JEE MAIN

COURSE
NUCLEUS

TEST CODE
11297

MOCK TEST-09

Class: **XII** ***Time:*** **3 Hours.** ***Max. Marks:*** **360**

IMPORTANT INSTRUCTIONS

1. The question paper consists of **'90'** objective type questions. There are **'30'** questions each in **Chemistry** , **Mathematics** and **Physics** respectively. **Please fill the OMR answer Sheet accordingly and carefully.**

2. Each question has four choices (1), (2), (3) and (4) out of which **ONLY ONE** is correct.

3. You will be **awarded 4 marks** for each question, if you have darkened only the bubble corresponding to the correct answer and zero mark if no bubble are darkened. In all other cases, **minus one (–1) mark** will be awarded.

4. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 3 above.

5. Use **Black or Blue Ball Point Pen** only for filling particulars.

6. Use of **Calculator, Log Table, Slide Rule and Mobile** is not allowed.

7. Rough work is to be done on the space provided at the bottom and in end of the booklet for this purpose in the Test Booklet only.

8. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator. However, the candidates are allowed to take away this Test Booklet with them.

9. Do not fold or make any stray marks on the Answer Sheet.

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USEFUL DATA

Atomic weights: Al = 27, Mg = 24, Cu = 63.5, Mn = 55, Cl = 35.5, O = 16, H = 1, P = 31, Ag = 108, N = 14, Li = 7, I = 127, Cr = 52, K=39, S = 32, Na = 23, C = 12, Br = 80, Fe = 56, Ca = 40, Zn = 65.5, Ti = 48, Ba = 137, U = 238, Co= 59, B =11, F = 19, He = 4, Ne = 20, Ar = 40 , Mo = 96

[Take : $ln\ 2 = 0.693$, $ln\ 1.1 = 0.095$, $ln\ 3 = 1.09$, $e = 1.6 \times 10^{-19}$, $m_e = 9.1 \times 10^{-31}$ kg]

Take: $\epsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$, $g = 10\ m/s^2$, $S_{water} = 1$ cal/gm °C, $L_{ice} = 80$ cal/gm. , $g = 10\ m/s^2$ unless otherwise stated

CHEMISTRY

Q.1 Consider the following process and respective energy change -

$M(s) \rightarrow M(g)$ .....(a)

$M(s) \rightarrow M^{2+}(g) + 2e^-$ .....(b)

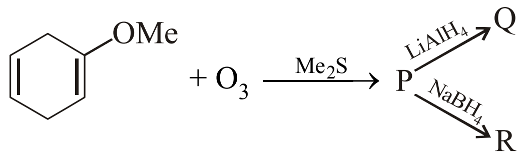
$M(g) \rightarrow M^+(g) + e^-$ .....(c)

$M^+(g) \rightarrow M^{2+}(g) + e^-$ .....(d)

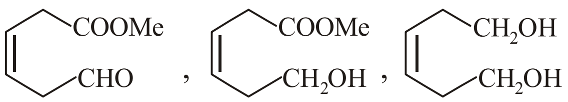
$M(g) \rightarrow M^{2+}(g) + 2e^-$ .....(e)

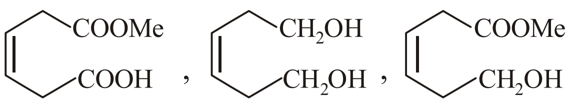
Energy required to change M(s) into $M^+(g)$ is -

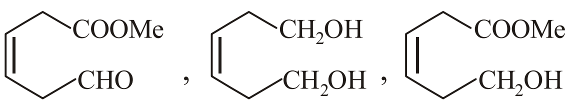
(1) a + c + d (2) a + c (3) a + e – c (4) e – c

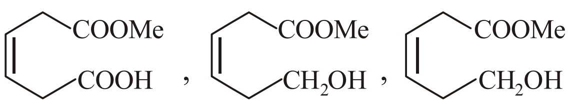
Q.2 

Product P, Q, R will be respectively:

(1) 

(2) 

(3) 

(4) 

SPACE FOR ROUGH WORK

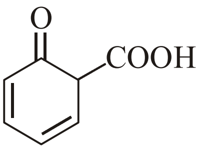
Q.3 In an unielectronic species, the number of revolution per second made by the electron in 4th orbit is twice of the number of revolutions per second made by the electron in 2nd orbit of H-atom. The unielectronic specie is,

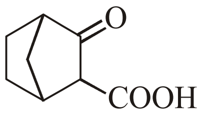
(1) H (2) He^{+} (3) Li^{2+} (4) Be^{3+}

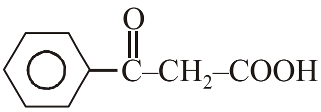
Q.4 Which metal gives H_2 gas on reaction with NaOH solution-

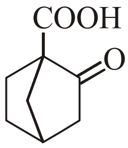
(1) Zn (2) Mg (3) Fe (4) Cu

Q.5 Which of the following will not undergo decarboxylation.

(1) 

(2) 

(3) 

(4) 

Q.6 50 ml aliquot of a H_2O_2 (aq.) solution is titrated against 200 ml of 0.2 M acidified $KMnO_4$ (aq.) solution. After the equivalence point, remaining $KMnO_4$ (aq.) solution requires 100 ml, 0.5 M $H_2C_2O_4$ (aq.) solution in acidic medium. The volume strength of H_2O_2 (aq.) solution is

(1) 1 V (2) 11.2 V (3) 5.6 V (4) 22.4 V

Q.7 The aqueous solution of slaked lime in excess water is known as -

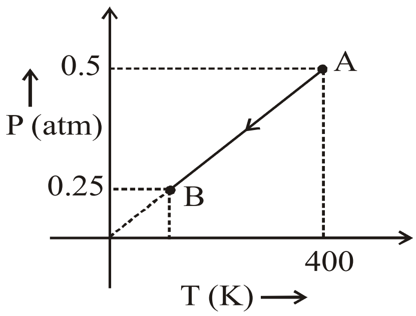
(1) Lime water (2) Lime stone (3) Milk of lime (4) Quick lime

Q.8 How many stereoisomers will be formed of 2-methyl hepta-3E, 5E-dienoic acid ?

(1) 4 (2) 6 (3) 7 (4) 8

SPACE FOR ROUGH WORK

Q.9 Two moles of Ar (g) are subjected to following change:



Considering ideal behaviour, the heat lost by the gas during process AB will be :

(1) –1200 Cal (2) 1200 Cal (3) –2400 Cal (4) 1000 Cal

Q.10 In which of the following process, the bond order has increased and the magnetic behaviours is changed?

(1) $C_2^+ \rightarrow C_2$ (2) $NO^+ \rightarrow NO$ (3) $O_2 \rightarrow O_2^+$ (4) $N_2 \rightarrow N_2^+$

Q.11 Which order of basicity is correct for given groups :

(I) NH_3 (II) $C_2H_5NH_2$ (III) $(C_2H_5)_2NH$ (IV) $(C_2H_5)_3N$

(1) IV > III > II >I (2) III > IV > II > I (3) III > II > IV > I (4) I > II > III > IV

Q.12 At 25°C, consider following reactions,

$$NO + SO_3 \rightleftharpoons NO_2 + SO_2 ; \quad K_C = 3$$
$$2SO_2 + O_2 \rightleftharpoons 2SO_3 ; \quad K_C = 16\ (mol / L)^{-1}$$

The value of equilibrium constant (K_C) for the reaction : $2NO_2 \rightleftharpoons 2NO + O_2$ will be :

(1) 12 mol / L (2) 144 mol/L (3) 9 mol / L (4) $\frac{1}{144}$ mol / L

Q.13 Which of the following is an oxidizing agent?

