

$= \pi \times 20 \times (15+3) + \pi(3)^2\ cm^2 = 360\pi + 9\pi\ cm^2$

$= 369\pi\ cm^2$

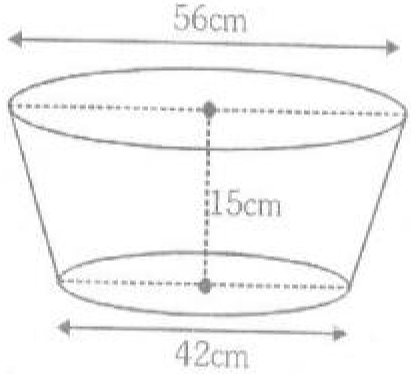
Note. The area of the metal sheet used = C.S. of larger cone – C.S. of smaller cone + Area of the base of the bucket

$= [\pi \times 25 \times 15 - \pi \times 5 \times 3 + \pi \times (3)^2]cm^2 = [375\pi - 15\pi + 9\pi]cm^2$

$= 369\pi\ cm^2$

Ex.22 A bucket is in the form of a frustum of a cone, depth is 15 cm and the diameters of the top and the bottom are 56 cm and 42 cm respectively. Find how many liters of water can the bucket hold ? (Take $\pi = 22/7$)

Sol. R = 28 cm
r = 21 cm
h = 15 cm



Capacity of the bucket $= \frac{1}{3}\pi h\ \{R^2 + r^2 + R\,r\}$

$= \frac{1}{3} \times 22 \times 15 \times \{(28)^2 + (21)^2 + (28)(21)\}\ cm^3$

$= \frac{22}{7} \times 5 \times \{784 + 441 + 588\}\ cm^3$

$= \frac{22}{7} \times 5 \times 1813\ cm^3 \qquad = 22 \times 5 \times 259\ cm^3$

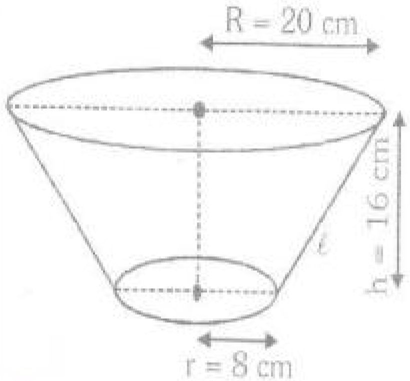
$= 28490\ cm^3 \qquad = \frac{28490}{1000}$ liters

$=$ 28.49 liters

Ex.23 A container made up of a metal sheet is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of the milk which can completely fill the container at the rate of Rs. 15 per liter and the cost of the metal sheet used, if it costs Rs. 5 per 100 cm^2. (Take π 3.14)

Sol. R = 20 cm, r = 8 cm, h = 16 cm

$\ell = \sqrt{h^2 + (R-r)^2} = \sqrt{256 + 144}\ cm = 20\,cm$



Volume of container $= \frac{1}{3}\pi h\ \{R^2 + r^2 + Rr\}$

$= \frac{1}{3} \times (3.14) \times 16\ \{400 + 64 + 160\}\ cm^3$

$= 3.14 \times \frac{16}{3}\ \{624\}\ cm^3$

$= 3.14 \times 16 \times 208\ cm^3$

$= 10449.92\,cm^3$

Therefore, the quantity of milk in the container $= \frac{10449.92}{1000}$ liters = 10.45 liters

Colt of milk at the rate of Rs. 15 per liter = Rs. $\{10.45 \times 15\}$ = Rs. 156.75

Surface area of the metal sheet used to make the container

$= \pi\,\ell(R + r) + \pi\,r^2 = \pi\ \{\ell(R + r) + r^2\}$

$= (3.14) \times \{20 \times 28 + 64\}\,cm^2$

$= (3.14) \times 624\,cm^2 = 1959.36\,cm^2$

Therefore, the cost of the metal sheet at rate of Rs. 5 per 100 cm^2

$$= \quad Rs.\frac{1959.36\times5}{100} = Rs.\,97.97 \text{ approx.}$$

Ex.24 The height of a cone is 40 cm. A small cone is cut off at the top by a plane parallel to the bases. It the volume of the small cone be $\frac{1}{64}$ of the volume of the given cone, at what height above the base is the section made

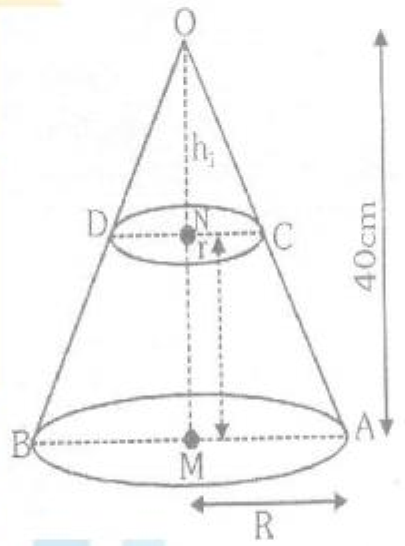
Sol. Let R be the radius of the given cone, r the radius of the small cone, h be the height of the frustum and h_1 be the height of the small cone.

In figure 13.49, ΔONC and ΔOMA are similar (ΔONC - ΔOMA)

$$\therefore \quad \frac{ON}{OM} = \frac{NC}{MA} \quad \Rightarrow \quad \frac{h_1}{40} = \frac{r}{R}$$

$$\Rightarrow \quad h_1 = \left(\frac{r}{R}\right)40 \qquad \ldots(i)$$

We are given that $\frac{Volume\ of\ small\ cone}{Volume\ of\ given\ cone} = \frac{1}{64}$



$$\Rightarrow \quad \frac{\frac{1}{3}\pi r^2 \times h_1}{\frac{1}{3}\pi R^2 \times 40} = \frac{1}{64}$$

$$\Rightarrow \quad \frac{r_2}{R_2}\times\frac{1}{40}\times\left\{\left(\frac{r}{R}\right)40\right\} = \frac{1}{64} \qquad \text{(By 1)}$$

$$\Rightarrow \quad \left(\frac{r}{R}\right)^3 = \frac{1}{64} = \left(\frac{1}{4}\right)^3 \Rightarrow \frac{r}{R} = \frac{1}{4} \qquad \ldots(2)$$

From (i) and (ii) $h_1 = \frac{1}{4}\times 40 = 10\,cm$

Therefore, $\quad h = 40 - h_1 = (40 - 10)$ cm

$\Rightarrow \quad h = 30$ cm

Ex.25 The radius of the base of a right circular cone is r. It is cut by a plane parallel to the base a height h from the base. The slant height of the frustum is $\sqrt{h^2+\frac{4}{9}r^2}$. Show that volume of the frustum is $\frac{13}{27}\pi r^2 h$.

Sol. In figure 13.50, $\ell=\sqrt{h^2+\frac{4}{9}r^2}$ is the slant height of frustum of the given cone having base radius r. O is the centre of the base and O' is the centre of the top of the frustum.

