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Secondary School Examination-2020

Marking Scheme - MATHEMATICS STANDARD

Subject Code: 041 Paper Code: 30/3/1, 30/3/2, 30/3/3

General instructions

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. **Evaluation is a 10-12 days mission for all of us. Hence, it is necessary that you put in your best effortsin this process.**
2. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them. In class-X, while evaluating two competency based questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, marks should be awarded.**
3. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
4. Evaluators will mark(√) wherever answer is correct. For wrong answer 'X"be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
5. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
6. If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
7. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
8. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
9. A full scale of marks **0-80** marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
10. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).
11. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong totaling of marks awarded on a reply.
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totaling on the title page.
    - Wrong totaling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
12. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
13. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
14. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.
15. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
16. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

QUESTION PAPER CODE 30/3/1

EXPECTED ANSWER/VALUE POINTS

SECTION – A

Question numbers 1 to 10 are multiple choice questions of 1 mark each.

You have to select the correct choice :

Q.No.		Marks
1.	The HCF of 135 and 225 is **(a)** 15 **(b)** 75 **(c)** 45 **(d)** 5 **Ans:** (c) 45	1
2.	The exponent of 2 in the prime factorization of 144, is **(a)** 2 **(b)** 4 **(c)** 1 **(d)** 6 **Ans:** (b) 4	1
3.	The common difference of an AP, whose n^{th} term is $a_n = (3n + 7)$, is **(a)** 3 **(b)** 7 **(c)** 10 **(d)** 6 **Ans:** (a) 3	1
4.	The value of λ for which $(x^2 + 4x + \lambda)$ is a perfect square, is **(a)** 16 **(b)** 9 **(c)** 1 **(d)** 4 **Ans:** (d) 4	1
5.	The value of k, for which the pair of linear equations $kx + y = k^2$ and $x + ky = 1$ have infinitely many solutions is **(a)** ± 1 **(b)** 1 **(c)** -1 **(d)** 2 **Ans:** (b) 1	1
6.	The value of p for which $(2p + 1)$, 10 and $(5p + 5)$ are three consecutive terms of an AP is **(a)** -1 **(b)** -2 **(c)** 1 **(d)** 2 **Ans:** (d) 2	1
	OR	
	The number of terms of an AP 5, 9, 13, … 185 is **(a)** 31 **(b)** 51 **(c)** 41 **(d)** 40 **Ans:** 1 mark should be given to each candidate.	1
7.	In Fig. 1, the graph of the polynomial p(x) is given. The number of zeroes of the polynomial is	

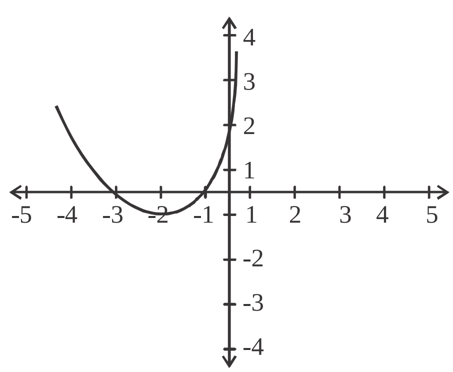
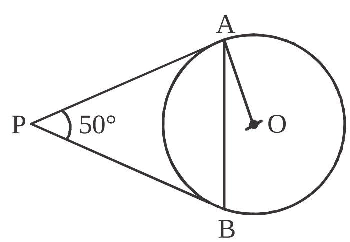
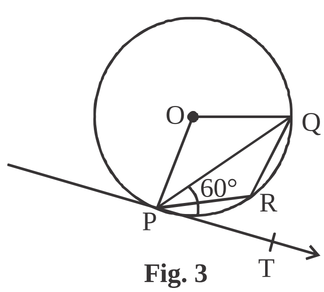


Fig. 1

	(a) 1 **(b)** 2 **(c)** 3 **(d)** 0	
	Ans: (b) 2	1
8.	If (a, b) is the mid-point of the line segment joining the points A(10, –6) and B(k, 4) and a – 2b = 18, the value of k is	
	(a) 30 **(b)** 22 **(c)** 4 **(d)** 40	
	Ans: (b) 22	1
9.	The value of k for which the points A (0, 1), B (2, k) and C(4, –5) are collinear is	
	(a) 2 **(b)** –2 **(c)** 0 **(d)** 4	
	Ans: (b) –2	1
10.	If $\Delta ABC \sim \Delta DEF$ such that AB = 1.2 cm and DE = 1.4 cm, the ratio of the areas of ΔABC and ΔDEF is	
	(a) 49 : 36 **(b)** 6 : 7 **(c)** 7 : 6 **(d)** 36 : 49	
	Ans: (d) 36 : 49	1
	In Q. Nos. 11 to 15, fill in the blanks. Each question is of 1 mark :	
11.	$\sqrt{2}$ times the distance between (0, 5) and (–5, 0) is __________.	
	Ans: 10	1
12.	The distance between two parallel tangents of a circle of radius 4 cm is __________.	
	Ans: 8 cm	1
13.	In Fig. 2, PA and PB are tangents to the circle with centre O such that $\angle APB = 50°$, then the measure of $\angle OAB$ is __________.	
	 **Fig. 2**	
	Ans: 25°	1
	OR	
	In Fig. 3, PQ is a chord of a circle and PT is tangent at P such that $\angle QPT = 60°$, then the measure of $\angle PRQ$ is __________.	
	 **Fig. 3**	
	Ans: 120°	1

14.	$\dfrac{3\cot 40^\circ}{\tan 50^\circ} - \dfrac{1}{2}\left(\dfrac{\cos 35^\circ}{\sin 55^\circ}\right) =$ ________.	
	Ans: $\dfrac{5}{2}$	**1**
15.	If $\cot\theta = \dfrac{7}{8}$, then the value of $\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} =$ ________.	
	Ans: $\dfrac{49}{64}$	**1**
	Q. Nos. 16 to 20 are short answer type questions of 1 mark each.	
16.	What is the value of $\left(\dfrac{1}{1+\cot^2\theta} + \dfrac{1}{1+\tan^2\theta}\right)$?	
	Ans: Given expression = $\sin^2\theta + \cos^2\theta$	**1/2**
	$= 1$	**1/2**
17.	Two right circular cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1, what is the ratio of their volumes?	
	Ans: $V_1 : V_2 = \dfrac{1}{3}\pi(3r)^2 h : \dfrac{1}{3}\pi r^2(3h)$	**1/2**
	$= 3 : 1$	**1/2**
18.	Using the empirical formula, find the mode of a distribution whose mean is 8.32 and the median is 8.05.	
	Ans: Mode $= 3 \times 8.05 - 2 \times 8.32$	**1/2**
	$= 7.51$	**1/2**
19.	The probability that it will rain tomorrow is 0.85. What is the probability that it will not rain tomorrow ?	
	Ans: Prob (no rain tomorrow) $= 1 - 0.85$	**1/2**
	$= 0.15$	**1/2**
20.	What is the arithmetic mean of first n natural numbers?	
	Ans: Sum of first n natural numbers = $\dfrac{n(n+1)}{2}$	**1/2**
	$\therefore$ Mean $= \dfrac{n+1}{2}$	**1/2**
	SECTION – B	
	Q. Nos. 21 to 26 carry 2 marks each.	
21.	Find the 11th term from the last term (towards the first term) of the AP 12, 8, 4, …, –84.	
	Ans: $l = -84$	**1/2**
	$d = -4$	**1/2**
	t_{11} (from the end) $= -84 + 40 = -44$	**1**

OR

Solve the equation : 1 + 5 + 9 + 13 + … + x = 1326

Ans: $\frac{n}{2}(1 + x) = 1326$ … (i) **1/2**

$x = 1 + (n - 1) \times 4$ … (ii) **1/2**

Solving (i) and (ii) x = 101 **1**

22. In Fig. 4 AB is a chord of circle with centre O, AOC is diameter and AT is tangent at A. Prove that $\angle BAT = \angle ACB$.

