JEE MAIN

COURSE
NUCLEUS

TEST CODE				
1	1	2	6	9

MOCK TEST-4

Class: **XII** ***Time:*** **3 Hours.** ***Max. Marks:*** **360**

IMPORTANT INSTRUCTIONS

1. The question paper consists of **'90'** objective type questions. There are **'30'** questions each in **Chemistry**, **Physics** and **Mathematics** respectively. **Please fill the OMR answer Sheet accordingly and carefully.**

2. Each question has four choices (1), (2), (3) and (4) out of which **ONLY ONE** is correct.

3. You will be **awarded 4 marks** for each question, if you have darkened only the bubble corresponding to the correct answer and zero mark if no bubble are darkened. In all other cases, **minus one (–1) mark** will be awarded.

4. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 3 above.

5. Use **Black or Blue Ball Point Pen** only for filling particulars.

6. Use of **Calculator, Log Table, Slide Rule and Mobile** is not allowed.

7. Rough work is to be done on the space provided at the bottom and in end of the booklet for this purpose in the Test Booklet only.

8. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator. However, the candidates are allowed to take away this Test Booklet with them.

9. Do not fold or make any stray marks on the Answer Sheet.

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USEFUL DATA

Atomic weights: Al = 27, Mg = 24, Cu = 63.5, Mn = 55, Cl = 35.5, O = 16, H = 1, P = 31, Ag = 108, N = 14, Li = 7, I = 127, Cr = 52, K=39, S = 32, Na = 23, C = 12, Br = 80, Fe = 56, Ca = 40, Zn = 65.5, Ti = 48, Ba = 137, U = 238, Co= 59, B =11, F = 19, He = 4, Ne = 20, Ar = 40 , Mo = 96

[Take : $ln\ 2 = 0.693$, $ln\ 1.1 = 0.095$, $ln\ 3 = 1.09$, $e = 1.6 \times 10^{-19}$, $m_e = 9.1 \times 10^{-31}$ kg]

Take: $\in_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$, $g = 10\ m/s^2$, $S_{water} = 1$ cal/gm °C, $L_{ice} = 80$ cal/gm. , $g = 10\ m/s^2$ unless otherwise stated

CHEMISTRY

Q.1 During the process of electrolytic refining of Copper which of the following may obtained as a anode mud ?

(1) Ag, Au (2) Ag, Zn (3) Fe, Ni (4) Zn, Ni

Q.2 How many moles of KOH are required per mole of $R–NH_2$ in the balanced reaction ?

$$R–NH_2 + CHCl_3 + KOH \xrightarrow{Heat} R–N \overset{\rightarrow}{=} C + KCl + H_2O$$

(1) 1 (2) 2 (3) 3 (4) 4

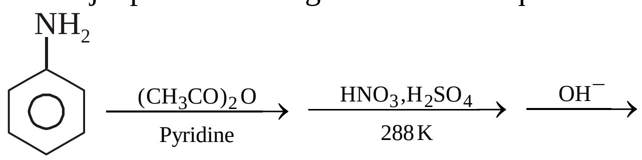
Q.3 An ionic solid 'AB' (M_w = 120 g/mol) has NaCl type structure. The shortest distance between cation and anion is 500 pm. If there is a schottky defect of 2% then the density of crystal in g / cm^3 will be : ($N_A = 6 \times 10^{23}$)

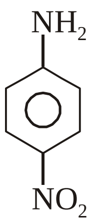
(1) 0.80 (2) 0.784 (3) 0.016 (4) None of these

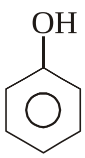
Q.4 Which of the following metal is not commercially extracted by self reduction method from their corresponding ore ?

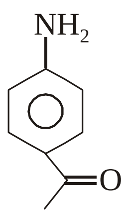
(1) Cu (2) Fe (3) Pb (4) Hg

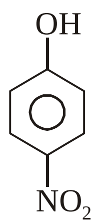
Q.5 What is the major product of the given reaction sequence ?



(1) 

(2) 

(3) 

(4) 

SPACE FOR ROUGH WORK

Q.6 At 27°C, the vapour pressure of an aqueous solution of urea is equal to the osmotic pressure of 5×10^{-3} M aqueous solution of glucose. If vapour pressure of pure water at 27°C is 114 torr then the mole fraction of urea in its solution is - [**Given :** R = 0.08 L-atm/mol-K]

(1) $\frac{4}{5}$ (2) 0.25 (3) $\frac{3}{4}$ (4) 0.2

Q.7 Select incorrect statement regarding silver extraction -
(1) Cupellation process is used when Lead - Silver alloy is rich in Silver and Lead is removed.
(2) Parke's and Pattinson's process is used when Lead Silver alloy is rich in lead.
(3) Zinc forms alloy with Lead, from which Lead is separated by distillation.
(4) Zinc forms alloy with Silver, from which Zinc is separated by distillation.

Q.8 What is the colour of the dye formed in the given reaction ?

$$PhN_2Cl + PhNH_2 \xrightarrow{H^+}$$

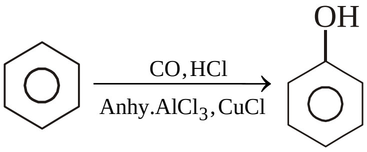
(1) Red (2) Orange (3) Blue (4) Yellow

Q.9 In a solid compound, X-particles are present in ccp position, Y-particle are in all octahedral voids and Z-particles are in alternate tetrahedral voids. If all void- particles along one of the body-diagonal are removed then the formula of compound will be given as -
(1) $X_4Y_3Z_3$ (2) $X_3Y_4Z_3$ (3) $X_4Y_4Z_3$ (4) $X_4Y_3Z_4$

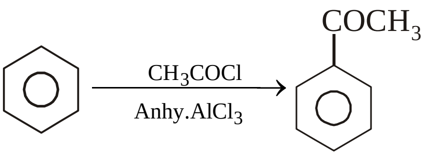
Q.10 During the extraction process of Copper, Silica is added to roasted ore, in order to remove.
(1) cuprous sulphide (2) ferrous oxide (3) ferrous sulphide (4) cuprous oxide

SPACE FOR ROUGH WORK

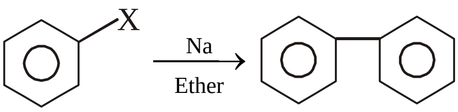
Q.11 Find the incorrect combination of reaction and reaction name.

(1) 

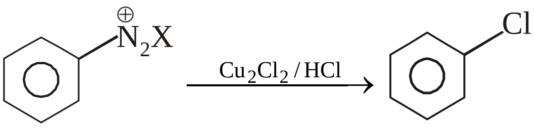
(Gatterman – Koch reaction)

(2) 

(Friedel Crafts Alkylation)

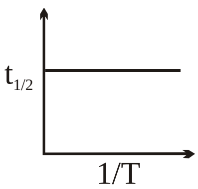
(3) 

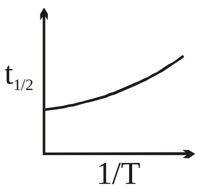
(Fittig reaction)

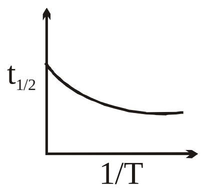
(4) 

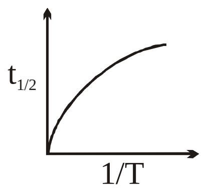
+ N_2 (Sandmeyer reaction)

Q.12 For a I[st] order reaction, the correct graph showing variation of half-life ($t_{1/2}$) with inverse of Kelvin-temperature (1/T) will be -

(1) 

(2) 

(3) 

(4) 

Q.13 During Froath Floatation process, use of depressant is :
(1) to remove gangue from the sulphide ore. (2) to seprate two sulphide ore selectively.
(3) as froath stabilizer. (4) to wet gangue particles.

Q.14 Which sugar is composed only of β-D-glucose units ?
(1) Starch (2) Cellulose (3) Lactose (4) Sucrose

SPACE FOR ROUGH WORK

Q.15 Due to non-stoichiometric defect, a sample of cuprous oxide is found to have a composition $Cu_{1.8}O$. The mole % of Cu^{2+} in total copper content of the crystal will be -
(1) 10% (2) 11.11 % (3) 88.88 % (4) 90 %

Q.16 A salt impart red colour to the borax bead in reducing flame, what could be the colour of the bead in oxidising flame.
(1) Blue (2) Green (3) Yellow (4) Voilet

Q.17 Which of the following is a condensation copolymer ?
(1) Styrene (2) Buna-S (3) Bakelite (4) Nylon-6

Q.18 The abnormal molecular mass of CH_3COOH when dissolved in benzene is found to be 80 g/mol. The percentage of CH_3COOH present in dimeric form in solution is -
(1) 50% (2) 12.75 % (3) 25 % (4) 33.33 %

Q.19 Which of the following pair of cations can be separated by using excess NH_3 solution ?
(1) Bi^{3+}(aq) and Al^{3+}(aq) (2) Al^{3+}(aq) and Zn^{2+}(aq)
(3) Hg^{2+}(aq) and Pb^{2+}(aq) (4) Cu^{2+}(aq) and Cd^{2+}(aq)

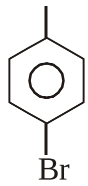
Q.20 Select the cationic detergent amongst the following.
(1) Sodium Palmitate (2) Sodium Stearate
(3) Cetyl trimethyl ammonium bromide (4) Sodium Lauryl sulphate

Q.21 A solution is prepared by mixing two volatile liquids. If its vapour pressure is lesser as compared to the vapoure pressure calculated from Raoult's law then the only incorrect option for the solution is -
(1) $\Delta H_{mix} > 0$ (2) $\Delta V_{mix} < 0$ (3) $\Delta S_{mix} > 0$ (4) $\Delta G_{mix} < 0$

Q.22 Filtrate obtained from group II basic radicals is heated with drops of dil. HNO_3 before adding reagent of group III, in order to -
(1) to remove HCl and H_2S (2) to convert Fe^{2+} into Fe^{3+}
(3) to oxidise Cr^{3+} to $Cr_2O_7^{2-}$ (4) to lower the pH of solution.

