Senior School Certificate Examination-2020
Marking Scheme - MATHEMATICS
Subject Code: 041 Paper Code: 65 (B)

***General instructions*:-**

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. **Evaluation is a 10-12 days mission for all of us. Hence, it is necessary that you put in your best efforts in this process.**
2. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them.**
3. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
4. Evaluators will mark(√) wherever answer is correct. For wrong answer 'X"be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. This is most common mistake which evaluators are committing.
5. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
6. If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
7. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
8. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
9. A full scale of marks 0 - 80 has to be used. Please do not hesitate to award full marks if the answer deserves it.
10. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).
11. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong totaling of marks awarded on a reply
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totaling on the title page.
    - Wrong totaling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
12. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
13. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
14. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.
15. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
16. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

QUESTION PAPER CODE 65(B)

EXPECTED ANSWER/VALUE POINTS

SECTION – A

Question Numbers 1 to 20 carry 1 mark each.

Question Numbers 1 to 10 are multiple choice type questions. Select the correct option.

Q.No. **Marks**

1. Total number of possible matrices of order 2×2 with each entry 2 or 3 is

(A) 4 **(B)** 8 **(C)** 16 **(D)** 32

Ans: (C) 16 — 1

2. The area of a triangle with vertices $(-2, 0)$, $(2, 0)$ and $(0, k)$ is 4 sq. units. The value of k is

(A) 4 **(B)** 2 **(C)** -4 **(D)** 6

Ans: (B) 2 — 1

3. The value of the expression $2 \operatorname{cosec}^{-1}2 + \cos^{-1}\left(\frac{1}{2}\right)$ is

(A) $\frac{\pi}{3}$ **(B)** $-\frac{\pi}{3}$ **(C)** $-\frac{2\pi}{3}$ **(D)** $\frac{2\pi}{3}$

Ans: (D) $\frac{2\pi}{3}$ — 1

4. $\int \frac{dx}{16 + 9x^2}$ is equal to

(A) $\frac{1}{4}\tan^{-1}\frac{3x}{4} + c$ **(B)** $\frac{1}{12}\tan^{-1}\frac{3x}{4} + c$ **(C)** $\frac{1}{3}\tan^{-1}\frac{3x}{4} + c$ **(D)** $\frac{1}{12}\tan^{-1}\frac{9x}{16} + c$

Ans: (B) $\frac{1}{12}\tan^{-1}\frac{3x}{4} + c$ — 1

5. The area of the trinagle whose two sides are represented by the vectors $\hat{i} + \hat{k}$ and $2\hat{i} + \hat{j} + \hat{k}$ is

(A) $\frac{\sqrt{3}}{2}$ **(B)** $\sqrt{3}$ **(C)** 3 **(D)** $\frac{3}{2}$

Ans: (A) $\frac{\sqrt{3}}{2}$ — 1

6. If the direction cosines of a line are a, a, a, then

(A) $a > 0$ **(B)** $a = 1$ or $a = -1$ **(C)** $0 < a < 1$ **(D)** $a = \frac{1}{\sqrt{3}}$ or $a = -\frac{1}{\sqrt{3}}$

Ans: (D) $a = \frac{1}{\sqrt{3}}$ or $a = -\frac{1}{\sqrt{3}}$ **1**

7. The plane $3x - 2y + 6z + 11 = 0$, makes an angle $\sin^{-1}(\alpha)$ with x-axis. The value of α is

(A) $-\frac{3}{7}$ **(B)** $\frac{3}{7}$ **(C)** $\frac{2}{7}$ **(D)** $\frac{\sqrt{3}}{2}$

Ans: (B) $\frac{3}{7}$ **1**

8. Let $f : \mathbf{R} \to \mathbf{R}$ be defined by $f(x) = 5x - 3$. Then, $f^{-1}(x)$ is given by

(A) $\frac{x+3}{5}$ **(B)** $\frac{x}{3} - 5$ **(C)** $\frac{x}{5} + 3$ **(D)** $\frac{x}{5} - 3$