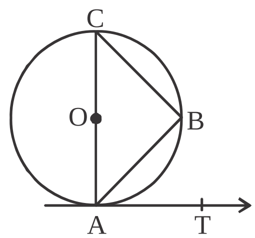


Fig. 4

Ans: $\angle BAC = 90° - \angle BAT$ … (i) **1/2**

In ΔBAC, $\angle B = 90°$ **1/2**

$\therefore \angle BCA = 90° - \angle BAC$

or $\angle ACB = \angle BAT$ (Using (i)) **1**

23. If $\tan\theta = \frac{3}{4}$, find the value of $\left(\frac{1-\cos^2\theta}{1+\cos^2\theta}\right)$

Ans: $\sec^2\theta = 1 + \frac{9}{16} = \frac{25}{16}$

$\therefore \cos^2\theta = \frac{16}{25}$ **1**

Hence $\frac{1-\cos^2\theta}{1+\cos^2\theta} = \frac{1-\frac{16}{25}}{1+\frac{16}{25}} = \frac{9}{41}$ **1**

OR

If $\tan\theta = \sqrt{3}$, find the value of $\left(\frac{2\sec\theta}{1+\tan^2\theta}\right)$

Ans: $\sec^2\theta = 1 + 3 = 4$

$\therefore \sec\theta = 2$ **1**

Hence $\frac{2\sec\theta}{1+\tan^2\theta} = \frac{2\times 2}{4} = 1$ **1**

24.	Read the following passage and answer the questions given at the end : Students of Class XII presented a gift to their school in the form of an electric lamp in the shape of a glass hemispherical base surmounted by a metallic cylindrical top of same radius 21 cm and height 3.5 cm. The top was silver coated and the glass surface was painted red. (i) What is the cost of silver coating the top at the rate of ₹ 5 per 100 cm^2 ? (ii) What is the surface area of glass to be painted red ?	
	Ans: (i) Surface Area of the top $= 2 \times \frac{22}{7} \times 21 \times 3.5 = 462\ cm^2$	1/2
	Cost of silver coating $= 462 \times \frac{5}{100} =$ Rs. 23.10	1/2
	(ii) Surface Area of glass $= 2 \times \frac{22}{7} \times 21 \times 21$	1/2
	$= 2772\ cm^2$	1/2
25.	Find the probability that a leap year selected at random will contain 53 Sundays and 53 Mondays.	
	Ans: 366 days = 52 weeks + 2 days	1/2
	Total possible outcomes are 7 (SM, MT, TW, WTh, ThF, FS, SS)	1
	Prob (having 53 Sundays & 53 Mondays) $= \frac{1}{7}$	1/2
26.	Find the value of p, if the mean of the following distribution is 7.5.	

Classes	2-4	4-6	6-8	8-10	10-12	12-14
Frequency (fi)	6	8	15	p	8	4

Ans:

Class	**Frequency (f)**	**x**	**fx**
2-4	6	3	18
4-6	8	5	40
6-8	15	7	105
8-10	p	9	9p
10-12	8	11	88
12-14	4	13	52
	41 + p		303 + 9p

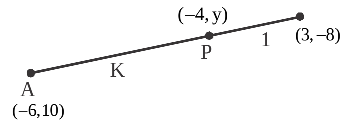
Correct table = 1

Mean $= 7.5 = \frac{303+9p}{41+p} \Rightarrow p = 3$ **1**

	SECTION – C	
	Q. Nos. 27 to 34 carry 3 marks each.	
27.	Find a, b and c if it is given that the numbers a, 7, b, 23, c are in AP.	
	Ans: a, 7, b, 23, c are in A.P	
	Let d be the common difference of AP.	
	$\therefore$ $a + d = 7$ … (i)	1/2
	$a + 3d = 23$ … (ii)	1/2
	Solving (i) & (ii) , $d = 8$	1/2
	$\therefore$ $a = -1, b = 15, c = 31$	1/2+1/2+1/2
	OR	
	If m times the m^{th} term of an AP is equal to n times its nth term, show that the $(m + n)^{th}$ term of the AP is zero.	
	Ans: Given $m[a + (m - 1)d] = n[a + (n - 1)d]$	1
	$\Rightarrow$ $a(m - n) + d(m^2 - m - n^2 + n) = 0$	
	$\Rightarrow$ $(m - n)[a + (m + n - 1)d = 0$	1
	$\because$ $m \neq n \Rightarrow a + (m + n - 1)d = 0$	1/2
	$\Rightarrow a_{m+n} = 0$	1/2
28.	Find the values of k, for which the quadratic equation $(k + 4)x^2 + (k + 1)x + 1 = 0$ has equal roots.	
	Ans: For equal roots $(k + 1)^2 - 4(k + 4) \times 1 = 0$	1
	$\Rightarrow$ $k^2 - 2k - 15 = 0$	1
	$\Rightarrow$ $(k + 3)(k - 5) = 0$	1/2
	$\Rightarrow$ $k = -3, 5$	1/2
29.	On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were $x - 2$ and $-2x + 4$ respectively. Find g(x).	
	Ans: $x^3 - 3x^2 + x + 2 = (x - 2) \times g(x) + (-2x + 4)$	1
	$\Rightarrow$ $(x - 2)g(x) = x^3 - 3x^2 + 3x - 2$	1/2
	$\Rightarrow$ $g(x) = \dfrac{(x-2)(x^2 - x + 1)}{(x-2)}$	1
	$= x^2 - x + 1$	1/2
	OR	
	If the sum of the squares of zeros of the quadratic polynomial $f(x) = x^2 - 8x + k$ is 40, find the value of k.	
	Ans: Let the zeroes of polynomial f(x) be α and β.	
	$\therefore$ $\alpha + \beta = 8$ and $\alpha\beta = k$	1/2+1/2
	$\because$ $\alpha^2 + \beta^2 = 40$	
	$\Rightarrow$ $(\alpha + \beta)^2 - 2\alpha\beta = 40$	1
	$\Rightarrow$ $64 - 2k = 40$	1/2
	$\Rightarrow$ $k = 12$	1/2

30. In what ratio does the point P(–4, y) divide the line segment joining the points A(–6, 10) and B(3, –8) if it lies on AB. Hence find the value of y.

Ans: Let AP : PB = k : 1



$\therefore \quad -4 = \dfrac{3k-6}{k+1}$ **1**

$\Rightarrow \quad k = \dfrac{2}{7}$ **1**

$\therefore \quad$ AP : PB = 2 : 7

Hence $y = \dfrac{-8k+10}{k+1} = \dfrac{-8\times\frac{2}{7}+10}{\frac{2}{7}+1} = 6$ **1**

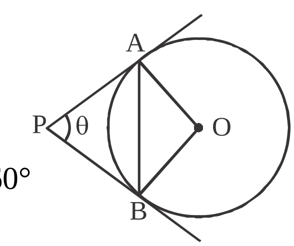
31. Prove that, a tangent to a circle is perpendicular to the radius through the point of contact.

Ans: Given, To prove, figure **1/2 × 3 = $1\frac{1}{2}$**

Correct proof **$1\frac{1}{2}$**

OR

Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.



cor. fig. 1/2

Ans: $\left.\begin{array}{l}\angle PAO = 90^\circ \text{ (radius} \perp \text{tangent)} \\ \angle PBO = 90^\circ\end{array}\right\}$ **1**

Now

$\angle PAO + \angle AOB + \angle OBP + \angle BPA = 360^\circ$

$\Rightarrow \; 90^\circ + \angle AOB + 90^\circ + \angle BPA = 360^\circ$ **1**

$\Rightarrow \; \angle AOB + \angle BPA = 180^\circ$

or $\angle AOB$ and $\angle BPA$ are supplementary. **1/2**

32. In a right triangle, prove that the square of the hypotenuse is equal to the sum of squares of the other two sides.

Ans: Correct given, To prove & figure **1/2 × 3 = $1\frac{1}{2}$**

Correct proof **$1\frac{1}{2}$**

33. If sin θ + cos θ = p and sec θ + cosec θ = q, show that $q(p^2 - 1) = 2p$.

	Ans: LHS = $q(p^2 - 1)$ = $(\sec\theta + \operatorname{cosec}\theta)((\sin\theta + \cos\theta)^2 - 1)$	
	$= \dfrac{\sin\theta + \cos\theta}{\sin\theta\cos\theta} \times 2\sin\theta\cos\theta$	1+1
	$= 2(\sin\theta + \cos\theta)$	1/2
	$= 2p = \text{RHS}$	1/2
34.	500 persons are taking dip into a cuboidal pond which is 80 m long and 50 m broad. What is the rise of water level in the pond, if the average displacement of the water by a person is 0.04 m^3 ?	
	Ans: Let the rise in the water level be h	
	$\therefore 500 \times .04 = 80 \times 50 \times h$	2
	$\Rightarrow h = \dfrac{500 \times .04}{80 \times 50}$	1/2
	$= .005$ m	1/2
	SECTION – D	
	Q. Nos. 35 to 40 carry 4 marks each.	
35.	Show that $(12)^n$ cannot end with digit 0 or 5 for any natural number n.	
	Ans: $12^n = (2^2 \times 3)^n = 2^{2n} \times 3^n$	2
	Since there is no factor of the form 5^m therefore 12^n can not end with digit 0 or 5 for any natural number n.	2
	OR	
	Prove that $\left(\sqrt{2} + \sqrt{5}\right)$ is irrational.	
	Ans: Let us assume $\sqrt{2} + \sqrt{5}$ is rational number	
	Let $\sqrt{2} + \sqrt{5} = m$ where m is rational	1
	$\Rightarrow \left(\sqrt{2} + \sqrt{5}\right)^2 = m^2$	1
	$\Rightarrow m^2 = 7 + 2\sqrt{10}$	
	$\Rightarrow \sqrt{10} = \dfrac{m^2 - 7}{2}$	1
	$\because$ m is rational	
	$\therefore \dfrac{m^2 - 7}{2}$ is also rational	
	but $\sqrt{10}$ is irrational	
	$\Rightarrow$ LHS $\neq$ RHS	
	It means our assumption was wrong.	
	Hence $\sqrt{2} + \sqrt{5}$ is an irrational number.	1

36. A train covered a certain distance at a uniform speed. If the train would have been 6 km/hr. faster, it would have taken 4 hours less than the scheduled time and if the train were slower by 6 km/hr., it would have taken 6 hrs. more than the scheduled time. Find the length of the journey.