SPACE FOR ROUGH WORK

Q.23 Indicate the IUPAC name of the following compound.



(1) 3-bromo-1-methyl benzene
(2) 4-bromo toluene
(3) p-ethyl bromo benzene
(4) 1-methyl-4-bromo benzene

Q.24 Specific activity of a radioactive substance (M = 226 amu) is 3.7×10^{10} distintegration per second per gram. The mean life of substance (years) is nearly -[$N_A = 6 \times 10^{23}$]
(1) 2270
(2) 2822
(3) 8228
(4) 8822

Q.25 H_2S gas when passed through a solution containing cations in presence of HCl, it precipitates the cations of second group of qualitative analysis but not those belongs to the fourth group. It is because :
(1) Presence of HCl decreases the sulphide ion concentration.
(2) Presence of HCl increases the sulphide ion concentration.
(3) K_{sp} of group II sulphides is more than that of group IV sulphides.
(4) Sulphides of group IV cations are unstable in HCl.

Q.26 Which compound gives a base insoluble product with benzene sulphonyl chloride?
(1) N-ethyl aniline
(2) p-toluidine
(3) Anisole
(4) Trimethyl amine

Q.27 AB_2 (aqueous) decomposes according to the first order reaction :

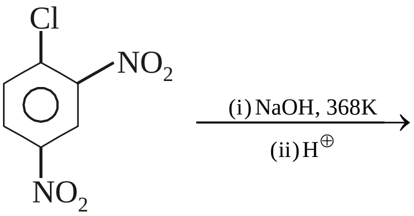
$$AB_2(aq) \rightarrow A(g) + 2B(l)$$

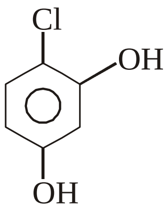
After 20 min the volume of A(g) collected during such a reaction is 20 ml and that collected after a very long time is 40 ml at same temperature and pressure. The rate constant for the reaction is:
(1) 1.435×10^{-2} min^{-1}
(2) 2.5×10^{-2} min^{-1}
(3) 3.46×10^{-2} min^{-1}
(4) 6.93 min^{-1}

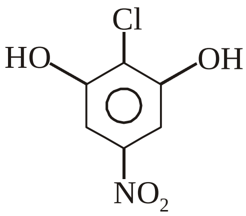
Q.28 On adding KI solution in excess to a solution of $CuSO_4$. We get precipitates 'P' and other liquid solution 'M'. Select **correct** pairs ?
(1) P is CuI and M is I_2 solution
(2) P is CuI_2 and M is I_2 solution
(3) P is CuI and M is KI_3 solution
(4) P is CuI_2 and M is KI_3 solution

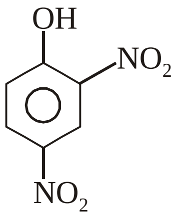
SPACE FOR ROUGH WORK

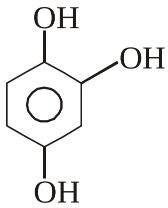
Q.29 What is the major product of the given reaction ?



(1) 

(2) 

(3) 

(4) 

Q.30 For the following sequential radioactive decay-

$${}^{Y}_{X}A \xrightarrow{-\alpha} B \xrightarrow{-\beta^-} C \xrightarrow{-\beta^-} D \xrightarrow{-\alpha} E$$

the only **correct** statement is -

(1) atomic number of E is (X +2)

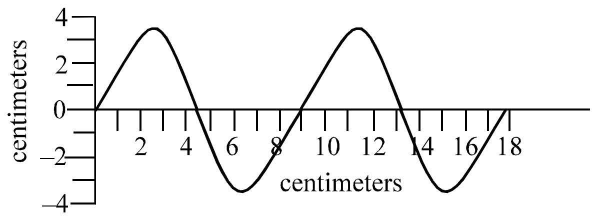
(2) A and E are isosters.

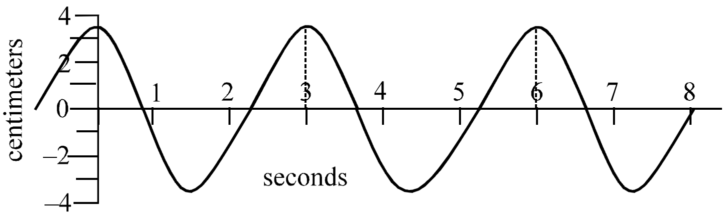
(3) Mass no. of D is (Y–8)

(4) B and D are isobars.

SPACE FOR ROUGH WORK

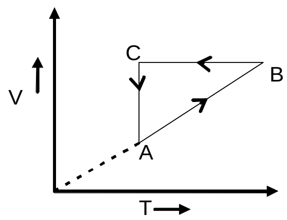
Q.31 A transverse wave is travelling along a horizontal string. The first picture shows the shape of the string at an instant of time. This picture is superimposed on a coordinate system to help you make any necessary measurements. The second picture is a graph of the vertical displacement of *one* point along the string as a function of time. How far does this wave travel along the string in one second?





(1) 0.3 cm (2) 3.0 cm (3) 9.0 cm (4) 27 cm

Q.32 A cyclic process of an enclosed gas of constant mass is represented by volume (V) against absolute temperature (T) as shown. If P represents pressure, the graph representing the same process can be



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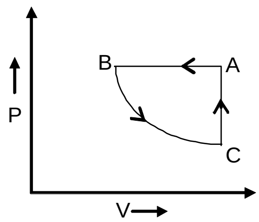

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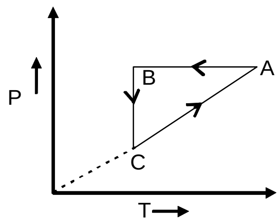

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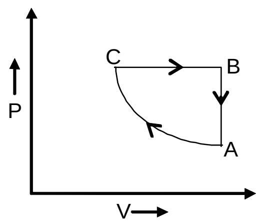

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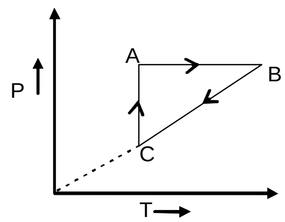
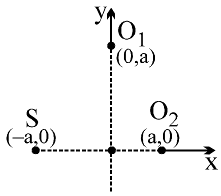


SPACE FOR ROUGH WORK

Q.33 A closed organ pipe is vibrating in its second overtone. The length of the pipe is 10cm and maximum amplitude of vibration of particles of the air in the pipe is 2mm. Then the amplitude of S.H.M. of the particles at 9cm from the open end is:

(1) $\sqrt{3}$ mm (2) $\sqrt{2}$ mm (3) $\frac{\sqrt{3}}{2}$ mm (4) none of these

Q.34 A sound source S and observers O_1, O_2 are placed as shown. S is always at rest and O_1, O_2 start moving with velocity v_0 at t = 0. At any later instant, let f_1 and f_2 represent apparent frequencies of sound received by O_1 and O_2, respectively. The ratio f_1/f_2 is



(1) zero (2) between 0 and 1 (3) 1 (4) > 1

Q.35 Equal masses of three liquids A, B and C have temperatures 10°C, 25°C and 40°C respectively. If A and B are mixed, the mixture has a temperature of 15°C. If B and C are mixed, the mixture has a temperature of 30°C,. If A and C are mixed the mixture will have a temperature of

(1) 16°C (2) 20°C (3) 25°C (4) 29°C

Q.36 A steel rod is 4.000 cm in diameter at 30 °C. A brass ring has an interior diameter of 3.992 cm at 30 °C. In order that the ring just slides onto the steel rod, the common temperature of the two should be nearly (α_{steel} = 11 × 10^{-6}/°C and α_{brass} = 19 × 10^{-6}/°C)

(1) 200 °C (2) 250 °C (3) 280 °C (4) 400 °C

Q.37 A hot liquid is kept in a big room. The logarithm of the numerical value of the temperature difference between the liquid and the room is plotted against time. The plot will be very nearly.

(1) a straight line (2) a circular arc (3) a parabola (4) exponential decay

Q.38 A body cools from 80°C to 50°C in 5 minutes. Calculate the time it takes to cool from 60°C to 30°C. The temperature of the surroundings is 20°C.

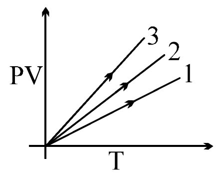
(1) 10 min (2) 15 min (3) 20 min (4) 5 min

SPACE FOR ROUGH WORK

Q.39 A sinusoidal wave (longitudinal or transverse) is propagating through a medium in the direction of –ve x-axis. The parameters of the waves are A, ω and k. The particle at $x = \lambda/4$ executes the motion $y(t) = A \sin\omega t$. Possible equation of the wave is

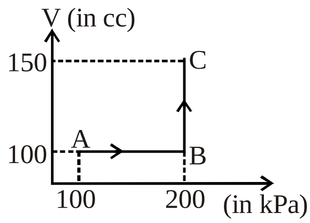
(1) $y(x, t) = A \sin[\omega t - kx + (\pi/2)]$
(2) $y(x, t) = A \sin[\omega t + kx + (\pi/2)]$
(3) $y(x, t) = A \sin[\omega t - kx - (\pi/2)]$
(4) $y(x, t) = A \sin[\omega t + kx - (\pi/2)]$

Q.40 PV versus T graph of equal masses of H_2, He and CO_2 is shown in figure. Choose the correct alternative



(1) 3 corresponds to H_2, 2 to He and 1 to CO_2
(2) 1 corresponds to He, 2 to H_2 and 3 to CO_2
(3) 1 corresponds to He, 3 to H_2 and 2 to CO_2
(4) 1 corresponds to CO_2, 2 to H_2 and 3 to He

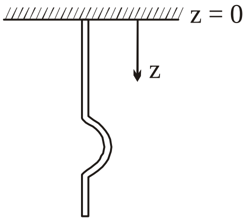
Q.41 A monoatomic gas is taken from A to C as shown in the figure. The temperature of gas at B is 27°C, then the change in internal energy of the gas is



(1) 15 J
(2) 20 J
(3) 30 J
(4) 25 J

SPACE FOR ROUGH WORK

Q.42 A rope hangs from a rigid support. A pulse is set by jiggling the bottom end. We want to design a rope in which velocity v of pulse is independent of z, the distance of the pulse from fixed end of the rope. If the rope is very long the desired function for mass per unit length $\mu(z)$ in terms of μ_0 (mass per unit length of the rope at the top (z = 0), g, v and z is :



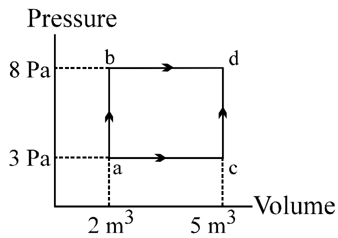
(1) $\mu(z) = \mu_0 e^{-[g/v^2]z}$

(2) $\mu(z) = \mu_0 e^{+[g/v^2]z}$

(3) $\mu(z) = \mu_0 \log_e\left(\frac{g}{v^2}\right)z$

(4) $\mu(z) = \mu_0 e + \left(\frac{v^2}{g}\right)z$

Q.43 In the Pressure versus Volume graph shown, in the process of going from a to b 60 J of heat is added, and in the process of going from b to d 20 J of heat is added. In the process of going from a to c to d, what is the total heat added?