(1) $[Mn(CO)_5]$ (2) $[Fe(CO)_5]$ (3) $[Mn_2(CO)_{10}]$ (4) $[Fe_2(CO)_5]$

Q.14 Select incorrect statement in the given:

(1) D-glucose and D-galactose are used to form lactose.

(2) Sucrose is the non-reducing sugar

(3) Glucose gives positive Tollen's test

(4) Glycine that is an amino acid is optically active.

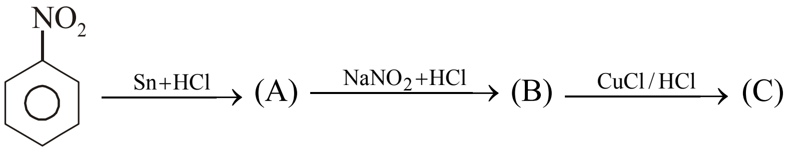
SPACE FOR ROUGH WORK

Q.15 In Cesium iodide crystal, iodide ions form ideal simple cubic lattice while Cs^+ ions occupied cubic voids. If radius of I^- ion is 500 pm then radius of Cs^+ ion is nearly

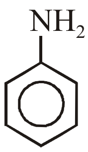
(1) 850 pm (2) 150 pm (3) 366 pm (4) 1000 pm

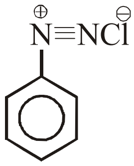
Q.16 Which of the following is not a common product of hydrolysis of XeF_2 and XeF_4 ?

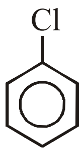
(1) Xe (2) XeO_3 (3) HF (4) O_2

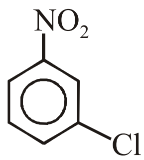
Q.17 

Final product C is :

(1) 

(2) 

(3) 

(4) 

Q.18 During electrophoresis of a sol -

(1) The ferric hydroxide (sol) particles get accumulated near the cathode
(2) The sol particles of metals and their sulphides are accumulted at cathode.
(3) Basic dye methylene blue are accumulted at anode.
(4) Neither DP nor DM particles move in an electric field.

Q.19 **Correct** order of bond angle is -

(1) $PH_4^+ > OF_2 > SF_2 > SbH_3$ (2) $OF_2 > SF_2 > PH_4^+ > SbH_3$
(3) $PH_4^+ > SF_2 > OF_2 > SbH_3$ (4) $SF_2 > OF_2 > PH_4^+ > SbH_3$

Q.20 $H_5C_2-\underset{}{\overset{H_3C}{CH}}-\overset{O}{\overset{\|}{C}}-NH_2 \xrightarrow[\Delta]{Br_2/KOH} P \xrightarrow[H_2O]{NaNO_2+HCl} Q \xrightarrow{PCC} R$

Final product R does not give

(1) Haloform reaction (2) Tautomerisation
(3) Tollen's Test (4) Fehling Test

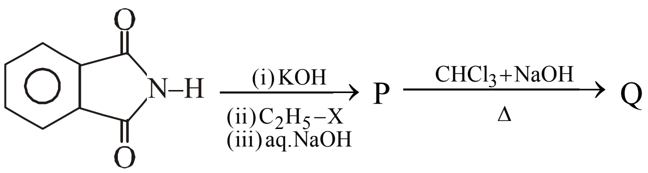
SPACE FOR ROUGH WORK

Q.21 At 25°C, if $E^o_{Sn^{2+}|Sn} = x$ volt and $E^o_{Sn^{4+}|Sn} = y$ volt then $E^o_{Sn^{2+}|Sn^{4+}}$ in volt will be :

(1) $x - y$ (2) $2x + 4y$ (3) $x - 2y$ (4) $4y - 2x$

Q.22 Boric acid polymerizes due to -

(1) the presence of hydrogen bond. (2) its acidic nature.
(3) its geometry (4) its monobasic nature.

Q.23 

Which of the given statement is incorrect :

(1) Product P reacts with grignard reagent to form hydrocarbon.
(2) Product P gives reaction with benzene sulphonyl chloride
(3) Product Q is a good smelling compound
(4) Product Q on acidic hydrolysis gives an amide.

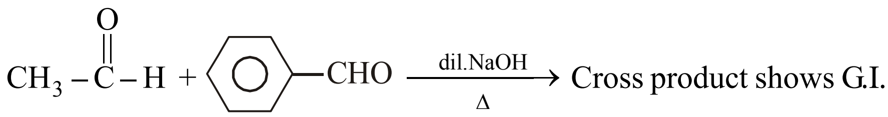
Q.24 At 373 K, the vapour pressure of pure water decreases by 190 torr when a certain amount of a non-volatile solute is dissolved. The mole fraction of solute in the solution is

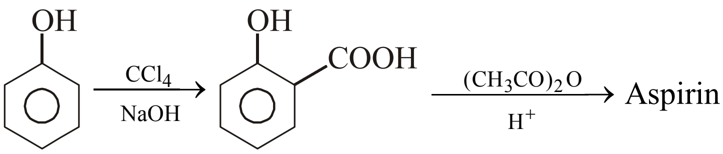
(1) 19 (2) 25 (3) 0.75 (4) 0.25

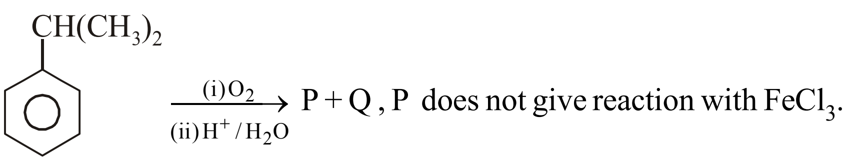
Q.25 Which one is mismatched ?

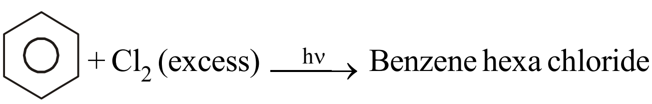
(1) Poling - refining of copper. (2) Cupellation - refining of silver
(3) Roasting - An oxidation process. (4) Smelting - An oxidation process.

Q.26 For which of the following reaction product is not correctly matched.

(1) 

(2) 

(3) 

(4) 

Q.27 A substance X decomposes following 1st order kinetics with a half life of 100 minutes. The fraction of initial concentration of X, which is reacted after 300 minutes will be :

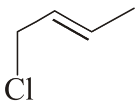
(1) $\frac{1}{8}$ (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) $\frac{7}{8}$

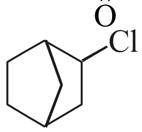
Q.28 Which of the following molecule have only one pπ-dπ bond and zero pπ-pπ bonds?