OO' = h (given)

AOB and COD are diameters of the lower and upper faces of the frustum. Draw DE $\perp$ OB. Let O' P = x

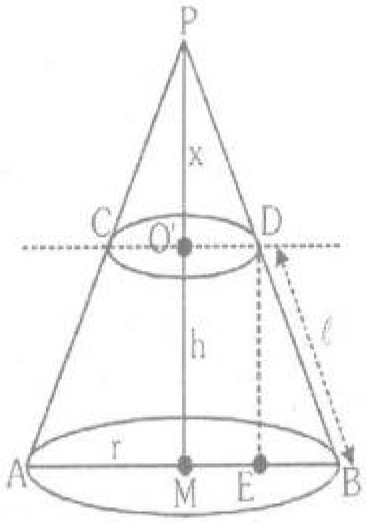
In right angled ΔDEB,

$$DB^2 = BE^2 + DE^2$$

$\Rightarrow \quad \ell^2 = BE + h^2 \quad (\because DE = OO' = h)$

$\Rightarrow \quad h^2+\frac{4}{9}r^2 = h^2 + BE^2 \qquad \Rightarrow \quad BE^2 = \frac{4}{9}r^2$

$\Rightarrow \quad BE = \frac{2}{3}r \qquad \Rightarrow \quad OE = r - \frac{2}{3}r = \frac{1}{3}r$



$\Rightarrow \quad O'D = \frac{1}{3}r$ is the radius of the top face of the frustum.

Now, $\Delta PO'D \sim \Delta POB$

$\Rightarrow \quad \frac{PO'}{O'D} = \frac{PO}{PB} \qquad \Rightarrow \quad \frac{x}{\frac{1}{3}r} = \frac{h+x}{r}$

$\Rightarrow \quad 3x = h + x \qquad \Rightarrow \quad x = \frac{1}{2}h.$

Volume of the frustum = Volume of the cone PAB – Volume of the cone PCD

$$=\frac{1}{3}\pi\times(r^2)\times OP-\frac{1}{3}\pi\times\left(\frac{1}{3}r\right)^2\times O'P$$

$$=\frac{1}{3}\pi r^2\times(h+x)-\frac{1}{27}\pi r^2\times x$$

$$=\frac{1}{3}\pi r^2\times\left(h+\frac{1}{2}h\right)-\frac{1}{27}\pi r^2\times\frac{1}{2}h$$

$$=\left(\frac{1}{2}\pi r^2-\frac{1}{54}\pi r^2\right)h=\frac{26}{54}\pi r^2 h=\frac{13}{27}\pi r^2 h$$

Hence, the required volume is $\frac{13}{27}\pi r^2 h$

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOOSE THE CORRECT OPTION IN EACH OF THE FOLLOWING

1. If the volume of a cube is 1728 cm^3, the length of its edge is equal to
(a) 12 cm (b) 14 cm (c) 16 cm (d) 24 cm

2. Two cubes each of 10 cm edge are joined end to. The surface area of the resulting cuboid is
(a) 1200 cm^2 (b) 1000 cm^2 (c) 800 cm^2 (d) 1400 cm^2

3. A rectangular sheet of paper 44 cm $\times$ 18 cm is rolled along its length and a cylinder is formed. The volume the cylinder so formed is equal to $(Take\, \pi = \frac{22}{7})$
(a) 2772 cm^3 (b) 2505 cm^3 (c) 2460 cm^3 (d) 2672 cm^3

4. If the radius and height of a cylinder are in ratio 5 : 7 and its volume is 550 cm^3, then its radius is equal to $(Take\, \pi = \frac{22}{7})$
(a) 6 cm (b) 7 cm (c) 5 cm (d) 10 cm

5. If the curved surface area of a solid right circular cylinder of height h and radius r is one-third of its total surface area, then
(a) $h = \frac{1}{3}r$ (b) $h = \frac{1}{3}r$ (c) $h = r$ (d) $h = 2r$

6. A hollow cylindrical pipe is 21 cm long. If its outer and inner diameters are 10 cm and 6 cm respectively, then the volume of the metal used in making the pipe is $(Take\, \pi = \frac{22}{7})$
(a) 1048 cm^3 (b) 1056 cm^3 (c) 1060 cm^3 (d) 1064 cm^3

7. If the radius and slant height of a cone are in the ratio 4 : 7 and is curved surface area is 792 cm^2, then its radius is $(Take\, \pi = \frac{22}{7})$
(a) 10 cm (b) 8 cm (c) 12 cm (d) 9 cm

8. If the radius of the base and the height of a right circular cone are respectively 21 cm and 28 cm, then the curved surface area of the cone is $(Take\, \pi = \frac{22}{7})$
(a) 3696 cm^2 (b) 2310 cm^2 (c) 2550 cm^2 (d) 2410 cm^2

9. A conical tent with base-radius 7 m and height 24 m is made from 5 m wide canvas. The length of the canvas used is $(Take\, \pi = \frac{22}{7})$
(a) 100 m (b) 105 m (c) 110 m (d) 115 m

10. The total surface area of a solid hemisphere of radius 3.5 m is covered with canvas at the rate of Rs. 20 per m^2. The total cost to cover the hemisphere is $(Take\, \pi = \frac{22}{7})$
(a) Rs. 2210 (b) Rs. 2310 (c) Rs. 2320 (d) Rs. 2420

11. If the volume of a vessel in the form of a right circular cylinder is $448\pi\ cm^3$ and its height is 7 cm, then the curved surface area of the cylinder is
(a) $224\pi\ cm^2$ (b) $212\pi\ cm^2$ (c) $112\pi\ cm^2$ (d) none of these

12. If the curved surface area of a right circular cone is $12320\ cm^2$ and its base-radius is 56 cm, then its height is $(Take \pi = \frac{22}{7})$
(a) 42 cm (b) 36 cm (c) 48 cm (d) 50 cm

13. If a solid metallic sphere of radius 8 cm is melted and recasted into spherical solid balls of radius 1 cm, then n =
(a) 500 (b) 510 (c) 512 (d) 516

14. If the diameter of a metallic sphere is 6 cm, it melted and a wire of diameter 0.2 cm is drawn, then the length of the wire made shall be
(a) 24 m (b) 28 m (c) 32 m (d) 36 m

15. If n coins each of diameter 1.5 cm and thickness 0.2 cm are melted and a right circular cylinder of height 10 cm and diameter 4.5 cm is made, then n =
(a) 336 (b) 450 (c) 512 (d) 545

16. A tent is in the form of a cylinder of diameter 8 m and height 2 m, surmounted by a cone of equal base and height 3 m. The canvas used for making the tent is equal to
(a) $36\pi\ m^2$ (b) $28\pi\ m^2$ (c) $24\pi\ m^2$ (d) $32\pi\ m^2$

17. A toy is in the form of a cone mounted on a hemisphere with same radius. The diameter of the base of the conical portion is 6 cm and its height is 4 cm. The surface area of the toy is
(a) $36\pi\ cm^2$ (b) $33\pi\ cm^2$ (c) $35\pi\ cm^2$ (d) $24\pi\ cm^2$

18. A frustum of a right circular cone is of height 16 cm with radii of its ends as 8 cm and 20 cm. The volume of the frustum is
(a) $3328\pi\ cm^3$ (b) $3228\pi\ cm^3$ (c) $3240\pi\ cm^3$ (d) $3340\pi\ cm^3$

19. A frustum of a right circular cone is of height 16 cm with radii of its ends as 8 cm and 20 cm has lateral surface area equal to
(a) $540\pi\ cm^2$ (b) $580\pi\ cm^2$ (c) $560\pi\ cm^2$ (d) $680\pi\ cm^2$

20. A solid metal cone with base-radius 12 cm and height 24 cm, is melted to form solid spherical balls, each of diameters 6 cm. The number of such balls made is
(a) 32 (b) 36 (c) 48 (d) none of these

OBJECTIVE				ANSWER					EXERCISE-4
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	A	B	A	A	B	B	C	B	C	B
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	C	A	C	D	B	A	B	A	C	A

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CONVERSION OF SOUPS

1. Two cubes of the side 10 cm are joined end to end. Find the surface area of the resulting rectangular shaped solid

2. Three cubes each of side 4 cm are joined end to end. Find the surface area of the resulting rectangular cuboid.

3. The six cube marked I, II, III, IV, V, VI each of side 3 cm are placed as shown in fig. It takes the shape of a cuboid. Find the surface area of the cuboid.