(1) 80 J (2) 65 J (3) 60 J (4) 56 J

Q.44 A glass tube of 1.0 meter length is filled with water. The water can be drained out slowly at the bottom of the tube. If a vibrating tuning fork of frequency 500Hz is brought at the upper end of the tube and the velocity of sound is 330m/s then the total number of resonances obtained will be

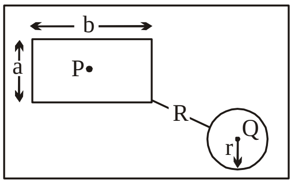
(1) 4 (2) 3 (3) 2 (4) 1

SPACE FOR ROUGH WORK

Q.45 A body of mass 25 kg is dragged on a rough horizontal floor for one hour with a speed of 2km/h. The coefficient of friction between the body and the surface in contact is 0.5 and half the heat produced is absorbed by the body. If specific heat of body is 0.1 cal g^{-1} $(^0C)^{-1}$, then the rise in temperature of body is

(1) 50 K (2) 23.8 K (3) 100 K (4) 11.9 K

Q.46 There is a rectangular metal plate in which two cavities in shape of rectangle and circle are made, as shown in figure, with dimensions. P & Q are centres of these cavities. On heating the plate which of the following quantities increases.



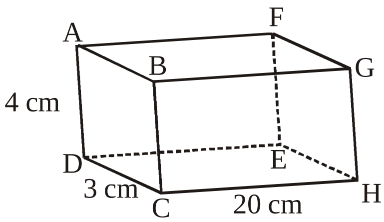
(1) πr^2 (2) ab (3) R (4) A, B, C are all correct

Q.47 A block of wood is floating in water at 0°C. The temperature of only water is slowly raised from 0°C to 10°C. With the rise in temperature of the water the volume of block above water level will be

(1) increase (2) decrease

(3) first increase and then decrease (4) first decrease and then increase

Q.48 A rectangular block of lead has dimensions 4 cm × 3 cm × 20 cm. A temperature difference of 100°C can be applied to any pair of opposite faces that we choose.



(1) The largest amount of heat flows if it flows parallel to line BG..
(2) The largest amount of heat flows if it flows parallel to line AD.
(3) The smallest amount of heat flows if it flows parallel to AF.
(4) The smallest amount of heat flows if it flows parallel to AB.

SPACE FOR ROUGH WORK

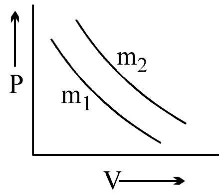
Q.49 Two identical solid spheres have the same constant temperature. One of the spheres is cut into two identical pieces. These two hemispheres are then separated. The intact sphere radiates an energy Q during a given time interval at temperature T_0. During the same interval, the two hemisphere radiate at total energy Q' at temperature T_0. Emissivity of all the surfaces is same. The ratio Q'/Q has value :

(1) 0.50 (2) 0.75 (3) 2.0 (4) 1.5

Q.50 An ideal gas obeys a law V^2P = constant. The gas is initially at temperature T and have volume V. As it expands to 2V the temperature becomes

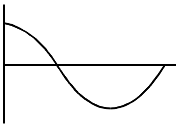
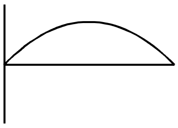
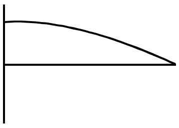
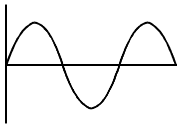
(1) 2T (2) 3T (3) T (4) T/2

Q.51 Two different isotherms representing the relationship between pressure P and volume V at a same temperature of the same ideal gas are shown for masses m_1 and m_2 of the gas respectively in the figure given, then:



(1) $m_1 > m_2$ (2) $m_1 = m_2$
(3) $m_1 < m_2$ (4) All of the above are possible

Q.52 Which of the figures, which show the pressure difference from regular atmospheric pressure for an organ pipe of length L closed at one end, corresponds to the 1st overtone for the pipe?

(1)  (2)  (3)  (4) 

Q.53 A car blowing a horn of frequency 350 Hz is moving normally towards a wall with a speed of 5 m/s. The beat frequency heard by a person standing between the car and the wall is (speed of sound in air = 350 m/s)

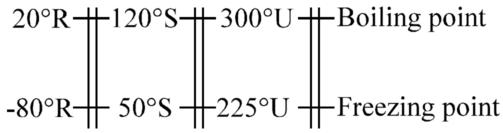
(1) zero (2) 3.5 Hz (3) 5 Hz (4) 10 Hz

Q.54 The quantity $\frac{pV}{kT}$ represents

(1) mass of the gas (2) kinetic energy of the gas
(3) number of molecules in the gas (4) number of moles of the gas

SPACE FOR ROUGH WORK

Q.55 Figure shows three temperature scales with the freezing and boiling point of water indicated. A change of 25 R^0, 25 S^0 and 25 U^0 is denoted by x_1, x_2, x_3 respectively. Which of the following is correct:



(1) $x_1 > x_2 > x_3$ (2) $x_2 < x_1 < x_3$ (3) $x_3 > x_2 > x_1$ (4) $x_2 > x_3 > x_1$

Q.56 One end of a conducting rod is maintained at temperature 50ºC and at the other end, ice is melting at 0ºC. The rate of melting of ice is doubled if :

(1) The temperature is made 200ºC and the area of cross-section of the rod is doubled.
(2) The temperature is made 100ºC and length of rod is made four times.
(3) Area of cross-section of rod is halved and length is doubled.
(4) The temperature is made 100ºC and the area of cross-section of rod and length both are doubled.

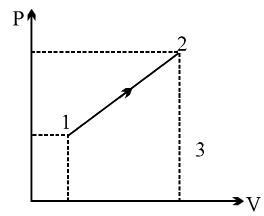
Q.57 Consider a solar system with planets that revolve around the sun in circular orbits. The temperature (T) of a planet having no atmosphere situated at a distance r from the sun varies as

(1) $T \propto r^4$ (2) $T \propto r^{4/3}$ (3) $T \propto r^3$ (4) $T \propto r^{-1/2}$

Q.58 The order of magnitude of the number of nitrogen molecules in an air bubble of diameter 2 mm under ordinary conditions (pressure = 1 atm; temperature = 27°C) is:

(1) 10^5 (2) 10^9 (3) 10^{13} (4) 10^{17}

Q.59 A process $1 \rightarrow 2$ using diatomic gas is shown on the P–V diagram on the right. $P_2 = 2P_1 = 10^6$ N/m^2, $V_2 = 4V_1 = 0.4$ m^3. The molar heat capacity of the gas in this process will be



(1) $\frac{35R}{12}$ (2) $\frac{25R}{13}$

(3) $\frac{35R}{11}$ (4) $\frac{22R}{7}$

Q.60 The efficiency of a carnot engine is 0.6. It rejects total 20 J of heat. The work done by the engine is

(1) 40 J (2) 50 J (3) 20 J (4) 30 J

SPACE FOR ROUGH WORK

Q.61 Let f(x) be a one-to-one function such that f(1) = 3, f(3) = 1, f '(1) = – 4 and f '(3) = 2. If $g = f^{-1}$, then the slope of the tangent line to $\frac{1}{g}$ at x = 1 is

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{-1}{9}$ (3) $\frac{-1}{18}$ (4) $\frac{1}{32}$

Q.62 The value of $\lim_{t\to 0} \ln\left(\frac{1}{t}\int_0^t (1+2\sin 3x)^{4/x}\,dx\right)$ is equal to

(1) 6 (2) 12 (3) 18 (4) 24

Q.63 If $g(x^3 + 1) = x^6 + x^3 + 2$, then the value of $g(x^2 - 1)$ is

(1) $x^4 - 3x^2 + 3$ (2) $x^4 + x^2 + 4$ (3) $x^4 - 3x^2 + 4$ (4) $x^4 + x^2 + 2$

Q.64 Suppose that f (0) = 0 and f ' (0) = 2, and let $g(x) = f\left(-x + f\left(f(x)\right)\right)$. The value of g ' (0) is equal to

(1) 0 (2) 1 (3) 6 (4) 8

Q.65 The value of the definite integral, $\int_1^{\infty} (e^{x+1} + e^{3-x})^{-1}\,dx$ is

(1) $\frac{\pi}{4e^2}$ (2) $\frac{\pi}{4e}$ (3) $\frac{1}{e^2}\left(\frac{\pi}{2} - \tan^{-1}\frac{1}{e}\right)$ (4) $\frac{\pi}{2e^2}$

Q.66 A line L is perpendicular to the curve $y = \frac{x^2}{4} - 2$ at its point P and passes through (10, –1). The coordinates of the point P are

(1) (2, –1) (2) (6, 7) (3) (0, –2) (4) (4, 2)