(1) ClO_3^{-1} (2) SO_2 (3) ClF_3 (4) SO_3^{2-}

Q.29 Find out correct order of reactivity of given compounds with respect to S_N2

(p) $Me-\underset{\underset{O}{||}}{C}-CH_2-Cl$

(q) 

(r) 

(s) $Me-O-CH_2-Cl$

(1) p > s > q > r (2) r > q > s > p (3) s > p > r > q (4) q > r > p > s

Q.30 At 25°C, K_{sp} values of four salts AB(s), MB_2(s), N_3C_2(s) and A_3C(s) are numerically equal. For dissolution of all salts in pure water, the only **correct** statement is -

(1) Solubility of all salts are same. (2) MB_2(s) is more soluble than A_3C(s).

(3) N_3C_2(s) is most soluble. (4) AB(s) is most soluble.

SPACE FOR ROUGH WORK

Q.31 If $y = \dfrac{ax^2 - 3x + 5}{5x^2 - 3x + a}$, then number of possible integral values of 'a' for which y may be capable of all values $\forall\, x \in R$ is

(1) 4 (2) 5 (3) 6 (4) none of these

Q.32 Let a^2, b^2 and c^2 be three distinct numbers in AP. If $ab + bc + ca = 1$ then $(b + c)$, $(c + a)$ and $(a + b)$ are in

(1) AP (2) GP (3) HP (4) none of these

Q.33 $\dfrac{\sin 20^\circ + \cos 20^\circ + \sin 50^\circ}{\cos 10^\circ \cdot \sin 35^\circ \cdot \cos 25^\circ}$ equals

(1) 4 (2) 2 (3) 3 (4) 15

Q.34 Let $f(x) = [x] + \sqrt{\{x\}} + 1$ and $g(x) = [x] + \{x\}^2 - 1$ then $\int_0^2 f(x)dx + \int_1^3 g(x)dx$ equals

Where [k] denotes greatest integer function less than or equal to k and { x } is fractional part function

(1) 2 (2) 3 (3) 6 (4) 0

Q.35 Let $\alpha_0, \alpha_1, \alpha_2, \ldots\ldots., \alpha_{n-1}$ be the n distinct n^{th} roots of the unity, then the value of $\sum_{r=0}^{n-1} \dfrac{\alpha_r}{3 - \alpha_r}$ is equal to

(1) $\dfrac{n}{3^n - 1}$ (2) $\dfrac{n-1}{3^n - 1}$ (3) $\dfrac{n+1}{3^n - 1}$ (4) $\dfrac{n+2}{3^n - 1}$

Q.36 The value of $\sqrt{7 + \sqrt{7 - \sqrt{7 + \sqrt{7 - \ldots\ldots.\infty}}}}$ is

(1) 5 (2) 4 (3) 3 (4) 2

SPACE FOR ROUGH WORK

Q.37 If $x \in \{1, 2, 3, \ldots\ldots, 9\}$ and $f_n(x) = x\,x\,x \ldots\ldots x$ (n-digits) then $\left(f_n(3)\right)^2 + f_n(2)$ is equal to

(1) $2f_{2n}(1)$ (2) $f_n^{\,2}(1)$ (3) $f_{2n}(1)$ (4) $f_{2n}(4)$

Q.38 Number of points having position vector $a\hat{i} + b\hat{j} + c\hat{k}$ where $a, b, c \in \{1, 2, 3, 4, 5\}$ such that $2^a + 3^b + 5^c$ is divisible by 4 is

(1) 70 (2) 140 (3) 210 (4) 280

Q.39 If n be an integer and x, y, z, w are distinct, the number of distinct terms in the expansion of $(x + y + z + w)^n$ is

(1) ${}^{n}C_2$ (2) ${}^{n+2}C_2$ (3) ${}^{n+3}C_n$ (4) ${}^{n}C_3$

Q.40 If $f(x) = \begin{vmatrix} x^2 + 3x & x - 1 & x - 3 \\ x + 1 & 2 - x & x - 3 \\ x - 3 & x + 4 & 3x \end{vmatrix}$, then $f'(0)$ is equal to

(1) – 39 (2) 64 (3) 24 (4) none of these

Q.41 If $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$, then $P^TQ^{2019}P$ is equal to

(1) $\begin{bmatrix} 1 & 2019 \\ 0 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} \frac{\sqrt{3}}{2} & 2019 \\ 0 & \frac{\sqrt{3}}{2} \end{bmatrix}$ (3) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{2019}{2} \\ \frac{-2019}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (4) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Q.42 Let A = {1, 3, 5, 7, 9} and B = {2, 4, 6, 8} be two set. An element (a, b) of their cartesian product A × B is chosen at random. The probability that (a + b) = 9 is

(1) $\frac{4}{5}$ (2) $\frac{3}{5}$ (3) $\frac{2}{5}$ (4) $\frac{1}{5}$

SPACE FOR ROUGH WORK

Q.43 Let $f : [0, 1] \rightarrow R$ be a function defined as $f(x) = x^3 - x^2 + 4x + 2\sin^{-1}x$, then number of integers in the range of the function $y = f(x)$ is

(1) 2 (2) 4 (3) 8 (4) 16

Q.44 $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\sin x - (\sin x)^{\sin x}}{1 - \sin x + \ln(\sin x)}$ is equal to

(1) 2 (2) 1 (3) 3 (4) 0

Q.45 Let $f(x) = x^3 + ax^2 + bx + 5\sin^2 x$ be an increasing function $\forall\ x \in R$, then which of the following must be CORRECT?

(1) $a^2 - 3b - 15 > 0$ (2) $a^2 - 3b + 15 < 0$ (3) $a^2 - 3b + 25 < 0$ (4) $a^2 - 3b + 15 > 0$

Q.46 The co-ordinate of the point on $y^2 = 8x$ which is closest from $x^2 + (y + 6)^2 = 1$ are

(1) (2, – 4) (2) (18, – 12) (3) (32, 16) (4) (32, – 16)

Q.47 The area bounded by $y = 2 - |2 - x|$; $y = \dfrac{3}{|x|}$ is

(1) $3\ln 5$ (2) $\dfrac{7\ln 2}{2}$ (3) $\dfrac{5 - 3\ln 3}{2}$ (4) none

Q.48 Solution of $\left(\dfrac{x + y - 1}{x + y - 2}\right)\dfrac{dy}{dx} = \left(\dfrac{x + y + 1}{x + y + 2}\right)$, if at x = 1, y = 1, is

(1) $\ln\left|\dfrac{(x - y)^2 - 2}{2}\right| = 2(x + y)$ (2) $\ln\left|\dfrac{(x - y)^2 - 2}{2}\right| = 2(x - y)$

(3) $\ln\left|\dfrac{(x - y)^2 + 2}{2}\right| = 2(x + y)$ (4) $\ln\left|\dfrac{(x - y)^2 + 2}{2}\right| = 2(x - y)$

SPACE FOR ROUGH WORK

Q.49 The four sides of a quadrilateral are given by the equation $xy(x-2)(y-3)=0$. The equation of the line parallel to $x-4y=0$ that divides the quadrilateral in two equal areas is

(1) $x-4y-5=0$ (2) $x-4y+5=0$ (3) $x-4y-1=0$ (4) $x-4y+1=0$

Q.50 If $\frac{\sin^4\theta}{5}+\frac{\cos^4\theta}{1}=\frac{1}{6}$, then $\sec^2\theta$ is equal to