I	II	II
I	V	V

4. A rectangular solid metallic cuboid 18 cm $\times$ 15 cm $\times$ 4.5 cm is melted and recast into solid cubes each of side 3 cm. How many solid cubes can be made ?

5. A rectangular solid metallic cuboid 32 cm $\times$ 27 cm $\times$ 15 cm is melted and recast into solid cubes each of side 6 cm. How many solid cubes can be made from the metal.

6. Two rectangular solid metallic cuboid 12 cm $\times$ 10 cm $\times$ 5 cm and 12 cm $\times$5 cm $\times$ 4 cm are melted together and recast into solid cubes each of side 2 cm. How many solid cubes can be made from the metal.

7. Three rectangular solid metallic cuboid 20 cm $\times$ 10 cm $\times$ 5 cm, 15 cm $\times$10 cm $\times$ 4 cm and 15 cm $\times$12 cm $\times$ 5 cm are melted together and recast into solid cubes each of side 2 cm. How many solid cubes can be made from the metal.

8. The side of a metallic cube 35 cm. The cube is melted and recast into 1000 equal solid dice. Determine the side of the dice.

9. Two solid metallic cube sides 40 cm and 30 cm are melted together into 160 equal solid cubical dice. Determine the side of the dice.

10. Three solid metallic cubes 60 cm, 50 cm and 30 cm are melted together and recast into 875 equal solid cubical dice. Determine the side of the dice.

11. The diameter of a metallic sphere is 6 cm. The sphere is melted and drawn into a wire of uniform circular cross-section. If the length of the wire is 36 m, find the radius of its cross-section.

12. The diameter of a metallic sphere is 18 cm. The sphere is melted and drawn into a wire having diameter of the cross-section as 0-4 cm. Find the length of the wire.

13. How many balls, each of radius 0.5 cm, can be made from a solid sphere of metal of radius 10 cm by melting the sphere ?

14. A spherical ball of lead 5 cm in diameter is melted and recast into three spherical balls. The diameters of two of these balls are 2 cm and $2\ (1405)^{1/3}$ cm. Find the diameter of the third ball.

15. How many bullets, can be made out of a solid cube of lead whose edge measures 44 cm cm and diameter of each bullet being 4 cm.

16. How many spherical lead shots each 4.2 cm in diameter can be obtained from a rectangular solid (cuboid)of lead with dimensions 66 cm, 42 cm, 21 cm.(Take $\pi = 22/7$)

17. How many spherical balls each of 5 cm in diameter can be cast from a rectangular block of metal 11$\times$ dm $\times$10 dm $\times$5 dm ? (1 dm = 10 cm)

18. A copper rod of diameter 1 cm and length 8 cm is drawn into a wire of length 32 m of uniform thickness (diameter). Find the thickness of the wire.

19. 56 circular plates, each of radius 5 cm and thickness 0.25 cm, are placed one above another to form a solid right circular cylinder. Find the curved surface and the volume of the cylinder so formed.

20. The diameter of a metallic sphere is 4.2 cm. It is melted and recast into a right circular cone of height 8.4 cm, Find the radius of the base of the cone.

21. A right circular metallic cone of height 20 cm and radius of base 5 cm is melted and recast into a sphere. Find the radius of the sphere.

22. A right circular cone of height 81 cm and radius of base 16 cm is melted and recast into a right circular cylinder of height 48 cm. Find the radius of the base of the cylinder.

23. A spherical shell of lead, whose external diameter is 24 cm, is melted and recast into a right circular cylinder, whose height is 12 cm and diameter 16 cm. Determine the internal diameter of the shell.

24. The internal and external radii of a metallic spherical shell are 4 cm and 8 cm, respectively. It is melted and recast into a solid right circular cylinder of height $9\frac{1}{3}$ cm. Find the diameter of the base of the cylinder.

25. A right circular cone is of height 3.6 cm and radius of its base 1.6 cm. It is melted and recast into a right circular cone with radius of its base 1.2 cm. Find the height of the cone so formed.

26. A solid metallic right circular cylinder 1.8 m high with diameter of its base 2 m is melted and recast into a right circular cone with base of diameter 3 m. Find the height of the cone.

27. Find the umber of coins, 1.5 cm in diameter and 0.2 cm thickness, to be melted to form a right circular cylinder o f height 10 cm and diameter 4.5 cm.

28. A conical vessel whose internal radius is 5 cm and height 24 cm is full of water. The water is emptied into a cylinder vessel with internal radius 10 cm. Find the height to which the water rises.

29. A well, whose diameter is 4 m, has been dug 16 m deep and the earth dug out is used to form an embankment 8 m wide around it. Find the height of the embankment.

30. A well, whose diameter is 3.5 m, has been dug 16 m deep and the earth dug out is used to form a platform 27.5 m by 7 m just near the site of the well. Find the height of the platform. (Take $\pi = 22/7$)

31. The base radius and height of a right circular solid cone are 12 cm and 24 cm respectively. It is melted and recast into spheres of diameter 6 cm each. Find the number of spheres so formed.

32. The internal and external diameter of a hollow hemispherical shell are 6 cm and 10 cm respectively. It is melted and recast into a solid cone of base diameter 14 cm. Find the number of spheres so formed.

33. A solid metallic sphere of diameter 28 cm is melted and recanted into a number of smaller cones, each of diameter $4\frac{2}{3}$ cm and height 3 cm. Find the number of cones so formed.

34. A conical flask is full of water. The flask has base-radius 3 cm and height 15 cm. The water is poured into a cylindrical glass tube of uniform inner radius 1.5 cm, placed vertically and closed at the lower end. Find the height of water in the glass tube.

35. Find the depth of a cylindrical tank of radius 10.5 m, if its capacity is equal to that of a rectangular tank of size 15 m $\times$ 11 m $\times$ m $\times$ 10.5 m. (Take $\pi = 22/7$)

36. A rectangular tank 28 m ling and 22 m wide is required to receive entire water from a full cylindrical tank of internal diameter 28 m and depth 4 m. Find the least height of the tank that will serve the purpose(Take $\pi = 22/7$)

37. A conical flask is full of water. The flask has base-radius a and height 2 a. The water is poured into a cylindrical flask of base-radius $\frac{2a}{3}$ Find the height of water in the cylindrical flask.

38. A sphere of diameter 2 a is dropped into a cylindrical vessel partly filled with water. The diameter of the base of the vessel is $\frac{8a}{3}$ If the sphere is completely submerged, by how much will the level of water rise ?

39. The rain water from a roof 44 m $\times$ 20 m drains into a cylindrical vessel having diameter 2 m and height 2.8 m. If the vessel is just full, find the rainfall in cm.

40. An agricultural field is in the form of a rectangle of length 20 m and width 14 m. A 10 m deep well of diameter 7 m is dug in a corner of the field and the earth taken out of the well is spread evenly over the remaining part of the field. Find the rise in its level. (Take $\pi = 22/7$)

41. 600 persons took dip in a rectangular tank which is 60 m long and 40 m broad. What is the rise in the level of water in the tank, if the average displacement of water by a person is 0.04 m^3 ?