SPACE FOR ROUGH WORK

Q.67 If $f(x) = \max.\left(x^4, x^2, \frac{1}{81}\right)$ $\forall\ x \in [0, \infty)$, then the sum of the square of reciprocal of all the values of x where f(x) is non-differentiable, is equal to

(1) 1 (2) 81 (3) 82 (4) $\frac{82}{81}$

Q.68 Given: $f(x) = 4 - \left(\frac{1}{2} - x\right)^{2/3}$, $g(x) = \begin{cases} \frac{\tan[x]}{x} & , x \neq 0 \\ 1 & , x = 0 \end{cases}$

$h(x) = \{x\}$, $k(x) = 5^{\log_2(x+3)}$

then in [0, 1], Lagranges Mean Value Theorem is NOT applicable to

(1) f, g, h (2) h, k (3) f, g (4) g, h, k

[**Note :** where [x] and {x} denote the greatest integer and fractional part function of x respectively]

Q.69 If the function $f(x) = ax\,e^{-bx}$ has a local maximum at the point (2, 10), then

(1) a = 5; b = 0 (2) a = 5e, b = 1/2 (3) $a = 5e^2$, b = 1 (4) none

Q.70 Suppose, $f(x, n) = \sum_{k=1}^{n} \log_x\left(\frac{k}{x}\right)$, then the value of x satisfying the equation $f(x, 10) = f(x, 11)$, is

(1) 9 (2) 10 (3) 11 (4) none

Q.71 $\lim_{n\to\infty} \sum_{r=1}^{r=4n} \frac{\sqrt{n}}{\sqrt{r}\left(3\sqrt{r} + 4\sqrt{n}\right)^2}$ is equal to

(1) $\frac{1}{35}$ (2) $\frac{1}{14}$ (3) $\frac{1}{10}$ (4) $\frac{1}{5}$

SPACE FOR ROUGH WORK

Q.72 Number of integral solutions of the equation $\text{sgn}\left(\sin^{-1}\left[\frac{\pi x}{6}\right]\right) = 1$, is

[**Note :** where [x] denotes the greatest integer less than or equal to x and sgn x denotes signum function of x.]

(1) 2 (2) 3 (3) 5 (4) 7

Q.73 The area bounded by the curve $y = x^2 + 4x + 5$, the axes of co-ordinates & the minimum ordinate is

(1) $3\frac{2}{3}$ (2) $4\frac{2}{3}$ (3) $5\frac{2}{3}$ (4) $\frac{8}{3}$

Q.74 The differential equation of all parabolas having their axis of symmetry coinciding with the axis of x has its order and degree respectively

(1) (2, 1) (2) (2, 2) (3) (1, 2) (4) (1, 1)

Q.75 Number of roots of the equation $x^2 - 2x - \log_2|1 - x| = 3$ is

(1) 4 (2) 2 (3) 1 (4) 0

Q.76 Let F(x) be the primitive of $\frac{3x+2}{\sqrt{x-9}}$ w.r.t. x. If F(10) = 60 then the value of F(13), is

(1) 66 (2) 132 (3) 248 (4) 264

Q.77 If $f(x) = \begin{cases} x^2\left\{e^{\frac{1}{x}}\right\}; & x \neq 0 \\ k; & x = 0 \end{cases}$

is continuous at x = 0, then

[**Note :** {x} denotes fractional part of x.]

(1) f(x) is differentiable at x = 0

(2) k = 1

(3) f(x) is continuous but not differentiable at x = 0

(4) f(x) is continuous every where in its domain.

SPACE FOR ROUGH WORK

Q.78 $\int \frac{\cos^3 x+\cos^5 x}{\sin^2 x+\sin^4 x}\, dx$ is equal to

(1) $\sin x - 6\tan^{-1}(\sin x) + C$
(2) $\sin x - 2\sin^{-1} x + C$
(3) $\sin x - 2(\sin x)^{-1} - 6\tan^{-1}(\sin x) + C$
(4) $\sin x - 2(\sin x)^{-1} + 5\tan^{-1}(\sin x) + C$

[**Note :** where C is constant of integration.]

Q.79 Point 'A' lies on the curve $y = e^{-x^2}$ and has the coordinate (x, e^{-x^2}) where $x > 0$. Point B has the coordinates (x, 0). If 'O' is the origin then the maximum area of the triangle AOB is

(1) $\frac{1}{\sqrt{e}}$
(2) $\frac{1}{\sqrt{8e}}$
(3) $\frac{1}{\sqrt{2e}}$
(4) $\frac{1}{\sqrt{4e}}$

Q.80 Let $f(x) = \begin{cases} 3\sin x + a^2 - 10a + 30, & x \notin Q \\ 4\cos x, & x \in Q \end{cases}$.

Which one of the following statement is correct ?

(1) f (x) is continuous for all x when a = 5.
(2) f (x) is discontinuous for all x.
(3) f (x) is continuous for all $x = 2n\pi - \tan^{-1}\left(\frac{3}{4}\right)$, $n \in I$ when a = 5.
(4) f (x) is continuous for all $x = 2n\pi - \tan^{-1}\left(\frac{4}{3}\right)$, $n \in I$ when a = 5.

Q.81 If $x = \tan^{-1} 1 - \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\frac{1}{2}$; $y = \cos\left(\frac{1}{2}\cos^{-1}\left(\frac{1}{8}\right)\right)$ then :

(1) $x = \pi y$
(2) $y = \pi x$
(3) $\tan x = -(4/3)\, y$
(4) $\tan x = (4/3)\, y$

Q.82 The domain of the derivative of the function $f(x) = \begin{cases} e^{-|x|+1}, & \text{if } |x| \le 1 \\ \left|\,\left|\,|x|-4\right|-2\right|, & \text{if } |x| > 1 \end{cases}$ is

(1) R – {–2, 0, 2}
(2) R – {– 4, –2, –1, 0, 1, 2, 4}
(3) R – {–6, – 4, –2, 0, 2, 4, 6}
(4) R – {–6, – 4, –2, –1, 1, 2, 4, 6}

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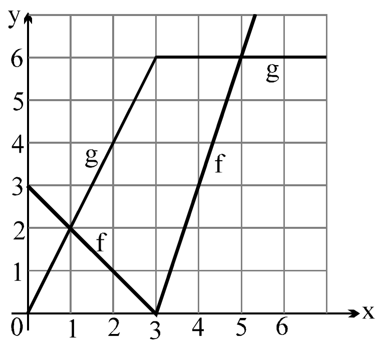
Q.83 If f and g are the functions whose graphs are shown, let



$P(x) = f(x)\,g(x),\ Q(x) = \dfrac{f(x)}{g(x)}$ and $C(x) = f(g(x))$.

The value of $(P'(2) - C'(2))Q'(2)$ equals

(1) 3/2 (2*) 3

(3) – 3 (4) – 6

Q.84 Let $G(x) = \int e^x \left(\int_0^x f(t)\,dt + f(x) \right) dx$ where f(x) is continuous on R. If f(0) = 1, G(0) = 0 then G'(0) equals

(1) 1 (2) 2 (3) 3 (4) 4

Q.85 $\lim\limits_{n\to\infty} \dfrac{e^n}{\left(1+\dfrac{1}{n}\right)^{n^2}}$ equals

(1) 1 (2) $\dfrac{1}{2}$ (3) e (4) $\sqrt{e}$

Q.86 If $\dfrac{dy}{dx} = (e^y - x)^{-1}$ where y (0) = 0, then y is expressed explicitly as

(1) $\dfrac{1}{2}\ln(1 + x^2)$ (2) $\ln(1 + x^2)$ (3) $\ln\left(x + \sqrt{1+x^2}\right)$ (4) $\ln\left(x + \sqrt{1-x^2}\right)$

Q.87 The area enclosed by g(x), x = –3, x = 5 and x-axis where g(x) is the inverse of $f(x) = x^3 + 3x + 1$, is

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(1) $\frac{5}{2}$ (2) $\frac{7}{2}$ (3) $\frac{9}{2}$ (4) $\frac{11}{2}$

Q.88 Let $J = \int_0^{e-1} \frac{1}{x+1} \exp.\left(\frac{x^2+2x-1}{2}\right)dx$ and $K = \int_1^{e} x\,ln\,x \exp.\left(\frac{x^2-2}{2}\right)dx$.

The value of $(J + K)$ is equal to

(1) $\left(\sqrt{e}\right)^{e^2+1}$ (2) $\left(\sqrt{e}\right)^{e^2-1}$ (3) 0 (4) $\left(\sqrt{e}\right)^{e^2-2}$

Q.89 The graphs $y = 2x^3 - 4x + 2$ and $y = x^3 + 2x - 1$ intersect in exactly 3 distinct points. The slope of the line passing through two of these points

(1) is equal to 4 (2) is equal to 6 (3) is equal to 8 (4) is not unique

Q.90 The value of $\lim_{n\to\infty}\left(\sum_{r=1}^{n}\left(\frac{2r+5}{r^2+r}\right)\left(\frac{3}{5}\right)^{r+1}\right)$ is equal to

(1) 2 (2) $\frac{9}{5}$ (3) $\frac{6}{5}$ (4) none

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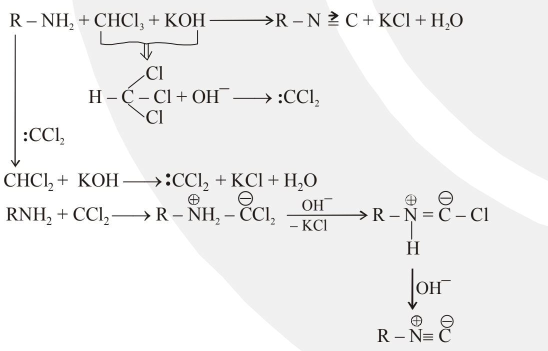
	IOC	OC	PC	IOC	OC	PC	IOC	OC	PC	IOC	OC	PC	IOC	OC	PC
Q.No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans	1	3	2	2	1	4	3	4	1	2	2	2	2	2	2
	IOC	OC	PC	IOC	OC	PC	IOC	OC	PC	IOC	OC	PC	IOC	OC	PC
Q.No.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans	1	3	4	2	3	1	2	2	1	1	1	3	3	3	4
Q.No.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**
Ans	2	4	2	4	1	3	1	1	4	1	3	1	2	2	4
Q.No.	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
Ans	4	3	3	4	4	3	1	1	3	4	4	4	4	4	4
Q.No.	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**
Ans	3	4	3	3	1	4	3	1	2	3	3	1	2	1	1
Q.No.	**76**	**77**	**78**	**79**	**80**	**81**	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**
Ans	2	1	3	2	3	3	3	2	1	4	3	3	4	3	2

HINTS & SOLUTIONS

CHEMISTRY

Q.1 Ag, Au (due to less electropositive)

Q.2



3 mole of KOH are required in carbylamine test.