(1) 5 (2) 1 (3) 6 (4) 2

Q.51 If the line $x\cos\alpha + y\sin\alpha = P$ cuts the circle $x^2+y^2=a^2$ at A and B $(0<P<a)$ then the equation of circle, whose one diameter is line segment AB, is

(1) $x^2+y^2-a^2+2P(x\cos\alpha+y\sin\alpha-P)=0$

(2) $x^2+y^2-a^2-2P(x\cos\alpha+y\sin\alpha-P)=0$

(3) $x^2+y^2-a^2-4P(x\cos\alpha+y\sin\alpha+P)=0$

(4) none of these

Q.52 If the chord of contact of tangents from a point P to the parabola $y^2=4ax$ touches the parabola $x^2=4by$, then the locus of P is a/an

(1) circle (2) parabola (3) ellipse (4) hyperbola

Q.53 The equation $(2x-y+1)^2+4(x+2y-3)^2=16$ represents a conic whose centre is

(1) $\left(\frac{7}{5},\frac{1}{5}\right)$ (2) $\left(\frac{1}{5},\frac{7}{5}\right)$ (3) $(1, 7)$ (4) none of these

Q.54 The equation of the common tangent to the curves $y^2=8x$ and $xy=-1$ is

(1) $3y=9x+2$ (2) $y=2x-1$ (3) $2y=x+8$ (4) $y=x+2$

Q.55 Let P be any arbitrary point on the circumcircle of a given equilateral triangle ABC of side length a, then $\left|\overrightarrow{PA}\right|^2+\left|\overrightarrow{PB}\right|^2+\left|\overrightarrow{PC}\right|^2$ is equal to

(1) $2a^2$ (2) $2\sqrt{3}\,a^2$ (3) a^2 (4) none of these

SPACE FOR ROUGH WORK

Q.56 If the lines $\vec{r} = \vec{a} + t\vec{b}$ and $\vec{r} = \vec{c} + \lambda\vec{d}$ are co-planar, then

(1) $(\vec{a} - \vec{b}) \cdot (\vec{c} \times \vec{d}) = 0$
(2) $(\vec{c} - \vec{d}) \cdot (\vec{a} \times \vec{b}) = 0$
(3) $(\vec{b} - \vec{d}) \cdot (\vec{a} \times \vec{c}) = 0$
(4) $(\vec{a} - \vec{c}) \cdot (\vec{b} \times \vec{d}) = 0$

Q.57 $\lim\limits_{n \to \infty} \sum\limits_{r=1}^{n} \frac{\pi}{n} \sin\left(\frac{\pi r}{n}\right)$ is equal to

(1) 1 (2) 2 (3) 3 (4) 4

Q.58 Number of points at which the function f(x) = [3 + 11sin x] [x $\in (0, \pi)$ and $[\cdot]$ is greatest integer function] is not differentiable is

(1) 11 (2) 22 (3) 21 (4) 23

Q.59 Sum of maximum and minimum values of $y = (\sin^{-1}x)^4 + (\cos^{-1}x)^4$ is

(1) $\frac{137\pi^4}{128}$ (2) $\frac{69\pi^4}{64}$ (3) $\frac{37\pi^4}{32}$ (4) $\frac{141\pi^4}{128}$

Q.60 If $f(x) = \int \sqrt{\frac{\cos x - \cos^3 x}{1 - \cos^3 x}}\, dx$ and $f\left(\frac{-\pi}{2}\right) = 0$, then $f\left(\frac{-\pi}{3}\right)$ is equal to

(1) $\frac{-\pi}{3}$ (2) $\frac{-\pi}{6}$ (3) $\frac{-2}{3}\sin^{-1}\left(\frac{1}{\sqrt{8}}\right)$ (4) $\frac{2}{3}\sin^{-1}\left(\frac{1}{\sqrt{8}}\right)$

SPACE FOR ROUGH WORK

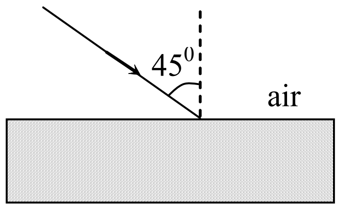
Q.61 The force exerted by a compression device is given by $F(x) = kx\,(x - \ell)$ for $0 \le x \le \ell$, where ℓ is the maximum possible compression, x is the compression and k is a constant. The work required to compress the device by a distance d will be maximum when :

(1) $d = \frac{\ell}{4}$ (2) $d = \frac{\ell}{\sqrt{2}}$ (3) $d = \frac{\ell}{2}$ (4) $d = \ell$

Q.62 Two uniform solid balls of same density and of radii r and 2r are dropped in air and fall vertically downwards. The terminal velocity of the ball with radius r is 1 cm s^{-1}, then the terminal velocity of the ball of radius 2r will be (neglect bouyant force on the balls.)

(1) 0.5 cm s^{-1} (2) 4 cm s^{-1} (3) 1 cm s^{-1} (4) 2 cm s^{-1}

Q.63 A ray of light enters into a transparent liquid from air as shown in the figure. The refractive index of the liquid varies with depth x from the topmost surface as $\mu = \sqrt{2} - \frac{1}{\sqrt{2}}x$ where x in meters. The depth of the liquid medium is sufficiently large. The maximum depth reached by the ray inside the liquid is



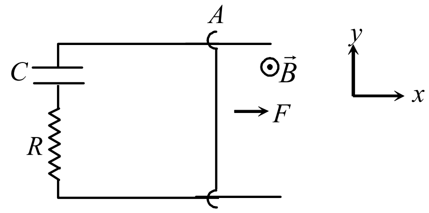
(1) $\sqrt{2}$ m (2) $\frac{1}{\sqrt{2}}$ m (3) 0.5 m (4) 1 m

Q.64 A point source is emitting sound in all directions. The ratio of distance of two points from the point source where the difference in loudness levels is 3 dB is: ($\log_{10} 2 = 0.3$)

(1) $\frac{1}{2}$ (2) $\frac{1}{\sqrt{2}}$ (3) $\frac{1}{4}$ (4) $\frac{2}{3}$

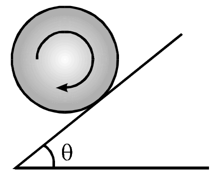
SPACE FOR ROUGH WORK

Q.65 A conducting rod AB moves parallel to x-axis in the x-y plane. A uniform magnetic field B pointing normally out of the plane exists throughout the region. A force F acts perpendicular to the rod, so that the rod moves with uniform velocity v. The force F is given by (neglect resistance of all the wires)



(1) $\frac{vB^2l^2}{R}e^{-t/RC}$ (2) $\frac{vB^2l^2}{R}$ (3) $\frac{vB^2l^2}{R}\left(1-e^{-t/RC}\right)$ (4) $\frac{vB^2l^2}{R}\left(1-e^{-2t/RC}\right)$

Q.66 A cylinder of mass m and radius R is spined to a clockwise angular velocity ω_o and then gently placed on an inclined plane for which coefficient of friction $\mu = \tan\theta$, θ is the angle of inclined plane with horizontal. The centre of mass of the cylinder will remain stationary for time:



(1) $\omega_o R/g\sin\theta$ (2) $2\omega_o R/3g\sin\theta$
(3) $2\omega_o R/5g\sin\theta$ (4) $\omega_o R/2g\sin\theta$

Q.67 N atoms of a radioactive element emit n alpha particles per second at an instant. Then the half - life of the element is

(1) $\frac{n}{N}$ sec. (2) 1.44 $\frac{n}{N}$ sec. (3) 0.69 $\frac{n}{N}$ sec. (4) 0.69 $\frac{N}{n}$ sec.