Ans: Let usual speed of train be x km/hr and distance covered be d km.

Therefore $\frac{d}{x} - \frac{d}{x+6} = 4 \quad \ldots(i)$ **1**

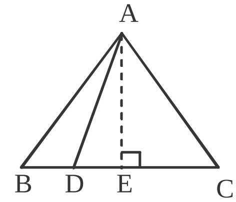
$\frac{d}{x-6} - \frac{d}{x} = 6 \quad \ldots(ii)$ **1**

Solving (i) and (ii) x = 30 and d = 720 **2**

$\therefore$ Length of journey = 720 km

37. In an equilateral triangle ABC, D is a point on the side BC such that

$BD = \frac{1}{3}BC$. Prove that $9\,AD^2 = 7\,AB^2$. **cor. fig 1/2**



Ans: Draw AE $\perp$ BC

$\because$ ΔABC is an equilateral Δ

$\therefore \quad BE = \frac{BC}{2}$ **1/2**

Now, $AD^2 = AE^2 + DE^2$ and $AB^2 = AE^2 + BE^2$ **1**

$\Rightarrow \quad AB^2 = AD^2 - DE^2 + BE^2$

$= AD^2 + (BE + DE)(BE - DE)$ **1/2**

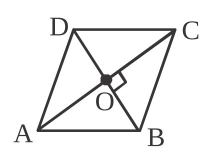
$= AD^2 + \frac{BC}{3} \times \left(\frac{BC}{2} + \frac{BC}{2} - \frac{BC}{3}\right)$

$= AD^2 + \frac{2}{9}BC^2 = AD^2 + \frac{2}{9}AB^2$ **1**

$\Rightarrow \quad 7AB^2 = 9AD^2$ **1/2**

OR

Prove that the sum of squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.



Ans: $AB^2 + BC^2 + CD^2 + AD^2$ **cor. fig 1/2**

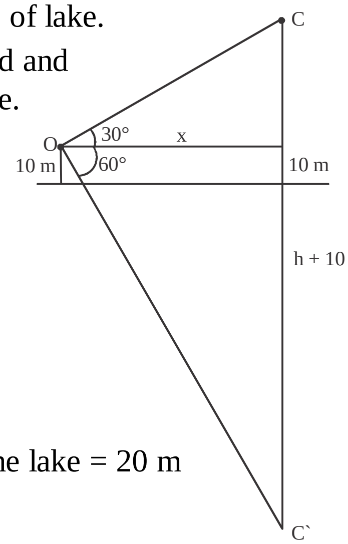
$= 4\,AB^2$ ($\because$ ABCD is a rhombus) **1**

$= 4\,(OA^2 + OB^2)$ **1**

$= 4\left(\frac{AC^2}{4} + \frac{BD^2}{4}\right)$ **1**

$= AC^2 + BD^2$ **1/2**

38. If the angle of elevation of a cloud from a point 10 metres above a lake is 30° and the angle of depression of its reflection in the lake is 60°, find the height of the cloud from the surface of lake.



Ans: Let C represents the position of cloud and C` represents its reflection in the lake. **cor. fig 1**

$\tan 30° = \frac{1}{\sqrt{3}} = \frac{h}{x}$ **1**

$\Rightarrow x = h\sqrt{3}$ … (i)

$\tan 60° = \sqrt{3} = \frac{h+20}{x}$ … (ii) **1**

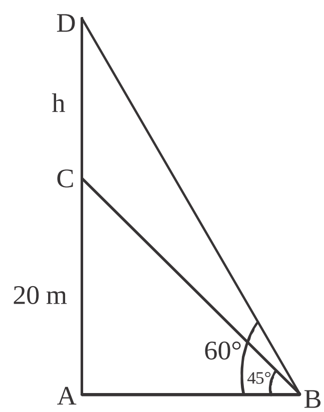
Solving (i) and (ii) h =10 **1/2**

$\therefore$ Height of cloud from surface of the lake = 20 m **1/2**

OR

A vertical tower of height 20 m stands on a horizontal plane and is surmounted by a vertical flag-staff of height h. At a point on the plane, the angle of elevation of the bottom and top of the flag staff are 45° and 60° respectively. Find the value of h.

Ans: Let AC be the tower and CD be the flag-staff. **cor. fig 1**



$\tan 45° = 1 = \frac{AC}{AB}$

$\Rightarrow AC = AB$ …(i) **1**

$\tan 60° = \sqrt{3} = \frac{AC + h}{AB}$

$\Rightarrow \sqrt{3}\,AB = AC + h$ …(ii) **1**

Using (i) and (ii)

$AC\left(\sqrt{3}-1\right) = h$

$\Rightarrow h = 20\left(\sqrt{3}-1\right)$ m **1**

39. A solid iron cuboidal block of dimensions 4.4 m × 2.6 m × 1 m is cast into a hollow cylindrical pipe of internal radius 30 cm and thickness 5 cm. Find the length of the pipe.

Ans: Internal radius of cylinder (r_2) = 30 cm = 0.30 m

Outer radius of cylinder (r_1) = 30 + 5 = 35 cm = 0.35 m **1**

Therefore $4.4 \times 2.6 \times 1 = \pi \times h \times \left((0.35)^2 - (.30)^2 \right)$ **2**

$= \pi \times h \times \frac{1}{100 \times 100} \times 65 \times 5$

$\Rightarrow \quad h = \frac{352}{\pi}$ m or 112 m **1**

40.	For the following frequency distribution, draw a cumulative frequency curve of 'more than' type and hence obtain the median value.		
	Ans: Plotting points (0, 100) (10, 95) (20, 80) (30, 60) (40, 37) (50, 20) (60, 9) and joining them.	**2** **$1\frac{1}{2}$**	
	Median = 34.3 (approx)	**1/2**	

Classes	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	5	15	20	23	17	11	9

QUESTION PAPER CODE 30/3/2
EXPECTED ANSWER/VALUE POINTS
SECTION – A

Question numbers 1 to 10 are multiple choice questions of 1 mark each.

You have to select the correct choice :

Q.No.		Marks
1.	In Fig. 1, the graph of the polynomial p(x) is given. The number of zeroes of the polynomial is	

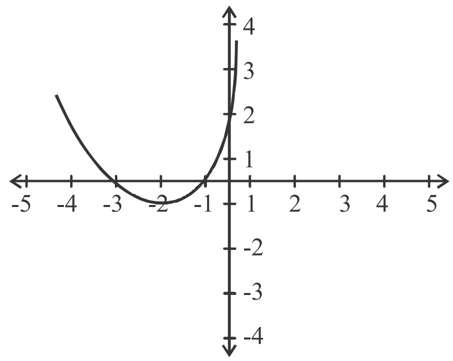
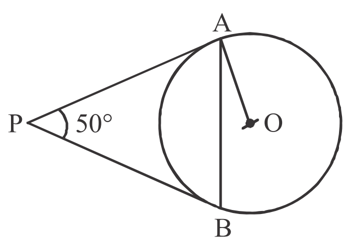
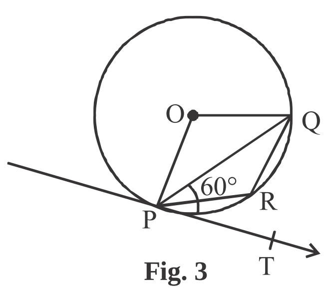


Fig. 1

Q.No.		Marks
	(a) 1 **(b)** 2 **(c)** 3 **(d)** 0	
	Ans: (b) 2	1
2.	If (a, b) is the mid-point of the line segment joining the points A(10, –6) and B(k, 4) and a – 2b = 18, the value of k is	
	(a) 30 **(b)** 22 **(c)** 4 **(d)** 40	
	Ans: (b) 22	1
3.	The value of k for which the points A (0, 1), B (2, k) and C(4, –5) are collinear is	
	(a) 2 **(b)** –2 **(c)** 0 **(d)** 4	
	Ans: (b) –2	1
4.	If $\Delta ABC \sim \Delta DEF$ such that AB = 1.2 cm and DE = 1.4 cm, the ratio of the areas of ΔABC and ΔDEF is	
	(a) 49 : 36 **(b)** 6 : 7 **(c)** 7 : 6 **(d)** 36 : 49	
	Ans: (d) 36 : 49	1
5.	The HCF of 135 and 225 is	
	(a) 15 **(b)** 75 **(c)** 45 **(d)** 5	
	Ans: (c) 45	1
6.	The exponent of 2 in the prime factorization of 144, is	
	(a) 2 **(b)** 4 **(c)** 1 **(d)** 6	
	Ans: (b) 4	1
7.	The common difference of an AP, whose n^{th} term is $a_n = (3n + 7)$, is	
	(a) 3 **(b)** 7 **(c)** 10 **(d)** 6	
	Ans: (a) 3	1