Ans: (A) $\frac{x+3}{5}$ **1**

9. If A and B be two events such that $P(A) = 0{\cdot}2$, $P(B) = 0{\cdot}4$ and $P(A \cap B) = 0{\cdot}08$, then $P(A \mid B)$ is

(A) 0·02 **(B)** 0.2 **(C)** 0·4 **(D)** 0·08

Ans: (B) 0.2 **1**

10. A bag contains 4 red and 3 black balls. If 2 balls are drawn from the bag at random without replacement, then the probability of getting exactly one red ball is

(A) $\frac{1}{7}$ **(B)** $\frac{2}{7}$ **(C)** $\frac{4}{7}$ **(D)** $\frac{3}{14}$

Ans: (C) $\frac{4}{7}$ **1**

Fill in the blanks in questions numbers 11 to 15

11. Matrices A and B will be inverse of each other only if __________.

Ans: $AB = BA = I$ **1**

12. The function $f(x) = |x - 3|$, $x \in \mathbf{R}$ is not differentiable at $x =$ __________.

Ans: 3 **1**

13. The function $f(x) = \log_e (\sin x)$ is strictly __________ on $\left(\frac{\pi}{2}, \pi\right)$.

Ans: decreasing **1**

OR

The approximate value of $\sqrt{26}$, using differentials, up to 2 places of decimal is __________.

Ans: 5.10 **1**

14. In an LPP, the linear function which has to be maximised or minimised is called a linear _______ function.

Ans: Objective **1**

15. A vector of magnitude 14 in the direction of the vector $\vec{a} = -2\hat{i} + 3\hat{j} + 6\hat{k}$ is ________.

Ans: $-4\hat{i} + 6\hat{j} + 12\hat{k}$ **1**

Question numbers 16 to 20 are very short answer type questions.

16. Find the value of the determinant $\begin{vmatrix} bc & 1 & a(b+c) \\ ca & 1 & b(c+a) \\ ab & 1 & c(a+b) \end{vmatrix}$.

Ans: Writing determinant as $(ab+bc+ca)\begin{vmatrix} bc & 1 & 1 \\ ca & 1 & 1 \\ ab & 1 & 1 \end{vmatrix}$ **1/2**

$= 0$ **1/2**

17. Evaluate :

$$\int_{-\pi/4}^{\pi/4} \left(x^3 + x\cos x + \tan^5 x\right) dx$$

Ans: $f(x) = x^3 + x\cos x + \tan^5 x$ is odd **1/2**

$\therefore$ Value of Integral = 0 **1/2**

18. Find :

$$\int \frac{6\cos x - 9\sin x}{6\cos x + 4\sin x} dx$$

Ans: Given integral $= \frac{3}{2}\int \frac{2\cos x - 3\sin x}{3\cos x + 2\sin x} dx$ **1/2**

$= \frac{3}{2}\log\left|3\cos x + 2\sin x\right| + C$ **1/2**

OR

Find : $\int \frac{(x-5)e^x}{(x-3)^3} dx$

Ans: Given integral $= \int \left[\frac{1}{(x-3)^2} - \frac{2}{(x-3)^3} \right] e^x dx$ **1/2**

$= \frac{e^x}{(x-3)^2} + C$ **1/2**

19. Find :

$\int \tan^2 (3x+5)\, dx$ m

Ans: Given integral $= \int \left(\sec^2 (3x+5) - 1 \right) dx$ **1/2**

$= \frac{\tan(3x+5)}{3} - x + C$ **1/2**

20. Form the differential equation representing the family of curves $y = a \cos (x + b)$, where a, b are arbitrary constants.

Ans: $\frac{dy}{dx} = -a \sin(x+b)$ **1/2**

$\frac{d^2y}{dx^2} = -a \cos(x+b) \Rightarrow \frac{d^2y}{dx^2} = -y$ **1/2**

OR

Find the integrating factor for the solution of the differential equation $y\, dx - (x + 2y^2)\, dy = 0$.