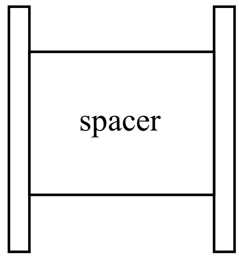
Q.68 A heavy nucleus having mass number 200 gets disintegrated into two small fragments of mass number 80 and 120. If binding energy per nucleon for parent atom is 6.5 M eV and for daughter nuclei is 7 MeV and 8 MeV respectively, then the energy released in the decay will be:
(1) 200 MeV (2) – 220 MeV (3) 220 MeV (4) 180 MeV

Q.69 The angular momentum of an electron in first orbit of Li^{++} ion is :

(1) $\frac{3h}{2\pi}$ (2) $\frac{9h}{2\pi}$ (3) $\frac{h}{2\pi}$ (4) $\frac{h}{6\pi}$

SPACE FOR ROUGH WORK

Q.70 A capacitor is to be designed to operate, with constant capacitance, in an environment of fluctuating temperature. As shown in the figure, the capacitor is a parallel plate capacitor with 'spacer' to change the distance for compensation of temperature effect. If α_1 be the co-efficient of linear expansion of plates and α_2 that of spacer, the condition for no change in capacitance with change of temperature is (The capacitance of the capacitor is equal to C and spacer have insulated ends)



(1) $\alpha_1 = \alpha_2$ (2) $\alpha_1 = 2\alpha_2$ (3) $2\alpha_1 = \alpha_2$ (4) $2\alpha_1 = 3\alpha_2$

Q.71 A balloon containing an ideal gas has a volume of 10 litre and temperature of 17º. If it is heated slowly to 75ºC, the work done by the gas inside the balloon is (neglect elasticity of the balloon and take atmospheric pressure as 10^5 Pa)
(1) 100 J (2) 200 J (3) 250 J (4) data insufficient

Q.72 A bird is flying up at angle $\sin^{-1}(3/5)$ with the horizontal. A fish in a pond looks at that bird. When it is vertically above the fish. The angle at which the bird appears to fly (to the fish) is: [$n_{water} = 4/3$]
(1) $\sin^{-1}(3/5)$ (2) $\sin^{-1}(4/5)$ (3) 45º (4) $\sin^{-1}(9/16)$

Q.73 A very small circular loop of area 5×10^{-4} m^2 and resistance 2 ohm is initially concentric and coplanar with a stationary loop of radius 0.1 m. If one ampere constant current is passed through the bigger loop and the smaller loop is rotated about its diameter with constant angular velocity ω. The current induced (in ampere) in the smaller loop will be :
(1) $\frac{\pi\omega}{2} \times 10^{-9} \cos\omega t$ (2) $\pi\omega \times 10^{-9} \sin\omega t$ (3) $\frac{\pi\omega}{2} \times 10^{-9} \sin\omega t$ (4) $\pi w \times 10^{-9} \sin\omega t$

Q.74 A metallic charged ring is placed in a uniform magnetic field with its plane perpendicular to the field. If the magnitude of field starts increasing with time, then :
(1) the ring starts translating
(2) the ring starts rotating about its axis
(3) the ring starts rotating about a diameter
(4) the ring remains at rest

SPACE FOR ROUGH WORK

Q.75 Axis of a solid cylinder of infinite length and radius R lies along y-axis it carries a uniformly distributed current ' i ' along +y direction. Magnetic field at a point $\left(\frac{R}{2}, y, \frac{R}{2}\right)$ is :

(1) $\frac{\mu_0 I}{4\pi R}(\hat{i}-\hat{k})$ (2) $\frac{\mu_0 i}{2\pi R}(\hat{j}-\hat{k})$ (3) $\frac{\mu_0 i}{4\pi R}\hat{j}$ (4) $\frac{\mu_0 I}{4\pi R}(\hat{i}+\hat{k})$

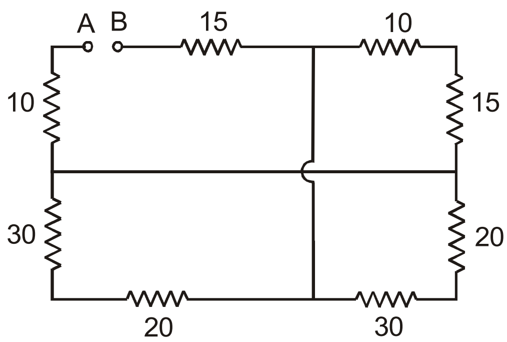
Q.76 A conducting disc of radius R rotates about its axis with an angular velocity ω. Then the potential difference between the centre of the disc and its edge is (no magnetic field is present) :

(1) zero (2) $\frac{m_e\omega^2 R^2}{2e}$ (3) $\frac{m_e\omega R^3}{3e}$ (4) $\frac{e m_e\omega R^2}{2}$

Q.77 A α particle is released from rest 10 cm from a large sheet carrying a surface charge density of -2.21×10^{-9} C/m^2. It will strike the sheet after the time. ($\in_0 = 8.84 \times 10^{-12}$ C^2/Nm^2)
(1) 4 μs (2) 2 μs (3) $2\sqrt{2}$ μs (4) $4\sqrt{2}$ μs.

Q.78 A thin rod of negligible mass and area of cross-section 2×10^{-6} m^2, suspended vertically from one end, has a length of 0.5 m at 200^{0}C. The rod is cooled to 0^{0}C, but prevented from contracting by attaching a mass at the lower end. The value of this mass is : (Young's modulus = 10^{11} N/m^2, Coefficient of linear expansion 10^{-5} K^{-1} and g = 10 m/s^2) :
(1) 20 kg (2) 30 kg (3) 40 kg (4) 50 kg

Q.79 The equivalent resistance of the circuit across points A and B is equal to :



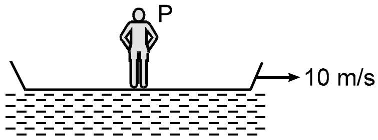
(1) 22.5 Ω (2) 25 Ω (3) 37.5 Ω (4) 75 Ω

SPACE FOR ROUGH WORK

Q.80 Two glass plates are separated by water. If surface tension of water is 75 dynes per cm and area of each plate wetted by water is 8 cm^2 and the distance between the plates is 0.12 mm, then the force applied to separate the two plates is

(1) 10^2 dynes (2) 10^4 dynes (3) 10^5 dynes (4) 10^6 dynes

Q.81 A person P of mass 50 kg stands at the middle of a boat of mass 100 kg moving at a constant velocity 10 m/s with no friction between water and boat and also the engine of the boat is shut off. With what velocity (relative to the boat surface) should the person move so that the boat comes to rest. Neglect friction between water and boat.