8.	The value of λ for which $(x^2 + 4x + \lambda)$ is a perfect square, is **(a)** 16 **(b)** 9 **(c)** 1 **(d)** 4 **Ans:** (d) 4	1
9.	The value of k, for which the pair of linear equations $kx + y = k^2$ and $x + ky = 1$ have infinitely many solutions is **(a)** ± 1 **(b)** 1 **(c)** -1 **(d)** 2 **Ans:** (b) 1	1
10.	The value of p for which $(2p + 1)$, 10 and $(5p + 5)$ are three consecutive terms of an AP is **(a)** -1 **(b)** -2 **(c)** 1 **(d)** 2 **Ans:** (d) 2	1
	OR The number of terms of an AP 5, 9, 13, … 185 is **(a)** 31 **(b)** 51 **(c)** 41 **(d)** 40 **Ans:** 1 mark should be given to each candidate.	1
	In Q. Nos. 11 to 15, fill in the blanks. Each question is of 1 mark :	
11.	$\dfrac{3\cot 40^\circ}{\tan 50^\circ} - \dfrac{1}{2}\left(\dfrac{\cos 35^\circ}{\sin 55^\circ}\right) =$ _________. **Ans:** $\dfrac{5}{2}$	1
12.	In Fig. 2, PA and PB are tangents to the circle with centre O such that $\angle APB = 50^\circ$, then the measure of $\angle OAB$ is _________.  **Fig. 2** **Ans:** 25°	1
	OR In Fig. 3, PQ is a chord of a circle and PT is tangent at P such that $\angle QPT = 60^\circ$, then the measure of $\angle PRQ$ is _________.  **Fig. 3** **Ans:** 120°	1

13.	The distance between two parallel tangents of a circle of radius 4 cm is __________. **Ans:** 8 cm	1
14.	The distance between the points $\left(-\frac{8}{5}, 2\right)$ and $\left(\frac{2}{5}, 2\right)$ is ____________. **Ans:** distance = 2	1
15.	If tan A = cot B, then A + B = ____________. **Ans:** A + B = 90°	1
	Q. Nos. 16 to 20 are short answer type questions of 1 mark each.	
16.	What is the arithmetic mean of first n natural numbers? **Ans:** Sum of first n natural numbers = $\frac{n(n+1)}{2}$ $\therefore$ Mean = $\frac{n+1}{2}$	1/2 1/2
17.	The probability that it will rain tomorrow is 0.85. What is the probability that it will not rain tomorrow ? **Ans:** Prob (no rain tomorrow) = 1 – 0.85 = 0.15	1/2 1/2
18.	Using the empirical formula, find the mode of a distribution whose mean is 8.32 and the median is 8.05. **Ans:** Mode = $3 \times 8.05 - 2 \times 8.32$ = 7.51	1/2 1/2
19.	Two right circular cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1, what is the ratio of their volumes? **Ans:** $V_1 : V_2 = \frac{1}{3}\pi (3r)^2 h : \frac{1}{3}\pi r^2 (3h)$ = 3 : 1	1/2 1/2
20.	If x = a sin θ and y = b cos θ, write the value of $(b^2 x^2 + a^2 y^2)$. **Ans:** $b^2 a^2 \sin^2\theta + a^2 b^2 \cos^2\theta$ $= a^2 b^2$	1/2 1/2
	SECTION – B	
	Q. Nos. 21 to 26 carry 2 marks each.	
21.	Read the following passage and answer the questions given at the end : Students of Class XII presented a gift to their school in the form of an electric lamp in the shape of a glass hemispherical base surmounted by a metallic cylindrical top of same radius 21 cm and height 3.5 cm. The top was silver coated and the glass surface was painted red. (i) What is the cost of silver coating the top at the rate of ₹ 5 per 100 cm^2 ? (ii) What is the surface area of glass to be painted red ?	

	Ans: (i) Surface Area of the top $= 2 \times \frac{22}{7} \times 21 \times 3.5 = 462\ cm^2$	1/2
	Cost of silver coating $= 462 \times \frac{5}{100} =$ Rs. 23.10	1/2
	(ii) Surface Area of glass $= 2 \times \frac{22}{7} \times 21 \times 21$	1/2
	$= 2772\ cm^2$	1/2
22.	If $\tan\theta = \frac{3}{4}$, find the value of $\left(\frac{1-\cos^2\theta}{1+\cos^2\theta}\right)$	
	Ans: $\sec^2\theta = 1+\frac{9}{16} = \frac{25}{16}$	
	$\therefore \cos^2\theta = \frac{16}{25}$	1
	Hence $\frac{1-\cos^2\theta}{1+\cos^2\theta} = \frac{1-\frac{16}{25}}{1+\frac{16}{25}} = \frac{9}{41}$	1
	OR	
	If $\tan\theta = \sqrt{3}$, find the value of $\left(\frac{2\sec\theta}{1+\tan^2\theta}\right)$	
	Ans: $\sec^2\theta = 1 + 3 = 4$	
	$\therefore \sec\theta = 2$	1
	Hence $\frac{2\sec\theta}{1+\tan^2\theta} = \frac{2\times 2}{4} = 1$	1
23.	Find the 11^{th} term from the last term (towards the first term) of the AP 12, 8, 4, …, –84.	
	Ans: $l = -84$	1/2
	$d = -4$	1/2
	t_{11} (from the end) $= -84 + 40 = -44$	1
	OR	
	Solve the equation : $1 + 5 + 9 + 13 + \ldots + x = 1326$	
	Ans: $\frac{n}{2}(1 + x) = 1326$ … (i)	1/2
	$x = 1 + (n-1) \times 4$ … (ii)	1/2
	Solving (i) and (ii) $x = 101$	1

24. Find the value of p, if the mean of the following distribution is 7.5.

Classes	2-4	4-6	6-8	8-10	10-12	12-14
Frequency (fi)	6	8	15	p	8	4

Ans:

Class	Frequence (f)	x	fx
2-4	6	3	18
4-6	8	5	40
6-8	15	7	105
8-10	p	9	9p
10-12	8	11	88
12-14	4	13	52
	41 + p		303 + 9p

Cor. tab = 1

Mean = $7.5 = \frac{303+9p}{41+p} \Rightarrow p = 3$ **1**

25. In a family of 3 children, find the probability of having at least one boy.

Ans: Total number of outcomes = 8 **1/2**

Number of Favourable outcomes = 7 **1/2**

Probability (having at least one boy) = $\frac{7}{8}$ **1**

26. In Fig. 4, PA is a tangent from an external point P to a circle with centre O. If $\angle POB = 115°$, find $\angle APO$.

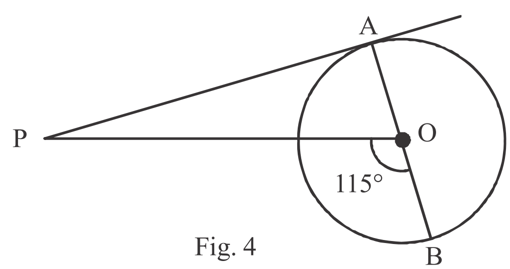


Fig. 4

Ans: $\angle POA = 180° - 115° = 65°$ **1**

$\because$ $OA \perp AP$ **1/2**

therefore $\angle APO = 90° - 65° = 25°$ **1/2**

SECTION – C

Q. Nos. 27 to 34 carry 3 marks each.

27. 500 persons are taking dip into a cuboidal pond which is 80 m long and 50 m broad. What is the rise of water level in the pond, if the average displacement of the water by a person is 0.04 m³ ?

Ans: Let the rise in the water level be h

$\therefore$ $500 \times .04 = 80 \times 50 \times h$ **2**

$\Rightarrow h = \frac{500 \times .04}{80 \times 50}$ **1/2**

$= .005$ m **1/2**

28. If $\sin\theta + \cos\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$, show that $q(p^2 - 1) = 2p$.

Ans: LHS $= q(p^2-1) = (\sec\theta + \operatorname{cosec}\theta)\left((\sin\theta+\cos\theta)^2 - 1\right)$

$$= \frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta} \times 2\sin\theta\cos\theta$$ **1+1**

$= 2(\sin\theta + \cos\theta)$ **1/2**

$= 2p = \text{RHS}$ **1/2**

29. Prove that, a tangent to a circle is perpendicular to the radius through the point of contact.

Ans: Given, To prove, figure **$1/2 \times 3 = 1\frac{1}{2}$**

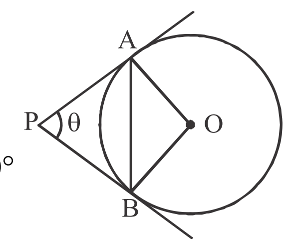
Correct proof **$1\frac{1}{2}$**

OR

Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

Ans: $\left.\begin{array}{l}\angle PAO = 90^\circ \text{ (radius} \perp \text{tangent)} \\ \angle PBO = 90^\circ\end{array}\right\}$ **cor. fig. 1/2**

1



Now

$\angle PAO + \angle AOB + \angle OBP + \angle BPA = 360^\circ$

$\Rightarrow 90^\circ + \angle AOB + 90^\circ + \angle BPA = 360^\circ$ **1**

$\Rightarrow \angle AOB + \angle BPA = 180^\circ$

or $\angle AOB$ and $\angle BPA$ are supplementary. **1/2**

30. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$ respectively. Find $g(x)$.