42. The largest sphere is curved cut of a cube whose edge is of length ℓ units. Find the volume of the sphere

43. The largest right circular cone is curved out of a cube whose edge is o length p units. Find the volume of the cone.

44. Two solid right circular cones have same height. The radii of their bases are 4 cm and 3 cm. They are melted and recast into a right circular cylinder of same height. Find the radius of the base of the cylinder. $\left(Take \frac{1}{\sqrt{3}} = .577\right)$

45. Water is being pumped out through a circular pipe whose diameter is p cm. If the flow of water is 14 p cm per second, how many litres of water are being pumped cut in one hour? $(Take\ \pi = 22/7)$

46. Water flow out through a circular pipe, whose internal diameter is $1\frac{1}{3}$ cm, at the rate of 0.63 m per second into a cylinder tank, the radius of whose base is 0.2 m. By how much will the level of water rise in one hour.

47. Water in a canal 4 m wide and 1.5 m deep is flowing with velocity 12 km per hour. How much area will it irrigate in 30 minutes, if 9 cm of standing water is required for irrigation?

48. Water flow at the rate of 15 m per minute through a cylindrical pipe having its diameter 1.2 cm. How much time will it tank to fill a conical vessel whose diameter of base is 40 cm and depth 81 cm ?

49. A hemispherical tank full of water is emptied by a pipe at the rate of $3\frac{4}{7}$ litres per second. How much time will it take to half-empty the tank, if the tank is 3 metres in diameter $(Take\ \pi = 22/7)$

50. A conical tank is full of water. Its base-radius is 1.75 m and height 2.25 m. It is connected with a pipe which empties it at the rate of 7 litres per second. How much time will it take to empty the tank completely ? $(Take\ \pi = 22/7)$

51. Two solid metallic right circular cones have same height h. The radii of their bases are r_1 and r_2. The two cones ae melted together and recast into a right circular cylinder of height h. Show that radius of the base of the cylinder is $\sqrt{\frac{1}{3}(r_1^2 + r_2^2)}$

52. The radii of the bases of two right circular solid metallic cones of same height h are r_1 and r_2. The cones are melted together and recast into a solid sphere of radius R. Show that $h = 4\left(\frac{R^3}{r_1^2 + r_2^2}\right)$

53. The radii of the solid metallic spheres are r_1 and r_2. The sphere are melted together and recast in a solid cone of height $(r_1 + r_2)$. Show that the radius of the cone is $2 \times \sqrt{r_1^2 + r_2^2 + r_1 r_2}$

54. The radii of a solid metallic sphere is r. A solid metallic cone of height h has base radius r. The two are melted together and recast into a solid right circular cone with base radius r. Prove that the height of the resulting cone is 4 r + h.

55. A solid metallic right circular cylinder and a solid metallic right circular cone are giver. The cylinder and cone both have same height h and same base radii r. The two solids are melted together and recast into a solid cylinder of radius $\frac{1}{2}$ r. Prove that the height of the cylinder is $\frac{16}{3}$ h.

SURFACE AREAS & VOLUMES OF COMBINATIONS OF SOLIDS

1. A solid is in the form of a cone mounted on a right circular cylinder both having same radii of their bases. Base of the cone is placed on the top base of the cylinder. If the radius of the base and height of the cone be 4 cm and 7 cm respectively and the height of the cylindrical part of the solid is 3.5 cm, find the volume of the solid.

2. A solid is in the form of a right circular mounted on a solid hemisphere of radius 14 cm. The radius of the base of the cylindrical part is 14 cm and the vertical height of the complete solid is 28 cm. Find :

(i) The volume of the solid

(ii) The surface area of the solid

(iii) Cost of painting the solid at the rate of Rs. 0.80 cm^2.

3. A solid is in the form of a cone of vertical height 9 cm mounted on the top base of a right circular cylinder of height 40 cm. The radius of the base of the cone and that of the cylinder are both equal to 7 cm. Find the weight of the solid if 1 cm^3 of the solid weight 4 gm.

4. A solid is in the form of a right circular cone mounted on a solid hemisphere with same radius is made from a piece of metal. The radius of the hemisphere is $\frac{1}{3}$ of the vertical height of the conical part. If the radius of the base of the cone is r, prove that the volume of the piece of metal is $\frac{5}{3}\pi r^3$

5. A solid wooden toy is in the shape of a right circular cone mounted on a solid hemisphere with same radius. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the volume of the wooden toy. $(Take\ \pi = 22/7)$

6. A solid toy is in the form of a hemisphere surmounted by a right circular cone. Height of the cone is 2 cm and diameter of the base is 4 cm. If a right circular cylinder circumscribes the solid, find how much more space it will cover. $(Take\ \pi = 3.14)$

7. A cylindrical tub of radius 5 cm and height 9.8 cm is full of water. A solid in the form of a right circular cone mounted on a hemisphere is immersed completely into the tub. If the radius of the hemisphere is 3.5 cm and the height of the conical part is 5 cm, find the volume of water left in the tub. $(Take\ \pi = 22/7)$

8. A cylindrical container of radius 6 cm and height 15 cm is filled with ice-cream. The ice-cream is to be distributed to 10 children in equal cones with hemisphere tops. If the height the height of the conical portion is 4 times the radius of its base, find the radius of the ice-cream cone.

9. A right circular cylinder having diameter 18 cm and height 20 cm is full of ice-cream. The ice-cream is to be filled in cones of height 12 cm and diameter 6 cm having hemispherical shape on the top. Find the number of such cones which can be filled with ice-cream.

10. A circus tent has cylindrical shape surmounted by a conical reef. The radius of the cylindrical base is 40 m. The heights of the cylindrical and conical portions are 6.3 m and 4.2 m, respectively. Find the volume of the tent. $(Take\ \pi = 22/7)$

11. A circus tent is cylindrical up to a height of 3 m and conical above it. If the diameter of the base is 105 m and the slant height of the conical part is 53 m, find the total cast of the canvas used to make the tent when the cost per square metre of the canvas is Rs. 10. $(Take\ \pi = 22/7)$

12. A tent of height 11 m is in the form of a right circular cylinder with diameter of base 30 m and height 3 m, surmounted by a right circular cone of the same base. Find the cost of the canvas of the tent at the rate of Rs. 25 per m^2. $(Take\ \pi = 22/7)$

13. A tent is in the form of a cylinder of diameter 15 m and height 2.4 m, surmounted by a cone of equal base and height 4 m. Find the capacity of the tent and the cost of the canvas at Rs. 50 per square metre. $(Take\ \pi = 22/7)$

14. A iron pillar has some part in the form of a right circular and remaining in the form of a right circular cone. The radius of the base of each of cone and cylinder is 8 cm. The cylindrical part is 240 cm high and the conical part is 36 cm high. Find the weight of the pillar if cone cubic cm of iron weight 7.8 gracs.

15. The interior of a building is in the form of a right circular of diameter 4.2 m and height 4 m, surmounted by a cone. The vertical height of the cone is 2.1 m. Find the outer surface area and volume of the building. $(Take\ \pi = 22/7)$

16. The in interior of a building is in the form of a right circular of diameter 4.3 m and height 3.8 m, surmounted by a cone whose vertical angel is right angle. Find the area of the surface and the volume of the building. $(Take\ \pi = 22/7)$

17. A vessel is in the form of a hemispherical bowl, surmounted by a hollow cylinder. The diameter of the hemisphere is 12 cm and the total height of the vessel is 16 cm. Find the capacity of the vessel. $(Take\ \pi = 22/7)$ Also find the internal surface area of the vessel by taking $\pi = 3.14$.

18. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 19 cm and diameter of the cylinder is 7 cm. Find the volume and total surface area of the solid. $(Take\ \pi = 22/7)$

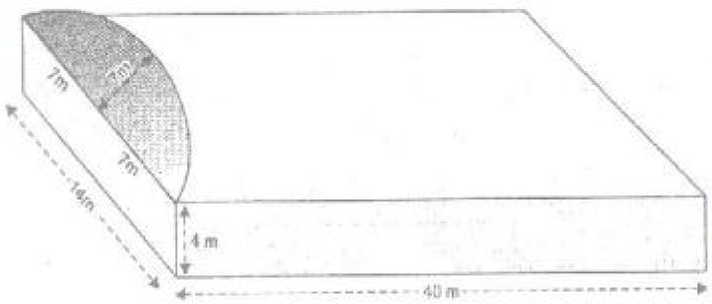
19. A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 108 cm and the diameter of the hemisphere ends is 36 cm, find the cost of polishing the surface of the solid at the rate of 10 paise per cm^2. $(Take\ \pi = 22/7)$

20. A solid toy is in the form of a right circular cylinder with a hemispherical shape at cone end and a cone at the other end. Their common diameter is 4.2 cm and the heights of the cylindrical and conical portions are 12 cm and 7 cm respectively. Find the volume of the solid. $(Take\ \pi = 22/7)$

21. A petrol tank is a cylinder of base diameter 28 cm and length 24 cm fitted with conical ends each of axis-length 9 cm. Determine the capacity of the tank.

22. Form a solid right circular cylinder with height h and radius of the base r, a right circular cone of the same height And same base is removed. Find the volume of the remaining solid.

23. A right circular cone with sides 12 cm and 16 cm is revolved around its hypotenuse. Find the volume of the double cone so formed.



24. A godown building as shown in figure is made in the form of a cuboidal base with dimensions 40 m × 14 m ×4 m, surmounted by a half cylindrical curved roof having same length as that of the base. The diameter of the cylinder is 14 m. Find the volume of the building and its total outer surface area.

25. Find the mass of a 3.5 m long lead pipe, if the external diameter of the pipe is 2.4 cm, thickness of the metal is 2 mm and mass of 1 cm^3 of lead is 11.4 g. $(Take\ \pi = 22/7)$

26. A cylindrical vessel of diameter 16 cm and height h cm is fixed symmetrically inside a similar vessel of diameter 20 cm and height h cm. The total space between the two vessels is filled with cork dust. How many cubic centimeters of cork dust is used.

27. The interior of a building is in the form of cylinder of radius 4 m and height 3.5 m, surmounted by a cone of vertical angle 90^0. Find the surface area of the interior of the building (excluding the flooring area of the building). Also find the cost of painting the interior of the building at the rate of Rs. 5 per m^2. Use $\pi = 22/7\ and \sqrt{2} = 1.414$

28. A solid is in the form of a cone of vertical height h mounted on a right circular cylinder of height 2 h and both having same radii of their bases. Base of the cone is placed on the top base of the cylinder. If V cube units be the volume of the solid, prove that the radius of the cylinder is $\sqrt{\frac{3V}{7\pi h}}$.

29. A solid is in the form of a cone of vertical height h mounted on the top base of a right circular cylinder of height $\frac{1}{3}h$. The circumference of the base of the cone and that of the cylinder are both equal to C. If V be the volume of the solid, prove that $C = 4\sqrt{\frac{3\pi V}{7h}}$.

30. A conical vessel of radius 12 cm and depth 16 cm is completely filled with water. A sphere is lowered into the water and its size is such that than when it touches inner curved surface of the vessel, it is just immersed upto the topmost point of the sphere. How much water over flows out of the vessel out of the total volume V cubic nuits.

FRUSTUM OF A RIGHT CIRCULAR CONE

1. A bucket of height 3 cm and made up of metal sheet is in the form of frustum of a right circular cone with radii of its lower and upper ends as 6 cm and 10 cm respectively. Calculate:

(i) the height of the cone of which the bucket is a part.
(ii) the volume of water which can be filled in the bucket.
(iii) the slant height of the bucket.
(iv) the area of the metal sheet required to make the bucket.

2. The radii of the circular ends of a frustum of a right circular cone are 5 cm and 8 cm and its lateral height (slant height) is 5 cm. Find the volume of the frustum. $(Take\ \pi = 22/7)$

3. The radii of the circular ends of a bucket frustum of a right circular cone are 14 cm an d2 cm and is thickness is 9 cm. Find the lateral surface of the frustum. $(Take\ \pi = 22/7)$

4. If the radii of the circular ends of a bucket 24 cm high are 5 cm and 15 cm respectively, find the inner surface area of the bucket (i.e., the area of the metal sheet required to make the bucket) $(Take\ \pi = 3.14)$

5. A bucket is in the form of a cone, its depth is 30 cm and the diameters of the top and the bottom are 42 cm and 14 cm respectively. Find how many litres of water can the bucket hold ? $(Take\ \pi = 22/7)$

6. A container made up of a metal sheet is in the form of a frustum of a cone of height 12 cm with radii of its lower and upper ends as 3 cm and 12 cm respectively. Find the cost of metal sheet used, if it costs Rs. 4 per 100 cm^2. $(Take\ \pi = 22/7)$

7. A vessel is in the form of a frustum of a cone of height 21 cm with radii of its lower and upper ends as 8 cm and 18 cm respectively. Find the cost of milk which can completely fill the vessel at the rate of Rs. 10 per litre.

8. The perimeters of the ends of a frustum are 48 cm and 36 cm. If the height of the frustum be 11 cm, fid the volume of the frustum. $(Take\ \pi = 22/7)$

9. The slant height of the frustum of a cone is 4 cm. If the perimeters of its circular bases be 18 cm and 6 cm, find the curved surface area of the frustum and also find the colt of painting its total surface at the rate of Rs. 12.50 per 100 cm^2.

10. The height of a cone is 30 cm. A frustum is cut off from this cone by a plane parallel to the base of the cone. If the volume of the frustum is $\frac{19}{27}$ of the volume of the cone, find the height of the frustum.

11. The height of a cone is 10 cm. The cone is divided into two parts by drawing a plane through the midpoint of the axis of the cone, parallel to the base. Compare the volume of the two parts.

12. A hollow cone is cut by a plane parallel to the base and upper part is removed. If the curved surface of the remainder is $\frac{15}{16}$ of the curved surface of the whole cone, find the ratio of the line-segments into which the cone's altitude is divided by the plane.

13. A right circular cone is cut by a plane parallel to the base of the cone and the upper portion is removed. If the curved surface of the frustum is $\frac{8}{9}$ of the curved surface of the whole given cone, prove that the height of the frustum is $\frac{2}{3}$ of the height of the whole cone.

14. The altitude of a right circular cone is trisected by two parallel planes, drawn parallel to the base of the cone. The cone is cut into three parts. The topmost part is a right circular cone, the middle one and last one at the bottom are two frustums. If V_1 be the volume of the small cone, V_2 be the volume of the middle portion frustum and V_3 be the volume of the frustum made at the bottom, prove the $V_1 : V_2 : V_3 = 1 : 7 : 19$.

15. A right circular cone is divided by a plane parallel to its base into a small cone of volume V_1 at the top and a frustum of volume V_2 as second part at the bottom. If $V_1 : V_2 = 1 : 3$, find the ratio of the height of the altitude of small cone and that of the frustum.