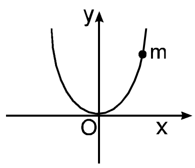


(1) 30 m/s towards right (2) 20 m/s towards right
(3) 30 m/s towards left (4) 20 m/s towards left

Q.82 If the apparent weight of the bodies at the equator is to be zero, then the earth should rotate with angular velocity

(1) $\sqrt{\frac{g}{R}}$ rad/sec (2) $\sqrt{\frac{2g}{R}}$ rad/sec (3) $\sqrt{\frac{g}{2R}}$ rad/sec (4) $\sqrt{\frac{3g}{2R}}$ rad/sec

Q.83 A bead of mass m is located on a parabolic wire with its axis vertical and vertex at the origin as shown in figure and whose equation is $x^2 = 4ay$. The wire frame is fixed and the bead can slide on it without friction. The bead is released from the point y = 4a on the wire frame from rest. The tangential acceleration of the bead when it reaches the position given by y = a is :



(1) $\frac{g}{2}$ (2) $\frac{\sqrt{3}g}{2}$ (3) $\frac{g}{\sqrt{2}}$ (4) $\frac{g}{\sqrt{5}}$

Q.84 N identical capacitors are connected in parallel to a potential difference V. These capacitors are then reconnected in series such that positively charged plate of one capacitor is connected to negatively charged plate of the other, their charges being left undisturbed. The potential difference obtained is :

(1) zero (2) (N - 1) V (3) N V (4) N^2V

SPACE FOR ROUGH WORK

Q.85 A small ball thrown at an initial velocity u directed at an angle $\theta = 37°$ above the horizontal collides inelastically ($e = 1/4$) with a vertical massive wall moving with a uniform horizontal velocity u/5 towards ball. After collision with the wall, the ball returns to the point from where it was thrown. Neglect friction between ball and wall. The time t from beginning of motion of the ball till the moment of its impact with the wall is ($\tan 37° = 3/4$)

(1) $\frac{3u}{5g}$ (2) $\frac{18u}{25g}$ (3) $\frac{54u}{125g}$ (4) $\frac{54u}{25g}$

Q.86 The output of an AC generator is given by : $E = E_m \sin(\omega t - \pi/4)$ and current is given by $i = i_m \sin(\omega t - 3\pi/4)$. The circuit contains a single element other than the generator. It is :

(1) a capacitor. (2) a resistor.

(3) an inductor. (4) not possible to decide due to lack of information.

Q.87 A particle undergoes SHM with a time period of 2 seconds. In how much time will it travel from its mean position to a displacement equal to half of its amplitude

(1) 1/2 sec (2) 1/3 sec (3) 1/4 sec (4) 1/6 sec.

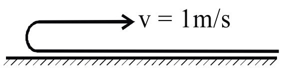
Q.88 Two points of a rod move with velocities 3 v & v perpendicular to the rod and in the same direction, separated by a distance 'r'. Then the angular velocity of the rod is:

(1) $\frac{3v}{r}$ (2) $\frac{4v}{r}$ (3) $\frac{5v}{r}$ (4) $\frac{2v}{r}$

Q.89 Inside a horizontally moving box, an experimenter (who is stationary relative to box) finds that when an object is placed on a smooth horizontal table and is released, it moves with an acceleration of 10 m/s^2. In this box if 1 kg body is suspended with a light string, the tension in the string in equilibrium position. (w.r.t. experimenter) will be. (Take $g = 10\ m/s^2$)

(1) 10 N (2) $10\sqrt{2}$ N (3) 20 N (4) zero

Q.90 A long, thin carpet is laid on a floor. One end of the carpet is bent back and then pulled backwards with constant unit velocity, just above the part of the carpet which is still at rest on the floor. The speed of centre of mass of the moving part is



(1) 1 m/s (2) $\frac{3}{4}$ m/s (3) $\frac{1}{2}$ m/s (4) $\frac{1}{4}$ m/s

SPACE FOR ROUGH WORK

ROUGH WORK

ROUGH WORK

ANSWER KEY

COURSE	JEE-MAIN MOCK TEST-9	TEST CODE
NUCLEUS	XII	1 1 2 9 7

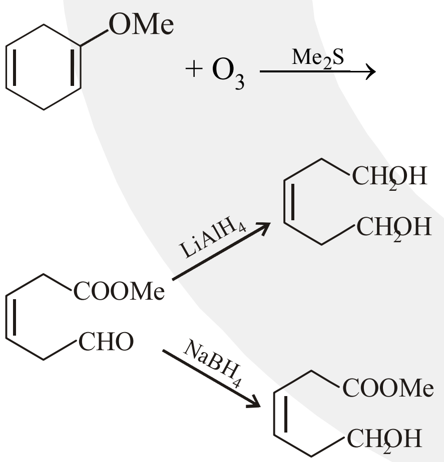
	IOC	OC	PC	IOC	OC	PC	IOC	OC	PC	IOC	OC	PC	IOC	OC	PC
Q.No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans	2	3	4	1	4	2	1	4	2	1	2	4	1	4	3
	IOC	OC	PC	IOC	OC	PC	IOC	OC	PC	IOC	OC	PC	IOC	OC	PC
Q.No.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans	2	3	1	1	3	3	1	3	4	4	3	4	4	1	3
Q.No.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**
Ans	2	3	1	3	1	3	3	1	3	2	1	4	3	1	2
Q.No.	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
Ans	1	3	2	2	3	2	4	2	4	1	4	2	3	1	4
Q.No.	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**
Ans	4	2	4	2	1	4	4	3	3	3	2	3	3	2	1
Q.No.	**76**	**77**	**78**	**79**	**80**	**81**	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**
Ans	2	4	3	3	3	1	1	3	3	3	3	4	4	2	2

HINTS & SOLUTIONS

CHEMISTRY

Q.1 $M(s) \xrightarrow{(a)} M(g) \xrightarrow{(c)} M^{+}(g)$

Q.2



Q.3 $\left(\dfrac{V_n}{2\pi r_n}\right) \propto \dfrac{Z^2}{n^3}$

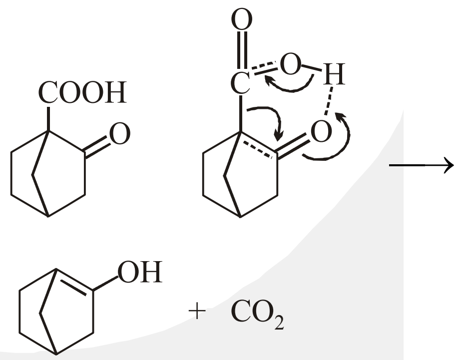
$\Rightarrow \quad \dfrac{Z^2}{4^3} = 2.\dfrac{1^2}{2^3}$

$\Rightarrow \quad Z^2 = 16$

$Z = 4$

Q.4 $Zn(\text{amphoteric}) + 2NaOH \rightarrow Na_2ZnO_2 + H_2 \uparrow$

Q.5



unstable structure double bond is at bridge head C

Q.6 $(Eq)_{KMnO_4} = (Eq)_{H_2O_2} + (Eq)_{H_2C_2O_4}$

$0.2 \times 200 \times 5 = (M \times 50 \times 2) + (0.5 \times 100 \times 2)$

$[H_2O_2] = 1M$

$\therefore$ volume strength $= 11.2 \times 1$ V

$= 11.2$ V

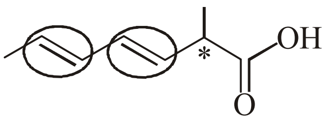
Q.7 $CaO(s) \longrightarrow Ca(OH)_2(s)$
(quick lime) (slaked lime)