Ans: $x^3 - 3x^2 + x + 2 = (x-2) \times g(x) + (-2x + 4)$ **1**

$\Rightarrow \quad (x-2)\, g(x) = x^3 - 3x^2 + 3x - 2$ **1/2**

$$\Rightarrow \quad g(x) = \frac{(x-2)(x^2 - x + 1)}{(x-2)}$$ **1**

$= x^2 - x + 1$ **1/2**

OR

If the sum of the squares of zeros of the quadratic polynomial $f(x) = x^2 - 8x + k$ is 40, find the value of k.

Ans: Let the zeroes of polynomial $f(x)$ be α and β.

$\therefore \quad \alpha + \beta = 8$ and $\alpha\beta = k$ **1/2+1/2**

$\because \quad \alpha^2 + \beta^2 = 40$

$\Rightarrow \quad (\alpha + \beta)^2 - 2\alpha\beta = 40$ **1**

$\Rightarrow \quad 64 - 2k = 40$ **1/2**

$\Rightarrow \quad k = 12$ **1/2**

31.	Find a, b and c if it is given that the numbers a, 7, b, 23, c are in AP. **Ans:** a, 7, b, 23, c are in A.P Let d be the common difference of AP. $\therefore\ a + d = 7 \quad \ldots (i)$ $a + 3d = 23 \quad \ldots (ii)$ Solving (i) & (ii), d = 8 $\therefore\ a = -1,\ b = 15,\ c = 31$	 1/2 1/2 1/2 1/2 + 1/2 + 1/2
	OR	
	If m times the m^{th} term of an AP is equal to n times its nth term, show that the $(m + n)^{th}$ term of the AP is zero. **Ans:** Given $m[a + (m - 1)d] = n[a + (n - 1)d]$ $\Rightarrow\ a(m - n) + d(m^2 - m - n^2 + n) = 0$ $\Rightarrow\ (m - n)\,[a + (m + n - 1)\,d = 0$ $\because\ m \neq n \Rightarrow a + (m + n - 1)\,d = 0$ $\Rightarrow a_{m+n} = 0$	 1 1 1/2 1/2
32.	Solve for x : $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30};\ x \neq -4, 7$ **Ans:** $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$ $\Rightarrow -11 \times 30 = 11\,(x^2 - 3x - 28)$ $\Rightarrow x^2 - 3x + 2 = 0$ $\Rightarrow (x - 2)(x - 1) = 0$ $\Rightarrow x = 2, 1$	 $1\frac{1}{2}$ 1 1/2
33.	Show that the points A (–1, 1), B(5, 7) and C(8, 10) are collinear. **Ans:** Points A (–1, 1), B(5, 7) and C(8, 10) are collinear. if Ar (ΔABC) = 0 $\text{Ar}(\Delta ABC) = \frac{1}{2}\left[(-1)(7-10) + 5(10-1) + 8(1-7)\right]$ $= \frac{1}{2}\left[3 + 45 - 48\right] = 0$ $\therefore$ Points A, B, C are collinear	 1 1 1
34.	If the areas of two similar triangles are equal, then prove that the triangles are congruent. **Ans:** Let the two triangles be ΔABC, ΔDEF such that $\Delta ABC \sim \Delta DEF$ and $\text{Ar}(\Delta ABC) = \text{Ar}(\Delta DEF)$ $\Rightarrow \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2} = \frac{\text{Ar}(ABC)}{\text{Ar}(DEF)}$ $\Rightarrow \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2} = 1$	 1/2 1/2 1

$\Rightarrow \quad AB = DE,\ BC = EF\ \&\ AC = DF$ 1/2

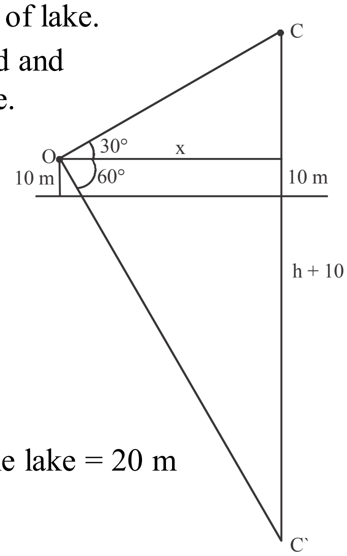
$\Rightarrow \quad \Delta ABC \cong \Delta DEF$ 1/2

SECTION – D

Q. Nos. 35 to 40 carry 4 marks each.

35. If the angle of elevation of a cloud from a point 10 metres above a lake is 30° and the angle of depression of its reflection in the lake is 60°, find the height of the cloud from the surface of lake.

Ans: Let C represents the position of cloud and C\` represents its reflection in the lake. cor. fig 1



$\tan 30° = \frac{1}{\sqrt{3}} = \frac{h}{x}$ 1

$\Rightarrow x = h\sqrt{3} \qquad \ldots \text{(i)}$

$\tan 60° = \sqrt{3} = \frac{h+20}{x} \ldots \text{(ii)}$ 1

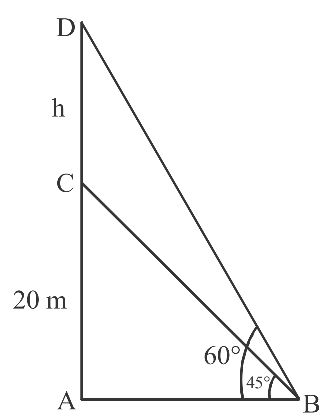
Solving (i) and (ii) h =10 1/2

$\therefore$ Height of cloud from surface of the lake = 20 m 1/2

OR

A vertical tower of height 20 m stands on a horizontal plane and is surmounted by a vertical flag-staff of height h. At a point on the plane, the angle of elevation of the bottom and top of the flag staff are 45° and 60° respectively. Find the value of h.

Ans: Let AC be the tower and CD be the flag-staff. cor. fig 1



$\tan 45° = 1 = \frac{AC}{AB}$

$\Rightarrow AC = AB \qquad \ldots \text{(i)}$ 1

$\tan 60° = \sqrt{3} = \frac{AC + h}{AB}$

$\Rightarrow \sqrt{3}\ AB = AC + h \qquad \ldots \text{(ii)}$ 1

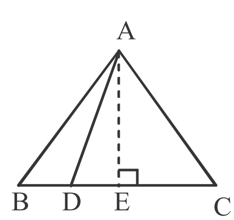
Using (i) and (ii)

$AC\left(\sqrt{3} - 1\right) = h$

$\Rightarrow h = 20\left(\sqrt{3} - 1\right) m$ 1

36. In an equilateral triangle ABC, D is a point on the side BC such that $BD = \frac{1}{3} BC$. Prove that $9\ AD^2 = 7\ AB^2$.

Ans: Draw $AE \perp BC$ cor. fig 1/2



$\because \Delta ABC$ is an equilateral Δ

$\therefore \quad BE = \frac{BC}{2}$ 1/2

Now, $AD^2 = AE^2 + DE^2$ and $AB^2 = AE^2 + BE^2$ **1**

$\Rightarrow \quad AB^2 = AD^2 - DE^2 + BE^2$

$= AD^2 + (BE + DE)(BE - DE)$ **1/2**

$$= AD^2 + \frac{BC}{3} \times \left(\frac{BC}{2} + \frac{BC}{2} - \frac{BC}{3}\right)$$

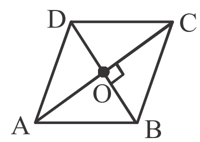
$$= AD^2 + \frac{2}{9}BC^2 = AD^2 + \frac{2}{9}AB^2$$ **1**

$\Rightarrow \quad 7AB^2 = 9AD^2$ **1/2**

OR

Prove that the sum of squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

Ans: $AB^2 + BC^2 + CD^2 + AD^2$ **cor. fig 1/2**



$= 4\,AB^2$ ($\because$ ABCD is a rhombus) **1**

$= 4\,(OA^2 + OB^2)$ **1**

$$= 4\left(\frac{AC^2}{4} + \frac{BD^2}{4}\right)$$ **1**

$= AC^2 + BD^2$ **1/2**

37. Show that $(12)^n$ cannot end with digit 0 or 5 for any natural number n.

Ans: $12^n = (2^2 \times 3)^n = 2^{2n} \times 3^n$ **2**

Since there is no factor of the form 5^m therefore 12^n can not end with digit 0 or 5 for any natural number n. **2**

OR

Prove that $\left(\sqrt{2} + \sqrt{5}\right)$ is irrational.