- **CONVERSION OF SOLIDS**

1. 1000 cm^2 **2.** 224 cm^2 **3.** 198 cm^2 **4.** 45 **5.** 60 **6.** 105 **7.** 20 **8.** 3.5 cm **9.** 2.5 cm
10. 2 cm. **11.** 0.1 cm **12.** 243 m **13.** 8000 **14.** 1 cm **15.** 2541 **16.** 1500 **17.** 8400 **18.** 0.05 cm
19. 1100 cm^3 **20.** 2.1 cm **21.** 5 cm **22.** 12. cm **23.** $8\,(18)^{1/3}$cm **24.** 16 cm **25.** 6.4 cm
26. 2.4 cm **27.** 450 **28.** 2 cm **29.** 75 cm **30.** 80 cm **31.** 32 **32.** 4 cm **33.** 672 **34.** 20 cm
35. 5 m **36.** 4 m **37.** $\frac{3}{2}$a **38.** $\frac{3a}{4}$ **39.** 1 cm **40.** 1.6 cm apprex. **41.** 1cm **42.** $\frac{\pi\ell^3}{6}$
43. $\frac{\pi p^3}{12}$ **44.** 2.885 cm **45.** 39.6 p^3 litres **46.** 2.52 m **47.** 400000 m^2 **48.** 20 min.
49. 16.5 min. **50.** $17\frac{3}{16}$ m.

- **SURFACE AREAS AND VOLUMES OF COMBINATIONS OF SOLIDS**

1. $293\frac{1}{3}cm^3$ **2.** (i) $14373\frac{1}{3}cm^3$ (ii) 3080 cm^2 (iii) Rs. 2464 **3.** 26.488 kg **5.** 266.11 cm3 **6.** 25.12 cm3
7. 616 cm^3 **8.** 3 cm **9.** 30 **10.** 48720 m^3 **11**. Rs. 97350 **12.** Rs. 27082.5 **13.** 660 m3 ; Rs. 15675
14. 395. 4 kg approx. **15.** 72. 4 m^2 ; 65. 142 m^3 **16.** 71.83 m^2, **17.** 1584 cm^3 ; 602.88 cm^2
18. 641 66 cm^3 ; 418 cm^2 **19.** Rs. 1221.94 **20.** 218.064 cm^3 **21.** 18480 cm^3 **22.** $\frac{2}{3}\pi r^2 h$
23. 798 .816 cm^3 **24.** 4320 m^3 ; 1096 m^2 **25.** 50518 kg **26.** $36\pi\, h cm3$ **27.** Rs. 159.104 m^2 ; Rs. 795.52
28. $\frac{3}{8}V$

- **FRUSTRUM OF A RIGHT CIRCULAR CONE**

1. 7.5 cm, 196 π cm^3, 5 cm 116 π cm^2. **2.** 540.57 cm^3 **3.** 753.6 cm^2 **4.** 1711.30 cm^2 **5.** 20.02 liters
6. Rs. 47.52 **7.** Rs. 117.04 **8.** 1554 cm3 **9.** Rs. 9.58 **10.** 10 cm **11.** 1 : 7 **12.** 1 : 3 **15.** 1 : $(4^{1/3} - 1)$

EXERCISE – 3 (FOR SCHOOL/BOARD EXAMS)

PEREVIOUS YEARS BOARD (CBSE) QUESTIONS

VERY SHORT ANSWER TYPE QUESTIONS

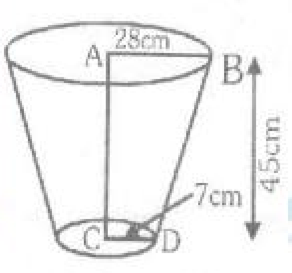
1. The surface area of a sphere is 616 cm^2. Find its radius. **[Foreign – 2008]**
2. A cylinder and a cone area of same base radius and of same height. Find the ratio of the volume of cylinder to that of the cone. **[Delhi – 2009]**
3. The slant height of the frustum of a cone is 5 cm. If the difference between the radii of its two circular ends is 4 cm. write the height of the frustum. **[AI – 2010]**

SHORT ANSWER TYPE QUESTIONS

1. A solid metallic sphere of diameter 21 cm is melted and recasted into a number of smaller cones, each of diameter 7 cm and height 3 cm. Find number of cones so formed. **[Delhi – 2004]**
2. A solid metallic sphere of diameter 28 cm is melted and recasted into a number of smaller cones, each of diameter $4\frac{2}{3}$ cm and height 3 cm. Find number of cones so formed. **[Delhi – 2004]**
3. A hemispherical bowl of internal diameter 30 cm contains some liquid. This liquid is to be filled into cylindrical shaped bottles each of diameter 5 cm and height 6 cm. Find the number of bottles necessary to empty the bowl. **[AI – 2004]**
4. Solid spheres of diameter 6 cm are dropped into a cylindrical beaker containing some water and are fully submerged. If the diameter of the beaker is 18 cm and the water rises by 40 cm, find the number of solid spheres dropped in the water. **[Foreign – 2004]**
5. A toy is in the form of a cone mounted on a hemisphere of common base radius 7 cm. The total height of the toy is 31 cm. Find the total surface area of the toy. [use $\pi = 22/7$] **[Delhi – 2007]**
6. A toy is in the form of a cone mounted on a hemisphere with same radius. The diameter of the base of the conical portion is 7 cm and total height of the toy is 14.5 cm. Find the volume of the toy. [use $\pi = 22/7$] **[AI – 2007]**

LONG ANSWER TYPE QUESTOINS

1. If the radii of the circular ends of a bucket, 45 cm high are 28 cm and 7 cm (as shown in given fig.), find the capacity of the bucket. **[AI – 2004]**



2. A hollow cone is cut by a plane parallel to the base and the upper is removed. If the curved surface of the remainder is $8/7^{th}$ of the curved surface of the whole cone, find the ratio of the line segments into which the cone's altitude is divided by the plane.

OR

If the radii of the ends of a bucket, 45 cm high, are 28 cm and 7 cm, find its capacity and surface area. **[Delhi – 2004C]**

3. A well, of diameter 3m, is dug 14 m deep. The earth taken out of it has been spread evenly all around it to a width of 4m, to form an embankment. Find the height of the embankment. [use $\pi = 22/7$] **[AI – 2004C]**

4. If the radii of the ends of a bucket, 45 cm high are 28 cm and 7 cm, determine the capacity and total surface area of the bucket. **[AI – 2005]**

5. The rain water form a roof 22 m $\times$ 20 m drains into a cylindrical vessel having diameter of base 2 m and height 3.5 m. If the vessel is just full, find the rainfall in cm. **[Delhi – 2006]**

6. Water flows at the rate of 10 m per minute through a pipe having its diameter as 5 mm? How much time will it take to fill a conical vessel whose diameter of base is 40 cm and depth 24 cm? **[Foreign – 2006]**

7. A sphere, of diameter 12 cm, is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level in the cylindrical vessel rises by $3\frac{5}{9}$ cm. Find the diameter of the cylindrical vessel.

OR

A solid right circular cone of diameter 14 cm and height 8 cm is melted to form a hollow sphere. If the external diameter of the sphere is 10 cm, find the internal diameter of the sphere. **[Delhi – 2007]**

8. A hemispherical bowl of internal diameter 36 cm is full of some liquid. This liquid is to be filled in cylindrical bottles of radius 3 cm and height 6 cm. Find the number of bottles needed to empty the bowl.

OR

Water flows out through a circular pipe whose internal radius is 1 cm, at the rate of 80 cm/second into an empty cylindrical tank, the radius of whose base is 40 cm. By how much will be level of water rise in the tank in half an hour. **[AI – 2007]**

9. A gulab jamun, when ready for eating, contains sugar syrup of about 30% of its volume. Find approximately how much syrup would be found in 45 such gulab jamuns, each shaped like a cylindrical with two hemispherical ends, if the complete length of each of them is 5 cm and it's diameter is 2.8 cm.