$\downarrow H_2O$

$Ca(OH)_2$
(white turbidity)
(milk of lime)

$\downarrow H_2O$

$Ca(OH)_2(aq)$
(lime water)

Q.8 

n = 3
$2^n = 2^3 = 8$

Q.9 Process is isochoric
$q = q_v = n.C_{v,m}\,\Delta T$
$= 2 \times \frac{3R}{2}\,(200 - 400)$
$= -1200$ Cal

Q.10 $C_2^+ \rightarrow C_2$
B.O. = 1.5 (para) B.O.=2(dia)

$C_2^+ = \sigma_{1s^2}\ \sigma^*_{1s^2}\ \sigma_{2s^2}\sigma^*_{2s^2}\ \pi_{2p_x{}^2} = \pi_{2p_y{}^1}$

Bond order $= \frac{N_b - N_a}{2}$

$= \frac{7-4}{2} = 1.5$ (Paramagnetic)

$C_2 = \sigma_{1s^2}\sigma^*_{1s^2}\sigma_{2s^2}\sigma^*_{2s^2}\ \pi_{2p_y^2} = \pi_{2p_y^2}$

Bond order $= \frac{N_b - N_a}{2}$

$= \frac{8-4}{2} = 2$ (diamagnetic)

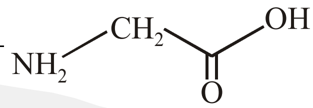
Q.11 Order of basicity
$(C_2H_5)_2NH > (C_2H_5)_3N > C_2H_5NH_2 > NH_3$

Q.12 $2NO + 2SO_3 \rightleftharpoons 2NO_2 + 2SO_2$; $K_C = (3)^2$
$2SO_2 + O_2 \rightleftharpoons 2SO_3$; $K_C = 16$

$2NO + O_2 \rightleftharpoons 2NO_2$; $K_C = 9 \times 16$

So, $2NO_2 \rightleftharpoons 2NO + O_2$; $K_C = \frac{1}{144}$

Q.13 $[Mn(CO)_5]$
EAN $= 25 + 5 \times 2$
$= 25 + 10 = 35$
$[Mn(CO)_5]$ requires 1 more electron to achieve 36 e^- so it act as oxidizing agent.

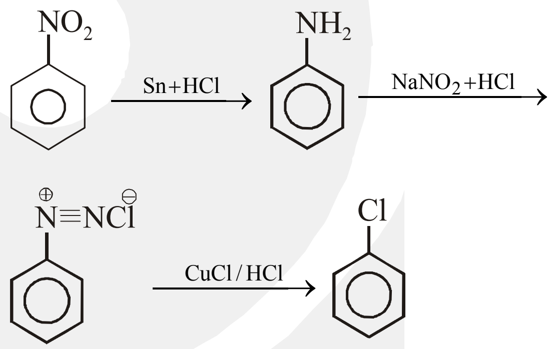
Q.14 Glycine – 

No chiral carbon is present here therefore optically inactive

Q.15 $\frac{r_{Cs^+}}{r_{Cl^-}} = 0.732$

Q.16 $XeF_2 + H_2O \rightarrow Xe + \frac{1}{2}O_2 + 2HF$

$2XeF_4 + 4H_2O \rightarrow Xe + \frac{1}{2}O_2 + 8HF + XeO_3$

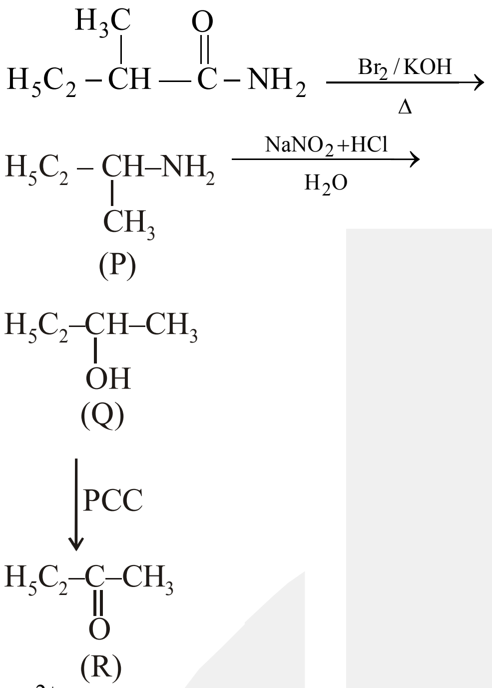
Q.17 

Q.18 Theory based

Q.19 $PH_4^+ > OF_2 > SF_2 > SbH_3$
(sp^3) (sp^3) (sp^3)
No hybridisation (Bond angle ≈ 90°)

BA α EN of CA

Q.20



Q.21 $Sn^{2+} + 2e^- \rightarrow Sn$; $\Delta G° = (-2.F.x)$

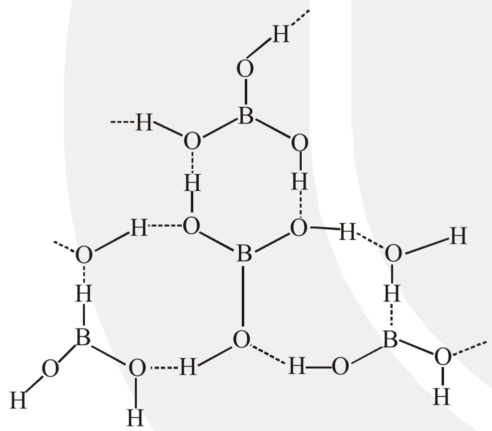
$Sn^{4+} + 4e^- \rightarrow Sn$; $\Delta G° = (-4.F.y)$

$Sn^{2+} \rightarrow Sn^{4+} + 2e^-$; $\Delta G° = (-2.F.E°)$

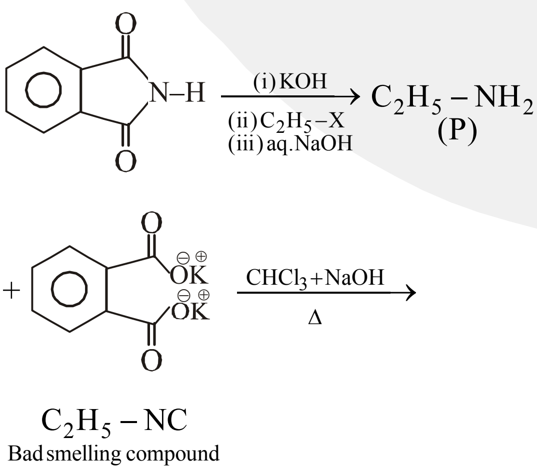
So, $(-2.F.E°) = (-2.F.x) - (-4.F.y)$

$\Rightarrow E° = (x - 2y)$

Q.22



Q.23



Q.24 AT 373K, $P^0_{H_2O} = 760$ torr

$\because P = P^0 . X_{H_2O}$

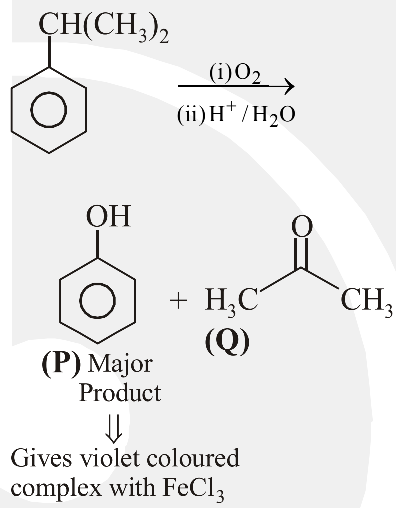
$570 = 760 . X_{H_2O}$

$\Rightarrow X_{solute} = \frac{1}{4}$

Q.25 Smelting - An oxidation process.