Ans: Given differential equation can be written as

$\frac{dx}{dy} - \frac{x}{y} = -y$ **1/2**

Integrating factor is $e^{\int -\frac{1}{y} dy} = \frac{1}{y}$ **1/2**

SECTION-B

Question numbers 21 to 26 carry 2 marks.

21. Show at the relation R in the set of all positive real numbers defined by $R = \{(a, b) : a \le b^3\}$ is neither symmetric nor transitive.

Ans: $(1, 2) \in R$ but $(2, 1) \notin R$ $\therefore$ R is not symmetriac **1**

$(9, 3) \in R$, $(3, 2) \in R$ but $(9, 2) \notin R$ $\therefore$ R is not transitive **1**

OR

Prove that :

$$\cos^{-1}\left(\frac{12}{13}\right)+\sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$$

Ans: LHS $= \tan^{-1}\frac{5}{12}+\tan^{-1}\frac{3}{4}$ **1/2+1/2**

$$= \tan^{-1}\frac{\frac{5}{12}+\frac{3}{4}}{1-\frac{5}{12}\cdot\frac{3}{4}} = \tan^{-1}\frac{56}{33}$$ **1/2**

$$= \sin^{-1}\frac{56}{65} = \text{RHS}$$ **1/2**

22. If $(x^2 + y^2)^2 = xy$, then find $\frac{dy}{dx}$.

Ans: Differentiating both sides w.r.t. x

$$2(x^2+y^2)\left(2x+2y\frac{dy}{dx}\right) = x\frac{dy}{dx}+y$$ **1+1/2**

Getting $\frac{dy}{dx} = \frac{y-4x^3-4xy^2}{4x^2y+4y^3-x}$ **1/2**

23. The radius of a right circular cylinder is increasing at the rate of 2cm/s and its height is decreasing at the rate of 8cm/s. Find the rate of change of its volume, when the radius is 3 cm and height is 6cm.

Ans: Let r be radius & h be the height

$$\frac{dr}{dt} = 2\text{ cm/s} \qquad \frac{dh}{dt} = -8\text{cm/s}$$ **1/2**

$$v = \pi r^2 h \quad \Rightarrow \frac{dv}{dt} = \pi\left[r^2\frac{dh}{dt}+2hr\frac{dr}{dt}\right]$$ **1**

$$\frac{dv}{dt}(\text{at } r=3, h=6) = 0$$ **1/2**

24. Show that the points with position vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$ are the vertices of a right angled triangle.

Ans: Let $A(2\hat{i}-\hat{j}+\hat{k})$, $B(\hat{i}-3\hat{j}-5\hat{k})$, $C(3\hat{i}-4\hat{j}-4\hat{k})$

$$\overrightarrow{AB} = -\hat{i}-2\hat{j}-6\hat{k}, \quad \left|\overrightarrow{AB}\right| = \sqrt{41}$$ **1/2**

$\overrightarrow{AC} = \hat{i} - 3\hat{j} - 5\hat{k}, \ |\overrightarrow{AC}| = \sqrt{35}$ **1/2**

$\overrightarrow{BC} = 2\hat{i} - \hat{j} + \hat{k}, \ |\overrightarrow{BC}| = \sqrt{6}$ **1/2**

Clearly $|\overrightarrow{AC}|^2 + |\overrightarrow{BC}|^2 = |\overrightarrow{AB}|^2$ **1/2**

$\therefore$ A, B, C are vertices of right angled triangle

OR

Using vectors, find the area of triangle ABC with vertices A(1, 1, 1) B(1, 2, 3) and C(2, 3, 1).

Ans: $\overrightarrow{AB} = \hat{j} + 2\hat{k}$ **1/2**

$\overrightarrow{AC} = \hat{i} + 2\hat{j}$ **1/2**

$\overrightarrow{AB} \times \overrightarrow{AC} = -4\hat{i} + 2\hat{j} - \hat{k}$ **1/2**

Required area $= \frac{1}{2}\sqrt{21}$ **1/2**

25. Find the angle between the lines.

$\vec{r} = \left(2\hat{j} - 3\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$ and

$\vec{r} = \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} - 6\hat{k}\right)$.