OR

A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice-cream. This ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice-cream. **[Delhi – 2008]**

10. A bucket made up of a metal sheet is in the form of a frustum of a cone of height 16 cm with diameters of it's lower and upper ends as 16 cm and 40 cm respectively. Find the volume of the bucket. Also find the cost of the bucket if the cost of metal sheet used is Rs. 20 per 100 cm^2 [use $\pi = 3.14$]

OR

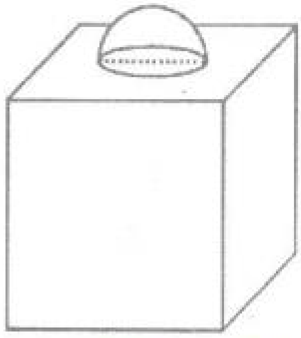
A farmer connects a pipe of internal diameter 20 cm from a canal in to a cylindrical tank in his field which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 6 km/hr, in how much time will be tank be fillrd? **[Delhi – 2008]**

11. A tent consists of a frustum of a cone, surmounted by a cone. If the diameter of the upper and lower circular ends of the frustum be 14 m and 26 m respectively, the height of the frustum be 8 m and the slant height of the surmounted conical portion be 12 m, find the area of canvas required to make the tent. (Assume that the radii of the upper circular end of the frustum and the base of surmounted conical portion are equal). **[AI – 2008]**

12. If the radii of the circular ends of a conical bucket, which is 16 cm high, are 20 cm and 8 cm, find the capacity and total surface area of the bucket. [use $\pi = 22 / 7$] **[Foreign – 2008]**

13. Form a solid cylinder whose height is 8 cm and radius 6 cm, a conical cavity of height 8 cm and of base radius 6 cm, is hollowed out. Find the volume of the remaining solid cored to two places of decimals. Also, find the total surface of the remaining solid. [Take $\pi = 3.1416$] **[Delhi – 2009]**

14. In figure, a decorative block which is made of two solids – a cube and a hemisphere. The base of the block is a cube with edge 5 cm and the hemisphere, fixed on the top, has a diameter of 4.2 cm. Find the total surface area of the block. [Take $\pi = 22/7$] **[AI – 2009]**



15. A spherical copper shell, of external diameter 18 cm is melted and recast into a solid cone of base radius 14 cm and height $4\frac{3}{7}$ cm. Find the inner diameter of the shell.

OR

A bucket is in the form of a frustum of a cone with a capacity of 12308.8 cm^3. The radii of the top and bottom circular ends of the bucket are 20 cm and 12 cm respectively. Find the height of the bucket and also the area of metal sheet used in making it. [use $\pi = 22/7$]

16. The rain-water collected on the roof of a building, of dimensions 22 m × 20 m, is drained into a cylindrical vessel having base diameter 2m and height 3.5 m. If the vessel is full up to the brim, find the height of rain-water on the roof. [use $\pi = 22/7$] **[AI – 2010]**

ANSWER KEY

- **VERY SHORT ANSWER TYPE QUESTIONS**

1. 7 cm **2.** 3 : 1 **3.** 3 cm

- **SHORT ANSWER TYPE QUESTIONS**

1. 126 **2.** 672 **3.** 60 **5.** 858 cm^2 **6.** 231 cm^3

- **ANSWER TYPE QUESTIONS**

1. 48510 cm^2 **2.** 1 : 2 or 48510 cm^3, 5616.38 cm^2 **3.** 1.125 m 4. 48510 cm^3, 8079.5 cm^2 **5.** 2.5 cm
6. 51.2 min 7. 18 cm or 6 cm **8.** 72 or 90 cm **9.** 338.184 cm^3 or 10 cones **10.** Rs. 391.87 or 50 minutes

11. (284π) m^2 **12.** $\frac{73216}{7} cm^3, \frac{13728}{7} cm^2$ **13.** 150.79 cm^3, 259.55 cm^2 **14.** 163.86 cm^2
15. 16 cm or cm ; 2160.32 cm^2 **16.** 2.5 cm

EXERCISE-4 FOR OLYMPIADS

1. One cubic metre piece of copper is melted and recast in to a square cross-section bar, 36 m long. An exact cube is cut off from this. If cubic metre of copper cost Rs. 108, then the cost of this cube is :
(A) 50 paisa (B) 75 paisa (C) One paisa (D) 1.50 paisa

2. If the surface areas of two spheres are in the ratio 4 : 9, then the ratio of their volume is :
(A) 8 : 25 (B) 8 : 26 (C) 8 : 27 (D) 8 : 28

3. In a shower 10 cm of rain fall. The volume of water that falls on 1.5 hectares of ground is :
(A) 1500 m^3 (B) 1400 m^3 (C) 1200 m^3 (D) 1000 m^3

4. The radius of base and the volume of a right circular cone are doubled. The ratio of the length of the larger cone to that of the smaller cone is :
(A) 1 : 4 (B) 1 : 2 (C) 2 : 1 (D) 4 : 1

5. A cone and a hemisphere have equal base diameter and equal volume. The ratio of their heights is :
(A) 3 : 1 (B) 2 : 1 (C) 1 : 2 (D) 1 : 3

6. If the lateral surface of a right circular cone is 2 times its base, then the semi-vertical angle of the cone must be :
(A) 15^0 (B) 30^0 (C) 45^0 (D) 60^0

7. The slant height of a conical tent made of canvas is $\frac{14}{3}$ m. The radius of tent is 2.5 m. The width of the canvas is 1.25 tube. If the height of the tube is 15 cm, then the diameter of the tube (in Rs.) is :
(A) 726 (B) 950 (C) 960 (D) 968

8. A hemispherical basin 150 cm in diameter holds water one hundred and twenty times as much a cylindrical m. If the rate of canvas per metre is Rs. 33, then the total cost of the canvas required for the tube (in cm) is :
(A) 23 (B) 24 (C) 25 (D) 26

9. A river 3 m deep and 60 m wide is flowing at the rate of 2.4 km/h. The amount of water running into the sea per minute is :
(A) 6000 m^3 (B) 6400 m^3 (C) 6800 m^3 (D) 7200 m^3

10. If a solid right circular cylinder is made of iron is heated to increase its radius and height by 1 % each, then the volume of the solid is increased by :
(A) 1.0 % (B) 3.03 % (C) 2.02 % (D) 1.2 %

11. If the right circular cone is separated into three solids of volumes V_1, V_2, and V_3 by two planes which are parallel to the base and trisects the altitude, then $V_1 : V_2 : V_3$ is :
(A) 1 : 2 : 3 (B) 1 : 4 : 6 (C) 1 : 6 : 9 (D) 1 : 7 : 19

12. Water flows at the rate of 10 m per minute from a cylindrical pipe 5 mm in diameter. A conical vessel whose diameter is 40 cm and depth 24 cm is filled. The time taken to fill the conical vessel is :
(A) 50 min (B) 50 min 12 sec. (C) 51 min 12 sec. (D) 51 min 15 sec.