Q.3 Shortest distance between cation and anion =

$\frac{a}{2} = 500$ pm

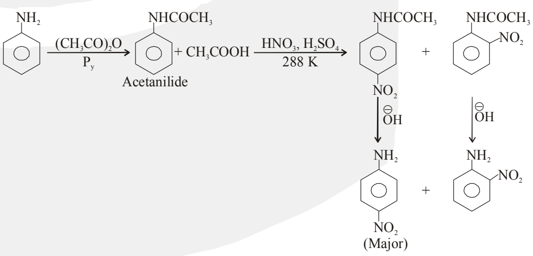
a = 1000 pm

$d_{ideal} = -$ ... $)(120)$ / ... $0^{-10})^3.6\times10^{23}$

... g/cm^3

... o, H... reduced by

Q.5



Q.6 Vapour pressure of aquoeus solution of urea

$= 5 \times 10^{-3} \times 0.08 \times 300$ $(\because \pi = CRT)$

= 0.12 atm

$= 0.12 \times 760 = 91.2$ torr

$$\text{R.L.V.P} = \frac{P^\circ - P}{P} = X_{solute}$$

$$\frac{114 - 91.2}{114} = 0.2 = X_{Solute}$$

Q.7 Ag is 3000 times more soluble in Zn in compression of Pb.

$$(Pb-Ag) + Zn \longrightarrow (Ag-Zn)\,Pb \xrightarrow[\text{separation}]{\text{after}} Zn-Ag$$

$$\downarrow \text{distillation}$$

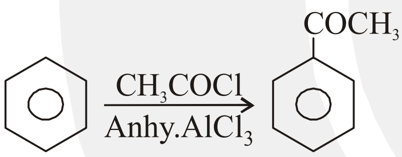
$$Ag + Zn\,(s)\uparrow$$

Q.8 

Yellow coloured dye

Q.9 $Z_x = 4$
$Z_y = 4$
$Z_z = 4$

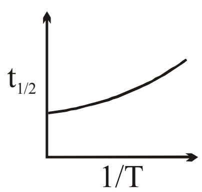
Q.10 $FeS + O_2\,(\text{limited}) \longrightarrow FeO + SO_2$
$FeO + SiO_2\,(\text{Flux}) \longrightarrow FeSiO_3\,(\text{Slag})$

Q.11 

It is Friedel Craft's acylation not Friedel crafts alkylation.

Q.12 $K = Ae^{-\frac{E_a}{RT}}$; $\frac{\ln 2}{t_{1/2}} = Ae^{\frac{-E_a}{RT}}$

$$t_{1/2} = \frac{\ln 2}{A} = e^{\frac{E_a}{RT}}$$



Q.13 Theory based

Q.14 Cellulose is a linear polymer of D-glucose units joined by β-glycosidic linkage.

Q.15 $\overset{+\frac{2}{1.8}}{Cu_{1.8}}O$ contains Cu^+ and Cu^{2+}

Let total Cu ions = 100
if $Cu^{2+} = x$
$\Rightarrow Cu^+ = (100 - x)$

so $$+\frac{2}{1.8} = \frac{x(+1) + (100 - x)(+2)}{100}$$

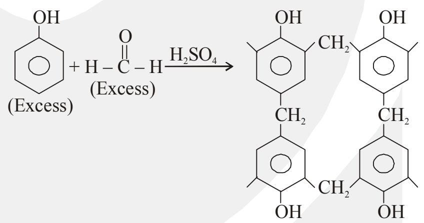
$$1000 = 1800 - 9x$$

$$x = \frac{800}{9} = 88.88\ \%$$

Q.16

d-block cation	**Bead colour in reducing flame**	**in oxidising flame**
Cu^{2+}	Red	Blue
Cr^{3+}	Green	Green
Fe^{3+}	Green	Yellow
Mn^{2+}	Colourless	Violet

Q.17 Bakelite is formed from a condensation reaction of phenol with formaldehyde.



Bakelite

Q.18 $$i = \frac{M_T}{M_O}$$

$$i = \frac{60}{80} = 1 + \alpha\left(\frac{1}{n} - 1\right)$$

$$0.75 = 1 + \alpha\left(\frac{1}{2} - 1\right)$$

$\alpha = 0.5$

$\% \alpha = 50 \%$

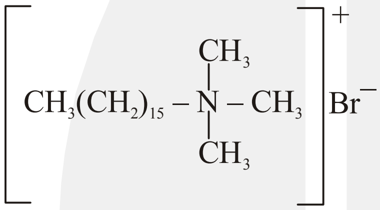
$2CH_3COOH \longrightarrow (CH_3COOH)_2$

$1 \qquad 1-\alpha \qquad \frac{\alpha}{2}$

$\% \text{ of } CH_3COOH = \frac{\left(\frac{\alpha}{2}\right)}{1-\frac{\alpha}{2}} \times 100$

in dimeric form $= \frac{0.5}{1.5} \times 100 = 33.3\ \%$

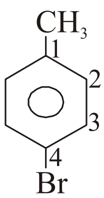
Q.19 $Bi^{3+} + \text{excess } NH_3 \longrightarrow Bi(OH)_3 \downarrow$ (white)
$Al^{3+} + \text{excess } NH_3 \longrightarrow Al(OH)_3 \downarrow$ (white)
$Zn^{2+} + \text{excess } NH_3 \longrightarrow Zn(NH_3)_3$ (clear)
$Hg^{2+} + \text{excess } NH_3 \longrightarrow HgO.HgNH_2 \downarrow$
$Pb^{2+} + \text{excess } NH_3 \longrightarrow Pb(OH)_2 \downarrow$
$Cu^{2+} + \text{excess } NH_3 \longrightarrow Cu(NH_3)_4^{2+}$ (clear)
$Cd^{2+} + \text{excess } NH_3 \longrightarrow Cd(NH_3)_4^{2+}$ (clear)

Q.20 

Cetyltrimethyl ammonium bromide is cationic detergent.

Q.21 If vapour pressure is less compared to that calculate from Raoult's law, then solution shows negative deviation and for that solution $\Delta V_{mix} < 0$; $\Delta S_{mix} > 0$; $\Delta G_{mix} < 0$; $\Delta H_{mix} > 0$

Q.22 Theory based

Q.23 

4– Bromo toluene

Q.24 $A = \lambda N$

$3.7 \times 10^{10} = \lambda \left(\frac{1}{226} \times 6 \times 10^{23}\right)$

$\lambda = \frac{3.7 \times 10^{10} \times 226}{6 \times 10^{23}}$

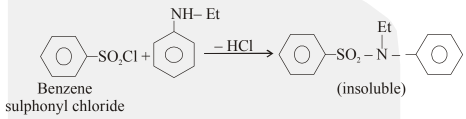
$t_{mean} = \frac{1}{\lambda}$

$= \frac{6 \times 10^{23}}{3.7 \times 10^{10} \times 226 \times 3600 \times 24 \times 365}$

≈ 2270 years

Q.25 Theory based

Q.26 Hinsberg test



Q.27 $AB_2(aq) \longrightarrow A(g) + 2B(l)$

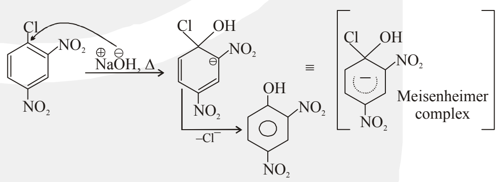
initial moles	a		
t = 20 min	a – x	x	2x
t = ∞	–	1	2a

$K = \frac{1}{t} \ln \frac{a}{a-x}$

$K = \frac{1}{20} \ln\left(\frac{40}{20}\right)$

$= \frac{0.693}{20} = 3.46 \times 10^{-2}\ \text{min}^{-1}$

Q.28 $CuSO_4 + \text{excess KI} \longrightarrow CuI \downarrow + KI_3 / I_2$
(white) (Brown)

Q.29 

Q.30 ${}^{Y}_{X}A \longrightarrow {}^{Y-4}_{X-2}B + {}^{4}_{2}He$

${}^{Y-4}_{X-2}B \longrightarrow {}^{Y-4}_{X-1}C + {}^{0}_{-1}e$

${}^{Y-4}_{X-1}C \longrightarrow {}^{Y-4}_{X}D + {}^{0}_{-1}e$

${}^{Y-4}_{X}D \longrightarrow {}^{Y-8}_{X-2}E + {}^{4}_{2}He$

PHYSICS

Q.31 From the graphs
$\lambda = 9cm$
T = 3 sec

$\Rightarrow \quad v = \frac{\lambda}{T} = \frac{9}{3}$ cm/sec = 3cm/sec.

Q.32 Combination of isoboric, isochoric & isothermal.