Ans: Let us assume $\sqrt{2} + \sqrt{5}$ is rational number

Let $\sqrt{2} + \sqrt{5} = m$ where m is rational **1**

$\Rightarrow \left(\sqrt{2} + \sqrt{5}\right)^2 = m^2$ **1**

$\Rightarrow m^2 = 7 + 2\sqrt{10}$

$\Rightarrow \sqrt{10} = \dfrac{m^2 - 7}{2}$ **1**

$\because$ m is rational

$\therefore \dfrac{m^2 - 7}{2}$ is also rational

but $\sqrt{10}$ is irrational

$\Rightarrow$ LHS $\neq$ RHS

It means our assumption was wrong. **1**

Hence $\sqrt{2} + \sqrt{5}$ is an irrational number.

38. For the following frequency distribution, draw a cumulative frequency curve of 'more than' type and hence obtain the median value.

Classes	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	5	15	20	23	17	11	9

Ans: Plotting points (0, 100) (10, 95) (20, 80) (30, 60) (40, 37) (50, 20), (60, 9) **2**

and joining them. **$1\frac{1}{2}$**

Median = 34.3 (approx) **1/2**

39. A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

Ans: Let the fraction be $\frac{x}{y}, y \neq 0$ **1/2**

Here $\frac{x-1}{y} = \frac{1}{3}$ **1**

and $\frac{x}{y+8} = \frac{1}{4}$ **1**

$\Rightarrow$ $3x - y = 3$ ...(i)

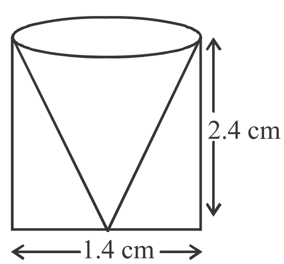
and $4x - y = 8$ ...(ii)

Solving (i) and (ii) $x = 5, y = 12$ **1/2+1/2**

$\therefore$ Fraction is $\frac{5}{12}$ **1/2**

40. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of same height and same diameter is hollowed out. Find the total surface area of the remaining solid. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Ans: Radius = 0.7 cm **1/2**



Total Surface Area = $2\pi rh + \pi r^2 + \pi r l$ **1**

Here r = 0.7 cm , h = 2.4 cm

$\therefore$ $l = \sqrt{.49 + 5.76} = 2.5$ cm **1**

TSA = $\frac{22}{7}[2 \times .7 \times 2.4 + .49 + 0.7 \times 2.5]$ **1**

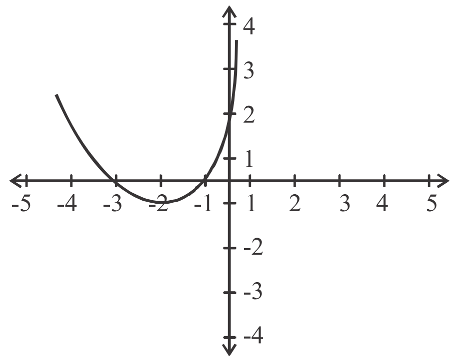
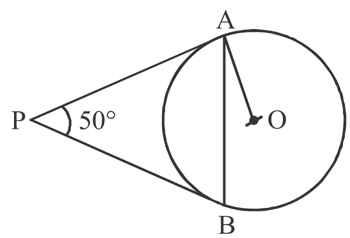
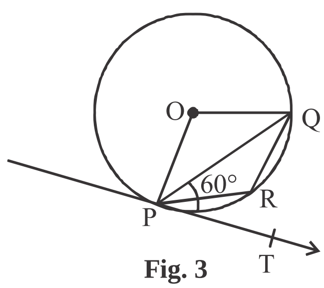
= 17.6 cm^2 **1/2**

QUESTION PAPER CODE 30/3/3
EXPECTED ANSWER/VALUE POINTS
SECTION – A

Question numbers 1 to 10 are multiple choice questions of 1 mark each.

You have to select the correct choice :

Q.No.		Marks
1.	The exponent of 2 in the prime factorization of 144, is	
	(a) 2 **(b)** 4 **(c)** 1 **(d)** 6	
	Ans: (b) 4	1
2.	The common difference of an AP, whose n^{th} term is $a_n = (3n + 7)$, is	
	(a) 3 **(b)** 7 **(c)** 10 **(d)** 6	
	Ans: (a) 3	1
3.	The HCF of 135 and 225 is	
	(a) 15 **(b)** 75 **(c)** 45 **(d)** 5	
	Ans: (c) 45	1
4.	If $\Delta ABC \sim \Delta DEF$ such that AB = 1.2 cm and DE = 1.4 cm, the ratio of the areas of ΔABC and ΔDEF is	
	(a) 49 : 36 **(b)** 6 : 7 **(c)** 7 : 6 **(d)** 36 : 49	
	Ans: (d) 36 : 49	1
5.	The value of λ for which $(x^2 + 4x + \lambda)$ is a perfect square, is	
	(a) 16 **(b)** 9 **(c)** 1 **(d)** 4	
	Ans: (d) 4	1
6.	The value of k, for which the pair of linear equations $kx + y = k^2$ and $x + ky = 1$ have infinitely many solutions is	
	(a) ± 1 **(b)** 1 **(c)** -1 **(d)** 2	
	Ans: (b) 1	1
7.	The value of k for which the points A (0, 1), B (2, k) and C(4, –5) are collinear is	
	(a) 2 **(b)** –2 **(c)** 0 **(d)** 4	
	Ans: (b) –2	1
8.	The value of p for which (2p + 1), 10 and (5p + 5) are three consecutive terms of an AP is	
	(a) –1 **(b)** –2 **(c)** 1 **(d)** 2	
	Ans: (d) 2	1
	OR	
	The number of terms of an AP 5, 9, 13, … 185 is	
	(a) 31 **(b)** 51 **(c)** 41 **(d)** 40	
	Ans: 1 mark should be given to each candidate.	1
9.	If (a, b) is the mid-point of the line segment joining the points A(10, –6) and B(k, 4) and a – 2b = 18, the value of k is	
	(a) 30 **(b)** 22 **(c)** 4 **(d)** 40	
	Ans: (b) 22	1

10.	In Fig. 1, the graph of the polynomial p(x) is given. The number of zeroes of the polynomial is  **Fig. 1** **(a)** 1 **(b)** 2 **(c)** 3 **(d)** 0 **Ans:** (b) 2	1
	In Q. Nos. 11 to 15, fill in the blanks. Each question is of 1 mark :	
11.	In Fig. 2, PA and PB are tangents to the circle with centre O such that $\angle APB = 50°$, then the measure of $\angle OAB$ is __________.  **Fig. 2** **Ans:** 25°	1
	OR In Fig. 3, PQ is a chord of a circle and PT is tangent at P such that $\angle QPT = 60°$, then the measure of $\angle PRQ$ is __________.  **Fig. 3** **Ans:** 120°	1
12.	$\dfrac{3\cot 40°}{\tan 50°} - \dfrac{1}{2}\left(\dfrac{\cos 35°}{\sin 55°}\right) =$ __________. **Ans:** $\dfrac{5}{2}$	1

13.	The distance between two parallel tangents of a circle of radius 4 cm is __________. **Ans:** 8 cm	1
14.	The distance of the point (–3, 4) from Y-axis is __________. **Ans:** 3	1
15.	Value of $\dfrac{2\tan^2 60°}{1+\tan^2 30°}$ is __________. **Ans:** $\dfrac{9}{2}$	1
16.	The probability that it will rain tomorrow is 0.85. What is the probability that it will not rain tomorrow ? **Ans:** Prob (no rain tomorrow) $= 1 - 0.85$ $= 0.15$	1/2 1/2
17.	What is the arithmetic mean of first n natural numbers? **Ans:** Sum of first n natural numbers $= \dfrac{n(n+1)}{2}$ $\therefore$ Mean $= \dfrac{n+1}{2}$	1/2 1/2
18.	Two right circular cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1, what is the ratio of their volumes? **Ans:** $V_1 : V_2 = \frac{1}{3}\pi (3r)^2 h : \frac{1}{3}\pi r^2 (3h)$ $= 3 : 1$	1/2 1/2
19.	Using the empirical formula, find the mode of a distribution whose mean is 8.32 and the median is 8.05. **Ans:** Mode $= 3 \times 8.05 - 2 \times 8.32$ $= 7.51$	1/2 1/2
20.	Evaluate (sec A + tan A) · (1 – sin A) for A = 60° **Ans:** $\left(2+\sqrt{3}\right)\left(1-\dfrac{\sqrt{3}}{2}\right)$ $= \dfrac{1}{2}$	1/2 1/2

SECTION – B

Q. Nos. 21 to 26 carry 2 marks each.

21. Find the value of p, if the mean of the following distribution is 7.5.

Classes	2-4	4-6	6-8	8-10	10-12	12-14
Frequency (fi)	6	8	15	p	8	4

Ans:

Class	Frequence (f)	x	fx
2-4	6	3	18
4-6	8	5	40
6-8	15	7	105
8-10	p	9	9p
10-12	8	11	88
12-14	4	13	52
	41 + p		303 + 9p

Correct table = 1

Mean = 7.5 = $\frac{303+9p}{41+p} \Rightarrow p = 3$ **1**

22. Read the following passage and answer the questions given at the end :

Students of Class XII presented a gift to their school in the form of an electric lamp in the shape of a glass hemispherical base surmounted by a metallic cylindrical top of same radius 21 cm and height 3.5 cm. The top was silver coated and the glass surface was painted red.