13. A cylinder circumscribes a sphere. The ratio of their volume is :
(A) 1 : 2 (B) 3 : 2 (C) 4 : 3 (D) 5 : 6

14. If form a circular sheet of paper of radius 15 cm, a sector of 144^0 is removed and the remaining is used to make a conical surface, then the angle at the vertex will be :
(A) $\sin^{-1}\left(\frac{3}{10}\right)$ (B) $\sin^{-1}\left(\frac{6}{5}\right)$ (C) $2\sin^{-1}\left(\frac{3}{5}\right)$ (D) $2\sin^{-1}\left(\frac{4}{5}\right)$

15. A right circular cone of radius 4 cm and slant height 5 cm is curved out from a cylindrical piece of wood of same radius and height 5 cm. The surface area of the remaining wood is:
(A) 84π (B) 70π (C) 76π (D) 50π

16. If h, s, V be the height, curved surface area and volume of a cone respectively, then $(3\pi Vh^3 + 9V^2 - s^2h^2)$ is

(A) 0 (B) π (C) $\frac{V}{sh}$ (D) $\frac{36}{V}$

17. If cone is cut into two parts by a horizontal plane passing through the mid point of its axis, the ratio of the volume of the upper part and the frustum is :

(A) 1 : 1 (B) 1 : 2 (C) 1 : 3 (D) 1 : 7

18. A cone, a hemisphere and a cylinder stand on equal bases of radius R and have equal heights H. Their whole surfaces are in the ratio:

(A) $(\sqrt{3}+1) : 3 : 4$ (B) $(\sqrt{2}+1) : 7 : 8$ (C) $(\sqrt{2}+1) : 3 : 4$ (D) None of these

19. If a sphere is placed inside a right circular cylinder so as to touch the top, base and the lateral surface of the cylinder. If the radius of the sphere is R, the volume of the cylinder is :

(A) $2\pi R^3$ (B) $8\pi R^3$ (C) $\frac{4}{3}\pi R^3$ (D) None of these

20. A cylinder is circumscribed about a hemisphere and a cone is inscribed in the cylinder so as to have its vertex at the centre of one end and the other end as its base. The volumes of the cylinder, hemisphere and the cone are respectively in the ratio of :

(A) $3 : \sqrt{3} : 2$ (B) 3 : 2 : 8 (C) 1 : 2 : 3 (D) 2 : 3 : 1

21. A hollow sphere of outer diameter 24 cm its cut into two equal hemisphere. The total surface area of one of the hemisphere is $1436\frac{2}{7}cm^2$. Each one of the hemisphere is filled with water. What is the volume of water that can be filled in each of the hemisphere?

(A) $3358\frac{2}{3}cm^3$ (B) $3528\frac{2}{3}cm^3$ (C) $2359\frac{2}{3}cm^3$ (D) $9335\frac{2}{3}cm^3$

22. A big cube of side 8 cm is formed by rearranging together 64 small but identical cubes each of side 2 cm. Further, if the corner cubes in the topmost layer of the big cube are removed, what is the change in total surface area of the big cube ?

(A) 16 cm^2, decreases (B) 48 cm^2, decreases

(C) 32 cm^2, decreases (D) Remains the same as previously

23. A large solid sphere of diameter 15 m is melted and recast into several small spheres of diameter 3 m. What is the percentage increase in the surface area of the smaller sphere over that of the large sphere?

(A) 200 % (B) 400 % (C) 500 % (D) Can't be determined

24. A cone is made of a sector with a radius of 14 cm and an angle of 60^0. What is total surface area of the cone ?

(A) 119.78 m^2 (B) 191.87 m^2 (C) 196.5 m^2 (D) None of these

25. If a cube of maximum possible volume is cut off from a solid sphere of diameter d, then the volume of the remaining (waste) material of the sphere would be equal to :

(A) $\frac{d^3}{3}\left(\pi - \frac{d}{2}\right)$ (B) $\frac{d^3}{3}\left(\frac{\pi}{2} - \frac{1}{\sqrt{3}}\right)$ (C) $\frac{d^3}{4}\left(\sqrt{2} - \pi\right)$ (D) None of these

26. A piece of paper is in the form of a right angle triangle in which the ratio of base and perpendicular is 3 : 4 and hypotenuse is 20 cm. What is the volume of the biggest cone that can be formed by taking right angle vertex of the paper as the vertex of the cone?

(A) 45.8 m^3 (B) 56.1 m^3 (C) 61.5 m^3 (D) 48 m^3

27. In a particular country the value of diamond is directly proportional to the surface area (exposed) of the diamond. For thieves steel a cubical diamond piece and then divide equally in four parts. What is the maximum percentage increase in the value of diamond after cutting it ?

(A) 50 % (B) 66.66 % (C) 100 % (D) None of these

28. In a bullet the gun powder is to be filled up inside the metallic enclosure. The metallic enclosure is made up of a cylindrical base and conical top with the base of radius 5 cm. The ratio of height of cylinder and cone is 3 : 2. A cylindrical hole is drilled through the metal solid with height two-third the height of metal solid. What should be the radius of the hole, so that the volume of the hole (in which gun powder is to be filled up) is one third the volume of metal solid after drilling ?

(A) $\sqrt{\frac{88}{5}}$ cm **(B)** $\sqrt{\frac{55}{8}}$ cm **(C)** $\frac{55}{8}$ cm **(D)** 33π cm

29. A cubical cake is cut into several smaller cubes by dividing each edge in 7 equal parts. The cake is cut from the top along the two diagonals forming four prisms. Some of them get cut and rest remained in the cubical shape. A complete cubical (smaller) cake was given to adults and the cut off part of a smaller cake is given to a child get the cake?

(A) 343 (B) 448 (C) 367 (D) 456

30. In a factory there are two identical solid blocks of iron. When the first block is melted and recast into spheres of equal radii Y, then 14 cc of iron was left. The volumes of the solid blocks and all the spheres are in integers. What is the volume (in cm^3) of each of the large sphere of radius '2r'?

(A) 176 (B) 12π (C) 192 (D) Data insufficient

31. Initially the diameter of a balloon is 28 cm. It can explode when the diameter becomes 5/2 times of the initial diameter. Air is blown at 156 cc/s. It is knows that the shape of balloon always remains spherical. In how many seconds the balloon will explode?

(A) 1078s (B) 1368s (C) 1087s (D) None of these

32. The radius of a cone is $\sqrt{2}$ times the height of the cone. A cube of maximum possible volume is cut from the same cone. What is the ratio of the volume of the cone to the volume of the cube?

(A) $2\sqrt{3}$ ft (B) $(2+\sqrt{3})$ ft (C) $(3+\sqrt{2})$ ft (D) $(2+2\sqrt{3})$ ft

34. A blacksmith has a rectangular sheet of iron. He has to cut out 7 circular discs from this sheet. What is the minimum possible width of the iron sheet if the radius of each disc is 1 ft?

(A) $\frac{1}{11}$ (B) $\frac{2}{17}$ (C) $\frac{3}{22}$ (D) None of these

35. Barun needs an open box of capacity 864 m^3. Actually where he lives, the rates of paints are soaring high so he wants to minimize the surface area of the box keeping the capacity of the box same as required. What is the base area and height of such a box?

(A) 36 m2, 24 m (B) 216 m2, 4 m (C) 144 m2, 6 m (D) None of these

36. There are two cylindrical containers of equal capacity and equal dimensions. If the radius of one of the containers a increased by 12 ft and the height of another container is increased by 12 ft, then the capacity of both the containers is equally increased by K cubic ft. If the actual heights of the container be 4 ft, then find the increased volume of each of the container :

(A) 1680 π cu ft (B) 2304 π cu ft (C) 1480 π cu ft (D) Can't be determined

OBJECTIVE						ANSWER KEY									EXERCISE-4
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	A	C	A	B	B	B	D	C	D	B	D	C	B	C	C
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	A	D	C	A	B	A	D	B	A	B	B	C	B	B	A
Que.	**31**	**32**	**33**	**34**	**35**	**36**									
Ans.	A	B	B	A	C	B									