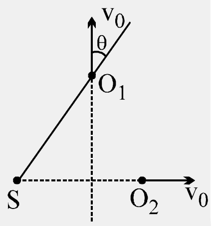
Q.33 $\frac{4L}{5} = \lambda \Rightarrow \lambda = 8cm$

thus 2 cm corresponds to $\Delta\phi = z/2$
1 cm corresponds to $\Delta\phi = z/4$

So $y = Asm\ \pi/4 = 2 \times \frac{1}{\sqrt{2}} = \sqrt{2}$

Q.34 $f_1 = f\,[\frac{v - v_0\cos\theta}{v}]$ ...(1)

$f_2 = f\,[\frac{v - v_0}{v}]$ ...(2)



$\therefore \quad \frac{f_1}{f_2} = \frac{v - v_0\cos\theta}{v - v_0} > 1$

Q.35 $ms_A\,(15 - 10) = ms_B\,(25 - 15)$
$s_A = 2s_B$
$ms_B\,(30 - 25) = ms_C\,(40 - 30)$
$s_B = 2s_C \qquad \Rightarrow s_A = 4s_C$
$ms_A\,(T - 10) = ms_C\,(40 - T)$
$\Rightarrow 4(T - 10) = 40 - T$
T = 16°C

Q.36 For ring just slides on to the steel rod the diameter of rod and ring should be equal to each other and suppose due to $\Delta\theta$ increment in temperature the diameter of both are equal then

$4\,(1 + \alpha_s\,\Delta\theta) = 3.992\,(1 + \alpha_{Brass}\,\Delta\theta)$

$4 + 4 \times 11 \times 10^{-6} \times \Delta\theta = 3.992 + 3.992 \times 20 \times 10^{-6} \times \Delta\theta$

$4 + 44 \times 10^{-6}\,\Delta\theta = 3.992 + 79.84 \times 10^{-6} \times \Delta\theta$

$0.008 = 35.84 \times 10^{-6}\,\Delta\theta$

$\frac{8 \times 10^3}{35.84} = \Delta\theta\ ;\ \Delta\theta = \frac{8000}{35.84} = 283$

so if temperature increased by 223°C then ring will start to slide and this temperature will equal to

$\theta = 30° + \Delta\theta = 30 + 253 = 283°C$

$\theta = 283°C \approx 280°C$

Q.37 From N.Law of collision
$\ell n\,(T - T_0) = -kt + \ell n\,(T_i - T_0)\ (y = -mx + x)$
equation of straight line.

Q.38 By average form of Newton's Law of cooling:

$\frac{80 - 50}{5} = k\left(\frac{80 + 50}{2} - 20\right)$ .... (i)

$\frac{60 - 30}{t} = k\left(\frac{60 + 30}{2} - 20\right)$ .... (ii)

Solving (i) and (ii) we get t = 9 minute
We should apply actual result.
By Newton's law of cooling :

$\frac{T_{initial} - T_{surrounding}}{T_{final} - T_{surrounding}} = e^{kt}$ when k is const.

$\frac{80 - 20}{50 - 20} = e^{k \times 5}$

$\Rightarrow \quad (2)^{1/5} = e^k$ .... (i)

$\frac{60 - 20}{30 - 20} = e^{kt}$

$\Rightarrow \quad (4)^{1/t} = e^k$ .... (ii)

From (i) and (ii) we get $2^{1/5} = 2^{2/t}$

$\Rightarrow \quad \frac{1}{5} = \frac{2}{t}$

$\Rightarrow \quad t = 10$ min.

Q.39 Let equation of wave as it is moving along – ve x-axis is
$y = A\sin(kx + \omega t + \alpha)$
But, $y(\lambda/4, t) = A\sin\omega t$
Comparing then
$kx + \alpha = 0 \Rightarrow \alpha = -\pi/2$

Q.40 $\frac{PV}{T} = \tan\theta = nR$

$\therefore$ slope α no. of moles

Q.41 $\Delta U = U_f - U_i$

$= \frac{3}{2} nR\Delta T = \frac{3}{2}[P_C V_C - P_A V_A]$

$= \frac{3}{2}$ [$150 \times 10^{-6} \times 200 \times 10^3 - 100 \times 10^{-6} \times 100 \times 10^3$]

= 30 J

Q.42 $\Sigma F_z = 0$

$(T + dT) + \mu g dz - T = 0$

$dT = -\mu g dz$ .... (i)

also $T = \mu v^2$

$dT = d\mu v^2 + 2v dv\, d\mu$

As v is independent of z

$dv = 0$

$dT = v^2 d\mu$ .... (ii)

from equation (1) and (2) we get

$$\mu\int \frac{d\mu}{\mu} = -\frac{g}{v^2}\int_0^z dz$$

or $\mu = \mu_0\, e^{-\left(g/v^2\right)z}$

Q.43 $Q_{abd} - Q_{acd}$

$= (W_{abd} - W_{acd}) + (DU_{abd} - DU_{acd})$

$= W_{abd} - W_{acd} + 0$

(internal energy change is same for two paths)

= area of abdca = 15 J]

$Q_{abd} = Q_{ab} + Q_{bd} = 60 + 20 = 80$ J

$Q_{acd} = Q_{abd} - 15 = 65$ J

Alternative :

From the First law of Thermodynamics, one has

$\Delta U_{a\to c\to d} = Q_{a\to c\to d} + W_{a\to c\to d} = (60\text{ J} + 20\text{ J}) + [-(8\text{Pa})(3\text{m}^3)] \Rightarrow 56$ J. Since energy is a state variable,

$\Delta U_{a\to c\to d} = Q_{a\to c\to d} + W_{a\to c\to d} \Rightarrow 56$ J

$= Q + [-(3\text{Pa})(3\text{m}^3)] \Rightarrow Q_{a\to c\to d} = 65$ J

Q.44 $\lambda = \frac{v}{f} = \frac{330}{500} = 0.66\text{ m} = \frac{4\ell}{2n-1}$

$\Rightarrow$ n = 3

Q.45 Friction force = 0.5 × 25 ×10 = 125 N

distance moved = 2×10^3

$\therefore$ work done against friction = 250×10^3 J

$\therefore$ Heat given to the body = 125×10^3 J

$\therefore T = \frac{125\times10^3}{25\times1000\times0.1\times4.2} = \frac{50}{42} = \frac{250}{21}$

= 11.9 K

Q.46 All dimension will increase

Q.47 To keep Buoyent force constant volume of submerged part must increase.

Q.48 $\frac{dQ}{dt} = \frac{kA}{\ell}(T_2 - T_1)$

$\frac{dQ}{dt}$ max if $\frac{A}{\ell}$ is max.

$\Rightarrow$ parallel to CD, AB, FG or EH.

$\frac{dQ}{dt}$ min. If $\frac{A}{\ell}$ is min.

$\Rightarrow$ parallel to CH / BG / AF / DE

$\Rightarrow$ [C]

Q.49 $u = \sigma e A T^4$ and σ_1e and T are constant

$\therefore \frac{u_2}{u_1} = \frac{A_2}{A_1} = \frac{(2\pi R^2 + \pi R^2)\times 2}{4\pi R^2} = \frac{3}{2}$

Q.50 PV × V = C

TV = C

$T' = \frac{T}{2}$

Q.51 $PV = \frac{m}{M}RT$

$V \alpha m$

$V_1 < V_2 \Rightarrow m_1 < m_2$

Q.52 Theree must be 3 half loops.

Q.53 Frequency observed by man is same as “observed” by wall and it reflects the same and as man and wall are relatively at rest, hence man observers same frequency of reflected sound. Hence no beat frequency

Q.54 $pV = N_A kT$

$N_A = \frac{pV}{kT}$

Q.55 $1° R = 1°C$

$1° S = \frac{100}{70} = \frac{10}{7}°C$

$1° U = \frac{100}{75} = \frac{4}{3}°C$

$1°S > 1°U > 1°R$

$\Rightarrow \quad x_2 > x_3 > x_1$

Q.56 $\frac{dQ}{dt} = \frac{dmL}{dt} = \frac{kA\Delta T}{L}$

Q.57 Power recived by earth from sun $\propto \frac{1}{r^2}$

Q.58 $\because \quad PV = nRT$

$10^5 \times \frac{4\pi}{3} r^2 = \frac{N}{N_V} RT$

Q.59 $P_2 = 2P_1 \quad V_2 = 4V_1 \quad n = 1$

$C = Cv + \frac{PdV}{dT}$

$dw = PdV = Area = \frac{1}{2}[(P_1 + P_2)(V_2 - V_1)]$

$= \frac{1}{2}(3P_1 \times 3V_1) = \frac{9}{2} P_1V_1$

$dT = T_2 - T_1 = \frac{P_2V_2}{R} - \frac{P_1V_1}{R}$

$= \frac{2P_1 \times 4V_1}{R} - \frac{P_1V_1}{R} = \frac{7P_1V_1}{R}$

$C = \frac{5}{2}R + \frac{9}{2}\frac{P_1V_1R}{7P_1V_1}$

$= \frac{5}{2}R + \frac{9R}{14} = \frac{44R}{14} = \frac{22R}{7}$ **Ans.**

Q.60 $0.6 = \frac{workdone}{Q_{input}} = \frac{Q_{input} - Q_{reject}}{Q_{input}}$

$= 1 - \frac{Q_r}{Q_i}$

$0.6 = 1 - \frac{20}{Q_i}$

$Q_i = 50$

$W = Q_i - Q_r = 30$ J

MATHEMATICS

Q.61 $f(x) = \frac{1}{g(x)}$; $f'(x) = \frac{-1}{g^2(x)} \cdot g'(x)$

$f'(1) = \frac{-1}{g^2(1)} g'(1) = \frac{-1}{9}\left(\frac{1}{f'(3)}\right)$

$= \frac{-1}{9}\left(\frac{1}{2}\right) = \frac{-1}{18}$. **Ans.**]

Q.62 $l = ln \lim_{t\to 0} \frac{\int_0^t (1+2\sin 3x)^{4/x} dx}{t}$

$= ln \lim_{t\to 0} (1+2\sin 3t)^{4/t}$

(using L'Hospital's rule)

$= ln\, e^{\lim_{t\to 0}\frac{4}{t}(2\sin 3t)} =$

$\lim_{t\to 0} \frac{2\cdot 3\cdot 4\cdot \sin 3t}{3t} = 24$ Ans.]