Ans: Let θ be angle between lines

$$\cos\theta = \left|\frac{\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)\cdot\left(2\hat{i} + 3\hat{j} - 6\hat{k}\right)}{\sqrt{3}\cdot\sqrt{7}}\right|$$ **1**

$$\theta = \cos^{-1}\frac{4}{\sqrt{21}}$$ **1**

26. Two dice are thrown once. Given that two numbers appearing on the dice are different, find the probability of the event 'the sum of numbers on the dice is 6'.

Ans: A : Sum of numbers is 6

B : Number appearing on dice are different

$A \cap B = (1, 5)\ (5, 1)\ (2, 4)\ (4, 2)$ **1**

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{4}{36}}{\frac{30}{36}} = \frac{2}{15}$$ **1/2+1/2**

SECTION-C

Question numbers 27 to 32 carry 4 marks each.

27. Consider $f : \mathbb{R}_+ \to [-5, \infty]$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \dfrac{\sqrt{y+6} - 1}{3}$, where $\mathbb{R}_+$ is the set of all non-negative real numbers.

Ans: One-One

Let $x_1, x_2 \in \mathbb{R}_+$ such that $f(x_1) = f(x_2)$

$\Rightarrow\ 9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$

$\Rightarrow\ x_1 = x_2$ or $x_1 + x_2 = -\dfrac{2}{3}$ rejected

$\therefore$ f is one-one **2**

For range let $f(x) = y$

i.e. $9x^2 + 6x - 5 = y$

which gives $x = \dfrac{\sqrt{y+6}-1}{3} \in R_+$ **1**

if $\sqrt{y+6} - 1 \geq 0$ i.e. $y \geq -5$

$\therefore$ Range = $[-5, \infty)$ = codomain **1/2**

f is onto

As f is one-one & onto $\therefore$ f is invertible & $f^{-1}(y) = \dfrac{\sqrt{y+6}-1}{3}$ **1/2**

28. If $y = (\cos x)^x$, find $\dfrac{dy}{dx}$.

Ans: Taking log on both sides gives $\log y = x \log \cos x$ **1**

differentiating w.r.t. x to get

$\dfrac{1}{y}\dfrac{dy}{dx} = -x \tan x + \log \cos x$ **2**

$\therefore\ \dfrac{dy}{dx} = (\cos x)^x (-x \tan x + \log \cos x)$ **1**

OR

If x = a cos θ and y = b sin θ, then prove that $\frac{d^2y}{dx^2} = -\frac{b^4}{a^2y^3}$.

Ans: $\frac{dx}{d\theta} = -a\sin\theta,\ \frac{dy}{d\theta} = b\cos\theta$ **1/2+1/2**

$\therefore\ \frac{dy}{dx} = -\frac{b}{a}\cot\theta$ **1**

$\frac{d^2y}{dx^2} = +\frac{b}{a}\operatorname{cosec}^2\theta \cdot -\frac{1}{a\sin\theta}$ **1**

$= -\frac{b}{a^2\sin^3\theta} \quad \left(\text{putting } \sin\theta = \frac{y}{b}\right)$ **1/2**

$= -\frac{b^4}{a^2y^3}$ **1/2**

29. Evaluate :

$$\int_1^2 \frac{x\,dx}{(x+1)(x+2)}$$

Ans: Writing $\frac{x}{(x+1)(x+2)}$ as $\frac{2}{x+2} - \frac{1}{x+1}$

Given integral= $2\int_1^2 \frac{dx}{x+2} - \int_1^2 \frac{dx}{x+1}$ **1**

$= 2|\log(x+2)|_1^2 - |\log(x+1)|_1^2$ **2**

$= 2\log\frac{4}{3} - \log\frac{3}{2}$ or $\log\frac{32}{27}$ **1**

30. Find the particular solution of the differential equation $\frac{dy}{dx}$ + y cot x = 2x + x^2 cot x, (x ≠ 0), given that y = 0, when x = $\frac{\pi}{2}$.