(i) What is the cost of silver coating the top at the rate of ₹ 5 per 100 cm^2 ?

(ii) What is the surface area of glass to be painted red ?

Ans: (i) Surface Area of the top = $2 \times \frac{22}{7} \times 21 \times 3.5 = 462 \text{ cm}^2$ **1/2**

Cost of silver coating = $462 \times \frac{5}{100}$ = Rs. 23.10 **1/2**

(ii) Surface Area of glass = $2 \times \frac{22}{7} \times 21 \times 21$ **1/2**

$= 2772 \text{ cm}^2$ **1/2**

23. Find the 11[th] term from the last term (towards the first term) of the AP 12, 8, 4, …, –84.

Ans: $l = -84$ **1/2**

$d = -4$ **1/2**

t_{11} (from the end) = –84 + 40 = –44 **1**

OR

Solve the equation : 1 + 5 + 9 + 13 + … + x = 1326

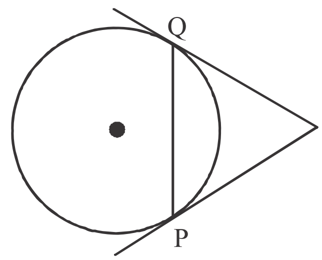
Ans: $\frac{n}{2}(1 + x) = 1326$ … (i) **1/2**

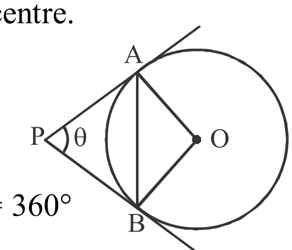
$x = 1 + (n - 1) \times 4$ … (ii) **1/2**

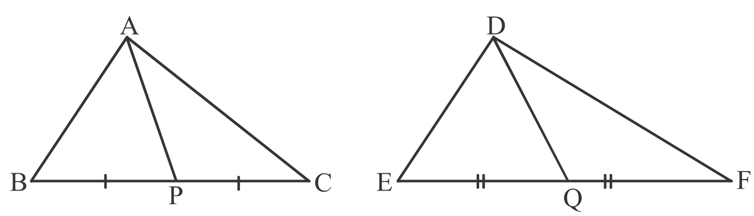
Solving (i) and (ii) x = 101 **1**

24. If $\tan\theta = \frac{3}{4}$, find the value of $\left(\frac{1-\cos^2\theta}{1+\cos^2\theta}\right)$

Ans: $\sec^2\theta = 1+\frac{9}{16} = \frac{25}{16}$

	$\therefore \cos^2\theta = \frac{16}{25}$	1
	Hence $\frac{1-\cos^2\theta}{1+\cos^2\theta} = \frac{1-\frac{16}{25}}{1+\frac{16}{25}} = \frac{9}{41}$	1
	OR	
	If $\tan\theta = \sqrt{3}$, find the value of $\left(\frac{2\sec\theta}{1+\tan^2\theta}\right)$	
	Ans: $\sec^2\theta = 1 + 3 = 4$	
	$\therefore \sec\theta = 2$	1
	Hence $\frac{2\sec\theta}{1+\tan^2\theta} = \frac{2\times 2}{4} = 1$	1
25.	Prove that the tangents at the extremities of any chord of a circle make equal angles with the chord.	Cor. fig. 1/2
		
	Ans: Here TQ = TP	1/2
	$\therefore$ ΔTQP is isosceles	1/2
	Hence $\angle TQP = \angle TPQ$	1/2
26.	Two dice are thrown together once. Find the probability of getting a sum of more than 9.	
	Ans: Total number of outcomes = 36	1/2
	Favourable outcomes are (5, 5), (4, 6), (6, 4), (6, 5), (5, 6), (6, 6) i.e. 6 outcomes.	1/2
	Prob. (sum > 9) = $\frac{6}{36} = \frac{1}{6}$	1
	SECTION – C	
	Q. Nos. **27** to **34** carry **3** marks each.	
27.	500 persons are taking dip into a cuboidal pond which is 80 m long and 50 m broad. What is the rise of water level in the pond, if the average displacement of the water by a person is 0.04 m^3 ?	
	Ans: Let the rise in the water level be h	
	$\therefore 500 \times .04 = 80 \times 50 \times h$	2
	$\Rightarrow h = \frac{500\times .04}{80\times 50}$	1/2
	$= .005$ m	1/2

28.	If $\sin\theta + \cos\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$, show that $q(p^2 - 1) = 2p$. **Ans:** LHS $= q(p^2 - 1) = (\sec\theta + \operatorname{cosec}\theta)((\sin\theta + \cos\theta)^2 - 1)$ $= \dfrac{\sin\theta + \cos\theta}{\sin\theta\cos\theta} \times 2\sin\theta\cos\theta$ $= 2(\sin\theta + \cos\theta)$ $= 2p = \text{RHS}$	1+1 1/2 1/2
29.	Prove that, a tangent to a circle is perpendicular to the radius through the point of contact. **Ans:** Given, To prove, figure Correct proof	$1/2 \times 3 = 1\frac{1}{2}$ $1\frac{1}{2}$
	OR	
	Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.  **Ans:** $\left.\begin{array}{l}\angle PAO = 90^\circ \text{ (radius} \perp \text{tangent)} \\ \angle PBO = 90^\circ\end{array}\right\}$ Now $\angle PAO + \angle AOB + \angle OBP + \angle BPA = 360^\circ$ $\Rightarrow 90^\circ + \angle AOB + 90^\circ + \angle BPA = 360^\circ$ $\Rightarrow \angle AOB + \angle BPA = 180^\circ$ or $\angle AOB$ and $\angle BPA$ are supplementary.	cor. fig. 1/2 1 1 1/2
30.	On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$ respectively. Find $g(x)$. **Ans:** $x^3 - 3x^2 + x + 2 = (x - 2) \times g(x) + (-2x + 4)$ $\Rightarrow \quad (x - 2)\,g(x) = x^3 - 3x^2 + 3x - 2$ $\Rightarrow \quad g(x) = \dfrac{(x-2)(x^2 - x + 1)}{(x-2)}$ $= x^2 - x + 1$	1 1/2 1 1/2
	OR	
	If the sum of the squares of zeros of the quadratic polynomial $f(x) = x^2 - 8x + k$ is 40, find the value of k. **Ans:** Let the zeroes of polynomial f(x) be α and β. $\therefore \quad \alpha + \beta = 8$ and $\alpha\beta = k$ $\because \quad \alpha^2 + \beta^2 = 40$ $\Rightarrow \quad (\alpha + \beta)^2 - 2\alpha\beta = 40$ $\Rightarrow \quad 64 - 2k = 40$ $\Rightarrow \quad k = 12$	1/2+1/2 1 1/2 1/2

31.	Find a, b and c if it is given that the numbers a, 7, b, 23, c are in AP.	
	Ans: a, 7, b, 23, c are in A.P	
	Let d be the common difference of AP.	
	$\therefore\ a + d = 7 \quad \ldots \text{(i)}$	1/2
	$a + 3d = 23 \quad \ldots \text{(ii)}$	1/2
	Solving (i) & (ii) d = 8	1/2
	$\Rightarrow a = -1,\ b = 15,\ c = 31$	1/2 + 1/2 + 1/2
	OR	
	If m times the m^{th} term of an AP is equal to n times its nth term, show that the $(m + n)^{th}$ term of the AP is zero.	
	Ans: Given $m[a + (m-1)d] = n[a + (n-1)d]$	1
	$\Rightarrow a(m-n) + d(m^2 - m - n^2 + n) = 0$	
	$\Rightarrow (m-n)\,[a + (m+n-1)\,d = 0$	1
	$\because m \neq n \Rightarrow a + (m+n-1)\,d = 0$	1/2
	$\Rightarrow a_{m+n} = 0$	1/2
32.	Find the values of k for which the points A(k + 1, 2k), B(3k, 2k + 3) and C(5k – 1,5k) are collinear.	
	Ans: Points A, B, C are collinear	
	$\Rightarrow (k+1)(2k+3-5k) + 3k(5k-2k) + (5k-1)(2k-2k-3) = 0$	1
	$\Rightarrow 6k^2 - 15k + 6 = 0$	1
	$\Rightarrow (k-2)(2k-1) = 0$	1/2
	$\Rightarrow k = 2, \frac{1}{2}$	1/2
33.	Prove that the ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding medians.	
	Ans: 	cor. fig. 1/2
	Here $\Delta ABC \sim \Delta DEF$	
	$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{2\,BP}{2\,EQ} = \frac{BP}{EQ}$ & $\angle B = \angle E$	
	$\therefore \Delta ABP \sim \Delta DEQ$	1
	$\Rightarrow \frac{AB}{DE} = \frac{AP}{DQ}$	1/2
	$\Rightarrow \frac{AB^2}{DE^2} = \frac{AP^2}{DQ^2}$	
	$\Rightarrow \frac{Ar(\Delta ABC)}{Ar(\Delta DEF)} = \frac{AP^2}{DQ^2} \quad (\because \Delta ABC \sim \Delta DEF)$	1

34. Find the value of k for which the quadratic equation $kx^2 + 1 - 2(k-1)x + x^2 = 0$ has equal roots. Hence find the roots of the equation.