Q.63 $g(x^3 + 1) = x^6 + x^3 + 2 = (x^3 + 1)^2 - x^3 + 1$

$=(x^3+1)^2-(x^3+1-1)+1=(x^3+1)^2-(x^3+1)+2$

Put $\quad x^3 + 1 = t$

So, $\quad g(t) = t^2 - t + 2$

$\Rightarrow \quad g(x^2 - 1) = (x^2 - 1)^2 - (x^2 - 1) + 2$

$= x^4 - 3x^2 + 4$. **Ans.**]

Q.64 $g(x) = f(-x + f(f(x)))$;

$f(0) = 0; \qquad f'(0) = 2$

$g'(x) = f'(-x + f(f(x)))\cdot[-1 + f'(f(x))\cdot f'(x)]$

$g'(0) = f'(f(0))\cdot[-1 + f'(0)\cdot f'(0)]$

$= f'(0)[-1 + (2)(2)]$

$= (2)(3) = 6$ **Ans.**]

Q.65 $I = \int_1^\infty \frac{dx}{(e\cdot e^x + e^3\cdot e^{-x})} = \int_1^\infty \frac{e^x dx}{e(e^{2x} + e^2)}$

(multiply N^r and D^r by e^x)

put $e^x = t \quad \Rightarrow \quad e^x dx = dt$

$I = \frac{1}{e}\int_e^\infty \frac{dt}{t^2 + e^2} = \frac{1}{e^2}\tan^{-1}\frac{t}{e}\Big|_e^\infty$

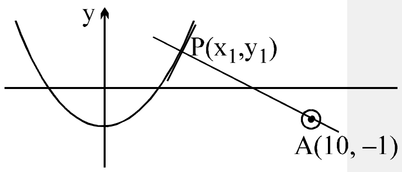
$= \frac{1}{e^2}\left[\frac{\pi}{2} - \frac{\pi}{4}\right] = \frac{\pi}{4e^2}$ Ans.]

Q.66 $\left.\frac{dy}{dx}\right|_P = \frac{2x_1}{4} = \frac{x_1}{2}$

$\Rightarrow$ slope of normal $= -\frac{2}{x_1}$

$\Rightarrow \quad -\frac{2}{x_1} = \frac{y+1}{x_1 - 10}$

$\Rightarrow 20 - 2x_1 = x_1y_1 + x_1$



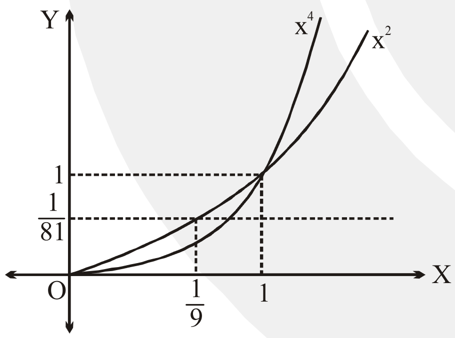
$\Rightarrow \quad 3x_1 + x_1y_1 = 20 \qquad ....(1)$

also $\quad y_1 = \frac{x_1^2}{4} - 2$

$\Rightarrow \quad 4y_1 = x_1^{\,2} - 8 \quad ....(2)$

only (D) satisfies (1) and (2) both.]

Q.67 Clearly $f(x) = \begin{cases} \frac{1}{81}, 0 \le x \le \frac{1}{9} \\ x^2, \frac{1}{9} < x \le 1 \\ x^4, x > 1 \end{cases}$



Clearly f(x) is non differentiable at $x = \frac{1}{9}, 1$

$\therefore$ sum of squares of reciprocals

$= 9^2 + 1 = 82$ Ans.]

Q.68 f is not differentiable at $x = \frac{1}{2}$

g is not continuous in [0, 1] at x = 0 & 1

h is not continuous in [0, 1] at x = 1

$k(x) = (x+3)^{\ln_2 5} = (x+3)^p$ where $2 < p < 3$

Q.69 f (2) = 10, hence $2ae^{-2b} = 10$

$\Rightarrow \quad ae^{-2b} = 5 \qquad ....(1)$

$f'(x) = a\,[e^{-bx} - bx\, e^{-bx}] = 0$

$f'(2) = 0$

$a(e^{-2b} - 2be^{-2b}) = 0$

$ae^{-2b}(1 - 2b) = 0$

$\Rightarrow \quad b = 1/2$ or $a = 0$ (rejected)

from (1) if $b = 1/2$; $\quad a = 5e$

$\therefore \quad a = 5e$ and $b = 1/2$ Ans.]

Q.70 $f(x, n) = \sum_{k=1}^{n} \log_x\left(\frac{k}{x}\right)$

$= \log_x\left(\frac{1}{x}\right) + \log_x\left(\frac{2}{x}\right) + ...\log_x\left(\frac{n}{x}\right) = \log_x\left(\frac{n!}{x^n}\right)$

given: f(x, 10) = f(x, 11)

$\Rightarrow \log_x\left(\frac{10!}{x^{10}}\right) = \log_x\left(\frac{11!}{x^{11}}\right) \Rightarrow \frac{10!}{x^{10}} = \frac{11!}{x^{11}}$

$\Rightarrow x = 11$ Ans.]

Q.71 $T_r = \frac{1}{\sqrt{\frac{r}{n}} \cdot n\left(3\sqrt{\frac{r}{n}} + 4\right)^2}$

$S = \frac{1}{n}\sum_{1}^{4n} \frac{1}{\left(3\sqrt{\frac{r}{n}} + 4\right)^2 \cdot \sqrt{\frac{r}{n}}}$

$= \int_0^4 \frac{dx}{\sqrt{x}(3\sqrt{x} + 4)^2}$

put $\quad 3\sqrt{x} + 4 = t$

$\Rightarrow \quad \frac{3}{2}\frac{1}{\sqrt{x}}dx = dt$

$= \frac{2}{3}\int_4^{10} \frac{dt}{t^2} = \frac{2}{3}\left[\frac{1}{t}\right]_{10}^{4} = \frac{2}{3}\left[\frac{1}{4} - \frac{1}{10}\right] = \frac{2}{3} \cdot \frac{6}{40} = \frac{1}{10}$

Q.72 We have $\sin^{-1}\left[\frac{\pi x}{6}\right] > 0 \Rightarrow \left[\frac{\pi x}{6}\right] = 1$

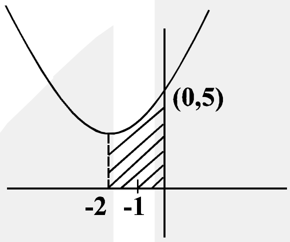
$\Rightarrow 1 \le \frac{\pi x}{6} < 2 \Rightarrow \frac{6}{\pi} \le x < \frac{12}{\pi}$

$\therefore$ x = 2, 3 only.

Hence two integral solution will satisfy above equation.]

Q.73 $y = x^2 + 4x + 5 = (x+2)^2 + 1$

$$A = \int_{-2}^{0}\left(x^2+4x+5\right)dx = \left.\frac{x^3}{3}+2x^2+5\right]_{-2}^{0}$$



$$= -\left[-\frac{8}{3}+8-10\right] = 2+\frac{8}{3} = \frac{14}{3} = 4\frac{2}{3}$$

Q.74 equation $(x-a)^2 + y^2 = (x-b)^2$ [S = (a , 0) ; D : x = b]

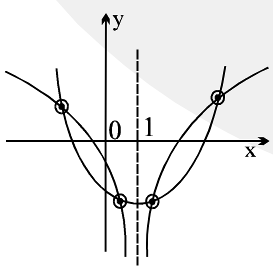
$$y^2 = (b^2 - a^2) + 2x(a - b)$$

differentiate twice to get $y\frac{d^2y}{dx^2} + \left[\frac{dy}{dx}\right]^2 = 0$;

$$y\frac{d^2y}{dx^x} + \left(\frac{dy}{dx}\right)^2 = 0.$$

Q.75 $x^2 - 2x - 3 = \log_2|1-x|$

4 points]



Q.76 $F(x) = \int \frac{3x+2}{\sqrt{x-9}}dx$; let $x - 9 = t^2$

$\Rightarrow$ dx = 2t dt

$\therefore$ $F(x) = \int\left(\frac{3(t^2+9)+2}{t}\cdot 2t\right)dt$

$= 2\int\left(29+3t^2\right)dt = 2\,[29t + t^3]$

$F(x) = 2\left[29\sqrt{x-9}+(x-9)^{3/2}\right] + C$

given $F(10) = 60 = 2\,[29 + 1] + C$

$\Rightarrow$ C = 0

$\therefore$ $F(x) = 2\left[29\sqrt{x-9}+(x-9)^{3/2}\right]$

$F(13) = 2\,[29 \times 2 + 4 \times 2]$

$= 4 \times 33 = 132$ Ans.]

Q.77 $\lim_{x\to 0} f(x) = 0$

$\left(\because \lim_{x\to 0} x^2 = 0 \text{ and } \left\{e^{1/x}\right\} \text{is a bounded function}\right)$

$\therefore$ k = 0

Now, $f'(0) = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$

$= \lim_{h\to 0} h\left\{e^{1/h}\right\} = 0 \Rightarrow f'(0) = 0$ **Ans.**

Note that f(x) is discontinuous at

$x = \pm\frac{1}{\ln 2}, \pm\frac{1}{\ln 3}$ and so on.]