Ans: Integrating factor is $e^{\int \cot x\,dx} = \sin x$ **1**

Solution is $y \cdot \sin x = \int \left(x^2\cos x + 2x\sin x\right)dx + C$ **1**

$$y \sin x = x^2 \sin x - \int 2x \sin x \, dx + \int 2x \sin x \, dx + C$$

$$y \sin x = x^2 \sin x + C \qquad \mathbf{1}$$

Putting $y = 0$, $x = \frac{\pi}{2}$ to get $C = -\frac{\pi^2}{4}$ **1/2**

$\therefore$ Particular solution is $y \sin x = x^2 \sin x - \frac{\pi^2}{4}$ **1/2**

31. A furniture firm manufactures chairs and tables, each requiring the use of three machines A, B and C. Production of one chair requires 2 hours on machine A, 1 hour on machine B and 1 hour on machine C. Each table requires 1 hour each on machines A and B and 3 hours on machine C. The profit obtained by sellling one chair is ₹ 300; while by selling one table, the profit is ₹ 600. The total time available per week on machine A is 70 hours, on machine B is 40 hours and on machine C is 90 hours. Formulate an LPP to determine the number of chairs and tables the firm should make per week in order to get maximum profit.

Ans: Let number of chairs be x and table be y.

Maximize profit $Z = 300x + 600y$ **1**

Subject to constraints

$$\left.\begin{aligned} 2x + y &\le 70 \\ x + y &\le 40 \\ x + 3y &\le 90 \\ x, y &\ge 0 \end{aligned}\right\} \qquad \mathbf{3}$$

32. A bag contains 2 white, 3 red and 4 blue balls. Two balls are drawn one-by-one without replacement from the bag. Find the probability distribution of the number of red balls. Also, find the mean of the number of red balls.

Ans: Let X denotes number of Red balls

X	0	1	2	
P(X)	$\frac{15}{36}$ or $\frac{5}{12}$	$\frac{18}{36}$ or $\frac{1}{2}$	$\frac{3}{36}$ or $\frac{1}{12}$	**1** + **2**
XP(X)	0	$\frac{18}{36}$	$\frac{6}{36}$	**1/2**

Mean $= \sum XP(X) = \frac{24}{36}$ or $\frac{2}{3}$ **1/2**

OR

A card from a pack of 52 playing cards is lost. From the remaining cards of the pack, two cards are drawn (one-by-one without replacement) and both are found to be diamonds. Find the probability of the lost card being a diamond card.

Ans: Let E_1 : Lost card is diamond

E_2 : Lost card is not a diamond

A : Two cards drawn are diamonds

$$P(E_1) = \frac{1}{4} \qquad P(E_2) = \frac{3}{4}$$ **1**

$$P(A/E_1) = \frac{^{12}C_2}{^{51}C_2} \qquad P(A/E_2) = \frac{^{13}C_2}{^{51}C_2}$$ **1**

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\frac{1}{4} \times \frac{^{12}C_2}{^{51}C_2}}{\frac{1}{4} \times \frac{^{12}C_2}{^{51}C_2} + \frac{3}{4} \times \frac{^{13}C_2}{^{51}C_2}}$$ **1**

$$= \frac{11}{50}$$ **1**

SECTION-D

Question numbers 33 to 36 carry 6 marks each.

33. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, show that $A^3 - 6A^2 + 9A - 4I = O$ and hence find A^{-1}.