Ans: Equation can be written as

$$(k+1)x^2 - 2(k-1)x + 1 = 0$$

For equal roots $4(k-1)^2 - 4(k+1) = 0$ **1**

$\Rightarrow \quad k^2 - 3k = 0$

$\Rightarrow \quad k(k-3) = 0$

$\Rightarrow \quad k = 0, 3$ **1**

For $k = 0$, equation is $x^2 + 2x + 1 = 0$

$\Rightarrow \quad x = -1, -1$ **1/2**

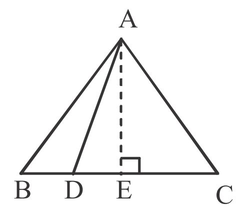
For $k = 3$, equation is $4x^2 - 4x + 1 = 0$

$\Rightarrow \quad x = \frac{1}{2}, \frac{1}{2}$ **1/2**

SECTION – D

Q. Nos. 35 to 40 carry 4 marks each.

35. In an equilateral triangle ABC, D is a point on the side BC such that $BD = \frac{1}{3}BC$. Prove that $9\,AD^2 = 7\,AB^2$.



Ans: Draw $AE \perp BC$ **cor. fig 1/2**

$\because$ ΔABC is an equilateral Δ

$\therefore \quad BE = \frac{BC}{2}$ **1/2**

Now, $AD^2 = AE^2 + DE^2$ and $AB^2 = AE^2 + BE^2$ **1**

$\Rightarrow \quad AB^2 = AD^2 - DE^2 + BE^2$

$= AD^2 + (BE + DE)(BE - DE)$ **1/2**

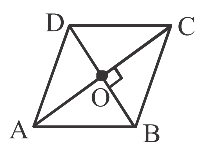
$= AD^2 + \frac{BC}{3} \times \left(\frac{BC}{2} + \frac{BC}{2} - \frac{BC}{3}\right)$

$= AD^2 + \frac{2}{9}BC^2 = AD^2 + \frac{2}{9}AB^2$ **1**

$\Rightarrow \quad 7AB^2 = 9AD^2$ **1/2**

OR

Prove that the sum of squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.



Ans: $AB^2 + BC^2 + CD^2 + AD^2$ **cor. fig 1/2**

$= 4\,AB^2$ ($\because$ ABCD is a rhombus) **1**

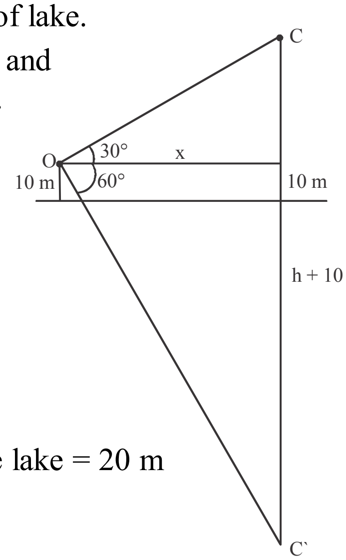
$= 4\,(OA^2 + OB^2)$ **1**

$= 4\left(\frac{AC^2}{4} + \frac{BD^2}{4}\right)$ **1**

$= AC^2 + BD^2$ **1/2**

36. If the angle of elevation of a cloud from a point 10 metres above a lake is 30° and the angle of depression of its reflection in the lake is 60°, find the height of the cloud from the surface of lake.

Ans: Let C represents the position of cloud and C\` represents its reflection in the lake. **cor. fig 1**



$$\tan 30° = \frac{1}{\sqrt{3}} = \frac{h}{x}$$ **1**

$\Rightarrow x = h\sqrt{3}$ … (i)

$$\tan 60° = \sqrt{3} = \frac{h+20}{x} \text{ … (ii)}$$ **1**

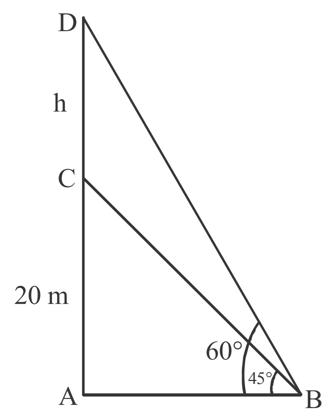
Solving (i) and (ii) h =10 **1/2**

$\therefore$ Height of cloud from surface of the lake = 20 m **1/2**

OR

A vertical tower of height 20 m stands on a horizontal plane and is surmounted by a vertical flag-staff of height h. At a point on the plane, the angle of elevation of the bottom and top of the flag staff are 45° and 60° respectively. Find the value of h.

Ans: Let AC be the tower and CD be the flag-staff. **cor. fig 1**



$$\tan 45° = 1 = \frac{AC}{AB}$$

$\Rightarrow AC = AB$ …(i) **1**

$$\tan 60° = \sqrt{3} = \frac{AC + h}{AB}$$

$\Rightarrow \sqrt{3}\, AB = AC + h$ …(ii) **1**

Using (i) and (ii)

$AC\left(\sqrt{3} - 1\right) = h$

$\Rightarrow h = 20\left(\sqrt{3} - 1\right)$ m **1**

37. Show that $(12)^n$ cannot end with digit 0 or 5 for any natural number n.

Ans: $12^n = (2^2 \times 3)^n = 2^{2n} \times 3^n$ **2**

Since there is no factor of the form 5^m therefore 12^n can not end with digit 0 or 5 for any natural number n. **2**

OR

Prove that $\left(\sqrt{2} + \sqrt{5}\right)$ is irrational.

Ans: Let us assume $\sqrt{2} + \sqrt{5}$ is rational number **1**

Let $\sqrt{2} + \sqrt{5} = m$ where m is rational

$\Rightarrow \left(\sqrt{2} + \sqrt{5}\right)^2 = m^2$ **1**

$\Rightarrow m^2 = 7 + 2\sqrt{10}$

	$\Rightarrow \sqrt{10} = \frac{m^2 - 7}{2}$	1
	$\because$ m is rational	
	$\therefore \frac{m^2 - 7}{2}$ is also rational	
	but $\sqrt{10}$ is irrational	
	$\Rightarrow$ LHS $\neq$ RHS	
	It means our assumption was wrong.	
	Hence $\sqrt{2} + \sqrt{5}$ is an irrational number.	1
38.	For the following frequency distribution, draw a cumulative frequency curve of 'more than' type and hence obtain the median value.	

Classes	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	5	15	20	23	17	11	9

	Ans: Plotting points (0, 100) (10, 95) (20, 80) (30, 60) (40, 37) (50, 20) (60, 9)	2
	and joining them.	$1\frac{1}{2}$
	Median = 34.3 (approx)	1/2
39.	If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?	
	Ans: Let the fraction be $\frac{x}{y}$, $y \neq 0$.	1/2
	Here $\frac{x+1}{y-1} = 1$.	1
	and $\frac{x}{y+1} = \frac{1}{2}$.	1
	$\Rightarrow 2x - y = 1 \ldots$ (i)	
	and $x - y = -2 \ldots$ (ii)	
	Solving (i) & (ii)	
	$x = 3, y = 5$	1/2+1/2
	$\therefore$ fraction is $\frac{3}{5}$	1/2

40.	A hemispherical depression is cut out from one face of a cuboidal block of side 7 cm such that the diameter of the hemisphere is equal to the edge of the cube. Find the surface area of the remaining solid.	
	Ans: Here r = $\frac{7}{2}$ cm	1/2
	Total Surface Area = $\left(5\times 7^2\right)+\left(7^2-\frac{49}{4}\pi\right)+2\times\pi\frac{49}{4}$	2
	$=\left(245+49+\frac{49}{4}\pi\right)\text{cm}^2$	1
	$=\left(294+49+\frac{49}{4}\pi\right)\text{cm}^2$	
	$=332.5$ cm^3 (approx)	1/2