Q.78 $\sin x = t$; $I = \int\frac{\left(1-t^2\right)\left(2-t^2\right)}{t^2\left(1+t^2\right)}dt$;

$f(t) = \int\frac{(y-1)(y-2)}{y(1+y)} = 1 + \frac{2(1-2y)}{y(y+1)}$; $y = t^2$

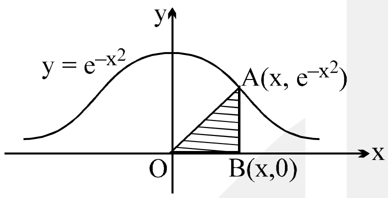
$= 1 + 6\left[\frac{1}{3y} - \frac{1}{y+1}\right]$; $\int\left(1+\frac{2}{t^2}-\frac{6}{1+t^2}\right)dt$]

Q.79 $A = \frac{x e^{-x^2}}{2}$;

$A' = \frac{1}{2}\left[e^{-x^2} - 2x^2 \cdot e^{-x^2}\right]$

$= \frac{e^{-x^2}}{2}\left[1 - 2x^2\right] = 0$

$\Rightarrow \ x = \frac{1}{\sqrt{2}}$ gives $A_{max.}$



$\therefore \quad A_{max} = \frac{e^{-1/2}}{2\sqrt{2}} = \frac{1}{\sqrt{8e}}$]

Q.80 f(x) will be continuous where
$3 \sin x + a^2 - 10a + 30 = 4\cos x$

or $\underbrace{a^2 - 10a + 30}_{\geq 5} = \underbrace{4\cos x - 3\sin x}_{\leq 5}$

or $(a - 5)^2 + 5 = 4\cos x - 3\sin x$

$\therefore$ $a = 5$ and $4 \cos x - 3 \sin x = 5$

$\Rightarrow \frac{4}{5}\cos x - \frac{3}{5}\sin x = 1$

or $\cos(x + \theta) = 1$, where $\tan\theta = \frac{3}{4}$

$\therefore \ x = 2n\pi - \theta = 2n\pi - \tan^{-1}\frac{3}{4}, \ n \in I.$

Q.81 $x = -\pi/4$; $y = \cos\frac{\theta}{2}$; where $\cos\frac{\theta}{2} = \frac{1}{8}$ and

$\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}} = \frac{3}{4}$]

Q.82 Solve graphically]

Q.83

$P'(x) = f(x)\, g'(x) + g(x)\, f'(x)$
$P'(2) = f(2)\, g'(2) + g(2)\, f'(2)$
$= (1)(2) + 4(-1)$
$= -2$

$Q'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{g^2(x)}$

$Q'(2) = \frac{(4)(-1) - (1)(2)}{16} = -\frac{6}{16} = -\frac{3}{8}$

$C'(x) = f'(g(x))g'(x)$
$C'(2) = f'(4) \cdot 2 = 3 \cdot 2 = 6$]

Q.84 Let $\int_0^x f(t)\,dt = T(x) \ \Rightarrow \ T'(x) = f(x)$

$\therefore$ On differentiating b.t.s. w.r.t. x, we get
$f(x) = T'(x)$
Hence

$G(x) = \int e^x \left(\int_0^x f(t)\,dt + f(x)\right) dx$

$= \int e^x \left(T(x) + T'(x)\right) dx = e^x\, T(x) + C$

$\Rightarrow \quad G(x) = e^x \int_0^x f(t)dt + C$

Now on differentiating

$G'(x) = e^x \int_0^x f(t)dt + e^x f(x)$

$\Rightarrow$ G'(0) = f(0) = 1 **Ans.**]

Q.85 $\lim_{n\to\infty} \frac{e^n}{\left(1+\frac{1}{n}\right)^{n^2}} = \lim_{n\to\infty} \frac{e^n}{e^{n^2 \ln\left(1+\frac{1}{n}\right)}}$

$= \lim_{n\to\infty} e^{n - n^2 \ln\left(1+\frac{1}{n}\right)}$; Put $n = \frac{1}{y}$

$= \lim_{y\to 0} e^{\frac{y - \ln(1+y)}{y^2}} = e^{\frac{1}{2}} = \sqrt{e}$ **Ans.**]

Alternatively: $L = \lim_{n\to\infty} \frac{e^n}{\left(1+\frac{1}{n}\right)^{n^2}}$

$\Rightarrow \ln L = \lim_{n\to\infty}\left(n - n^2 \ln\left(1+\frac{1}{n}\right)\right)$

Put $n = \frac{1}{y}$,

we get $\ln L = \lim_{y\to 0} \frac{y - \ln(1+y)}{y^2}$

$\Rightarrow \ln L = \lim_{y\to 0} \frac{y - \left(y - \frac{y^2}{2} + ...\right)}{y^2} = \frac{1}{2}$

$\Rightarrow L = e^{\frac{1}{2}} = \sqrt{e}$ **Ans.**]

Q.86 We have $\frac{dy}{dx} = (e^y - x)^{-1} \Rightarrow \frac{dx}{dy} = e^y - x$

$\Rightarrow \frac{dx}{dy} + x = e^y$; So I.F. $= e^{\int dy} = e^y$

$\therefore$ General solution is given by

$x e^y = \frac{1}{2}e^{2y} + C \Rightarrow x = \frac{e^y}{2} + Ce^{-y}$

As $y(0) = 0$, so $C = \frac{-1}{2}$

$\therefore x = \frac{e^y}{2} - \frac{1}{2}e^{-y} \Rightarrow e^y - e^{-y} = 2x$

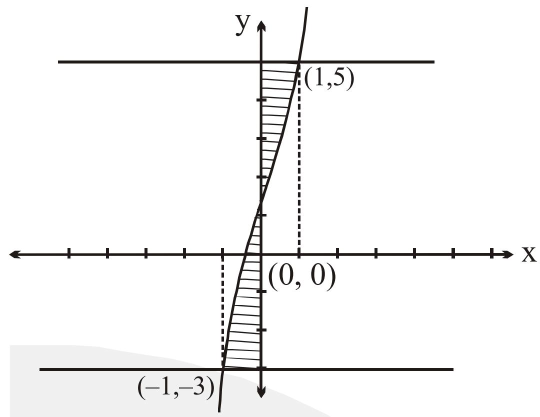
$\Rightarrow e^{2y} - 2xe^y - 1 = 0 \Rightarrow 2e^y = 2x \pm \sqrt{4x^2+4}$

But $e^y = x - \sqrt{x^2+1}$

(Rejected)

Hence $y = \ln\left(x + \sqrt{x^2+1}\right)$]

Q.87



Area

$= \int_{-1}^{0}\left((x^3+3x+1)-(-3)\right)dx + \int_{0}^{1}\left(5-(x^3+3x+1)\right)dx$

$= \frac{9}{2}$ **Ans.**]

Q.88 In the integral J, substitute $x + 1 = t$

$\Rightarrow dx = dt$ and $x^2 + 2x = (t^2 - 1)$

Now $J = \int_1^e \frac{e^{\frac{t^2-2}{2}}}{t}dt$ and $K = \int_1^e t\ln t\, e^{\frac{t^2-2}{2}}dt$

Hence $(J + K) = \int_1^e e^{\frac{t^2-2}{2}}\left(\frac{1}{t} + t\ln t\right)dt$

$= \left(e^{\frac{t^2-2}{2}}\ln t\right)_{t=1}^{t=e} = e^{\frac{e^2-2}{2}} = \left(\sqrt{e}\right)^{e^2-2}$

Q.89 Let (x_1, y_1) and (x_2, y_2) are two of these points given $y = x^3 + 2x - 1$ and $y = 2x^3 - 4x + 2$

$\therefore \quad y_1 = 2x_1^3 - 4x_1 + 2 \quad ....(1)$

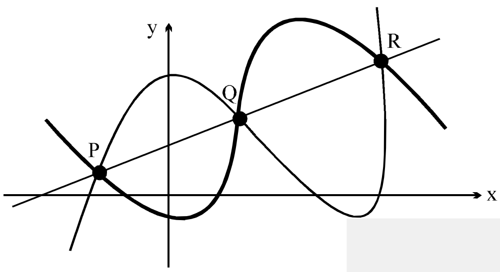
and $\quad 2y_1 = 2x_1^3 + 4x_1 - 2 \quad ....(2)$

(2) – (1)

$y_1 = 8x_1 - 4 \quad ....(3)$

|||ly $\quad y_2 = 8x_2 - 4 \quad ....(4)$

$y_2 - y_1 = 8(x_2 - x_1)$



$$\frac{y_2 - y_1}{x_2 - x_1} = 8$$ **Ans.**]

Q.90 $T_r = \frac{5(r+1) - 3r}{r(r+1)} \cdot \left(\frac{3}{5}\right)^{r+1}$

$$= \left(\frac{5}{r} - \frac{3}{r+1}\right)\left(\frac{3}{5}\right)^{r+1}$$

$$= \frac{5}{r} \cdot \frac{3}{5} \cdot \left(\frac{3}{5}\right)^{r} - \frac{3}{r+1}\left(\frac{3}{5}\right)^{r+1}$$

$$= 3\left[\frac{1}{r} \cdot \left(\frac{3}{5}\right) - \frac{1}{r+1}\left(\frac{3}{5}\right)^{r+1}\right]$$

$$\therefore \ S_n = \sum_{r=1}^{n} T_r$$

$$T_1 = 3\left[\frac{1}{1}\left(\frac{3}{5}\right)^1 - \frac{1}{2}\left(\frac{3}{5}\right)^2\right]$$

$$T_2 = 3\left[\frac{1}{2}\left(\frac{3}{5}\right)^2 - \frac{1}{3}\left(\frac{3}{5}\right)^3\right]$$

:

$$T_n = 3\left[\frac{1}{n}\left(\frac{3}{5}\right)^n - \frac{1}{n+1}\left(\frac{3}{5}\right)^{n+1}\right]$$

$$S_n = 3\left[\frac{3}{5} - \frac{1}{(n+1)}\left(\frac{3}{5}\right)^{n+1}\right]$$

$$\therefore \ \lim_{n\to\infty} S_n = \frac{9}{5}.$$ **Ans.**

Aliter: $T_r = \left(\frac{2r+5}{r(r+1)}\right)\left(\frac{3}{5}\right)^{r+1}$

$$= \left(\frac{5(r+1) - 3r}{r(r+1)}\right)\left(\frac{3}{5}\right)^{r+1}$$

$$= 3\left[\frac{1}{r}\left(\frac{3}{5}\right)^r - \frac{1}{r+1}\left(\frac{3}{5}\right)^{r+1}\right]$$

$$= 3(r_r - v_{r+1})$$

So, $\sum_{r=1}^{n} T_r = 3\left[\sum_{r=1}^{n} v_r - \sum_{r=1}^{n} v_{r+1}\right]$

$$\Rightarrow \ S_n = 3\,(v_1 - v_{n+1}) = \frac{9(n+1)5^n - 3^{n+2}}{(n+1)\,5^{n+1}}$$

So, $\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{9(n+1)5^n - 3^{n+2}}{(n+1)\,5^{n+1}} = \frac{9}{5}.$