Ans: $$A^2 = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$ **1**

$$A^3 = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$ **1**

$\therefore\ A^3 - 6A^2 + 9A - 4I$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix} + \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$= 0$ **2**

$A^3 - 6A^2 + 9A - 4I = 0 \Rightarrow A^2 - 6A + 9I - 4A^{-1} = 0$

$$\Rightarrow A^{-1} = \frac{1}{4}\left[A^2 - 6A + 9I\right]$$ **1**

$$\therefore A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$ **1**

OR

Using the properties of determinants,

prove that $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$

Ans: Applying $C_1 \to C_1 + C_2 + C_3$ LHS $= \begin{vmatrix} a+b+c & -a+b & -a+c \\ a+b+c & 3b & -b+c \\ a+b+c & -c+b & 3c \end{vmatrix}$ **2**

$$= (a+b+c)\begin{vmatrix} 1 & -a+b & -a+c \\ 1 & 3b & -b+c \\ 1 & -c+b & 3c \end{vmatrix}$$ **1**

$R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$

$$= (a+b+c)\begin{vmatrix} 1 & -a+b & -a+c \\ 0 & 2b+a & a-b \\ 0 & a-c & 2c+a \end{vmatrix}$$ **2**

Expanding to get

$= (a+b+c)(3ab+3bc+3ca)$ **1**

$= 3(a+b+c)(ab+bc+ca) = \text{RHS}$

34. Find the absolute maximum and absolute minimum values of a fucntion f given by $f(x) = 12x^{4/3} - 6x^{1/3}$, $x \in [-1, 1]$.

Ans: $f'(x) = 16x^{1/3} - \frac{2}{x^{-2/3}}$ **2**

$f'(x) = 0 \Rightarrow x = \frac{1}{8}$ **1**

Critical points are $-1, 0, \frac{1}{8}, 1$ **1**

$f(-1) = 18,\ f(0) = 0,\ f\left(\frac{1}{8}\right) = -\frac{9}{4},\ f(1) = 6$ **1**

absolute maximum value is 18 **1/2**

absolute minimum value is $-\frac{9}{4}$ **1/2**

35. Using integration, find the area of the region bounded by the lines $2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$.

Ans: Point of intersection of given lines are (2, 0), (1, 2) and (4, 3) **$1\frac{1}{2}$**

Required area $= \frac{1}{3}\int_1^4 (x+5)dx - 2\int_1^2 (2-x)dx - \frac{3}{2}\int_2^4 (x-2)dx$ **2**

$$= \left.\frac{(x+5)^2}{6}\right|_1^4 + \left.\frac{2(2-x)^2}{2}\right|_1^2 - \frac{3}{4}(x-2)^2\Big|_2^4$$ **$1\frac{1}{2}$**

$$= \frac{15}{2} - 1 - 3 = \frac{7}{2}$$ **1**

36. Prove that the line through A (0, –1, –1) and B(4, 5, 1) intersects the line throught C(3, 9, 4) and D(– 4, 4, 4).

Ans: dr's of AB : 4, 6, 2 **1**

dr's of CD : –7, –5, 0 **1**

Consider $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 3-0 & 9+1 & 4+1 \\ 4 & 6 & 2 \\ -7 & -5 & 0 \end{vmatrix}$ **3**

$= 30 - 140 + 110 = 0$

$\therefore$ lines intersect each other **1**

OR

Find the coordinates of the foot of perpendicular and the perpendicular distance from the point P(4, 3, 2) to the plane x + 2y + 3z = 2. Also, find the image of P in the plane.

Ans: Let foot of perpendicular be A

Equation of PA is $\frac{x-4}{1}=\frac{y-3}{2}=\frac{z-2}{3}=\lambda$ **1**

Coordinate of A (λ + 4, 2λ + 3, 3λ + 2) **1**

Putting in equation of plane we get λ = –1 **1**

Coordinate of A (3, 1, –1) **1**

distance AP = $\sqrt{(4-3)^2+(3-1)^2+(2+1)^2}$

= $\sqrt{14}$ **1**

For coordinate of image P′(x, y, z)

$\frac{x+4}{2}=3, \frac{y+3}{2}=1, \frac{z+2}{2}=-1$

to get P′(2, –1, –4) **1**