Smelting is a process of applying heat to ore in order to extract out a base metal. It is a form of extractive metallurgy. Smelting uses heat and a chemical reducing agent to decompose the ore, driving off other element as gases or slag leaving the metal base behind.

Q.26

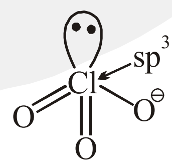


Q.27 $\frac{[A]_t}{[A]_0} = \frac{1}{2^n} = \frac{1}{2^3} = \frac{1}{8}$

Number of Half lives (n) $= \frac{300}{100} = 3$

Fraction reacted $= 1 - \frac{1}{8} = \frac{7}{8}$

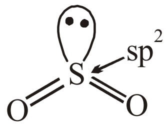
Q.28 (1) ClO_3^{-1}



2 pπ - dπ

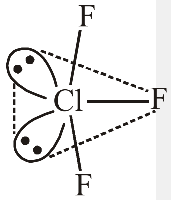
0 pπ - pπ

(2) SO_2



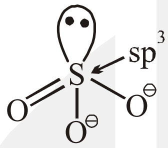
1 pπ - dπ
1 pπ - pπ

(3) ClF_3



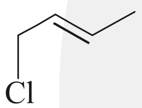
0 pπ - dπ
0 pπ - pπ

(4) SO_3^{2-}



1 pπ - dπ
0 pπ - pπ

Q.29 $Me-\underset{\underset{O}{\|}}{C}-CH_2-Cl > Me-O-CH_2-Cl >$



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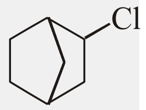


Q.30 For AB $= K_{sp} = s_1^2$

MB_2 $K_{sp} = 4\ s_2^3$

N_3C_2 $K_{sp} = 108\ s_3^5$

A_3C $K_{sp} = 27\ s_4^4$

All have same K_{sp}, So maximum solubility will be of $N_3C_2(s)$.

MATHEMATICS

Q.31 $y = \dfrac{ax^2-3x+5}{5x^2-3x+a}$

Range is R $\Rightarrow a \in (-8, -2)$

$a = -7, -6, -5, -4, -3$.

Q.32 a^2, b^2, c^2 AP

$\Rightarrow$ a^2+1, b^2+1, c^2+1 AP

$\Rightarrow a^2+ab+bc+ca,\ b^2+ab+bc+ca,\ c^2+ab+bc+ca$ AP

$\Rightarrow (a+b)(a+c),\ (a+b)(b+c),\ (b+c)(c+a)$ AP

$\Rightarrow \dfrac{1}{b+c}, \quad \dfrac{1}{c+a}, \quad \dfrac{1}{a+b}$ AP.

Q.33 $\dfrac{\sin 20°+\cos 20°+\sin 50°}{\cos 10°\cdot\sin 35°\cdot\cos 25°}$

$= \dfrac{\sin 160°+\sin 70°+\sin 130°}{\sin 80°\cdot\sin 35°\cdot\sin 65°} = 4$

Q.34 $f(x) = [x] + \sqrt{\{x\}} + 1$

$\Rightarrow f^{-1}(x) = g(x) = [x] + \{x\}^2 - 1$

$f(0) = 1$ and $f(2) = 3$ then

$\int_0^2 f(x)dx + \int_1^3 g(x)dx = 6$

Q.35 $(z^n-1) = (z-\alpha_0)(z-\alpha_1)\ \ldots\ldots\ (z-\alpha_{n-1})$

$\ln(z^n-1) = \ln(z-\alpha_0) + \ln(z-\alpha_1)\ \ldots\ldots + \ln(z-\alpha_{n-1})$

$\dfrac{n\cdot z^{n-1}}{z^n-1} = \sum_{r=0}^{n-1}\dfrac{1}{z-\alpha_r}$

$\Rightarrow \dfrac{n\cdot 3^{n-1}}{3^n-1} = \sum_{r=0}^{n-1}\dfrac{1}{3^n-\alpha_r}$

Now,

$\sum_{r=0}^{n-1}\dfrac{\alpha_r}{3-\alpha_r} = \sum_{r=0}^{n-1}\left(\dfrac{3}{3-\alpha_r}-1\right) = \dfrac{n\cdot 3^n}{3^n-1} - n = \dfrac{n}{3^n-1}$.

Q.36 $y = \sqrt{7+\sqrt{7-\sqrt{7+\sqrt{7-\ldots\ldots\infty}}}}$

$\Rightarrow y^2 = 7 + \sqrt{7-y} \Rightarrow y = 3$

Q.37 $f_n^{\,2}(3) + 2f_n(1) = 9f_n^{\,2}(1) + 2f_n(1)$

$= 9\left(\dfrac{10^n-1}{9}\right)^2 + \dfrac{2(10^n-1)}{9}$

$= \dfrac{10^{2n}-1}{10-1} = f_{2n}(1)$.

Q.38 $2^a + 3^b + 5^c = 2^a + (4-1)^b + (4+1)^c$

$= 2^a + 4k + (-1)^b + 1$

Case-1: a = 1 ⇒ b ∈ even and c is any number
number of ways = 10.
Case-2: a ≠ 1 ⇒ b ∈ odd and c is any number
number of ways = 4 × 3 × 5 = 60.

Q.39 In multinomial, by beggar's method
Total number of distinct terms = ${}^{n+r-1}C_{r-1}$.
So, ${}^{n+4-1}C_{4-1} = {}^{n+3}C_3 = {}^{n+3}C_n$.

Q.40 $f(x) = \begin{vmatrix} x^2+3x & x-1 & x-3 \\ x+1 & 2-x & x-3 \\ x-3 & x+4 & 3x \end{vmatrix}$

f '(0) =

$$\begin{vmatrix} 3 & -1 & -3 \\ 1 & 2 & -3 \\ 1 & 4 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 & -3 \\ 1 & -1 & -3 \\ -3 & 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & -1 & 1 \\ 1 & 2 & 1 \\ -3 & 4 & 3 \end{vmatrix}$$

= 36 + 3 – 6 + 9 + 6 + 6 + 10 = 60.

Q.41

$$PP^T = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$\Rightarrow Q^2 = PAP^TPAP^T = PA^2P^T$

$\Rightarrow P^TQ^{2019}P = A^{2019} = \begin{bmatrix} 1 & 2019 \\ 0 & 1 \end{bmatrix}$.

Q.42 n (A × B) = n(S) = 20.
a + b = 9 ⇒ {(1, 8), (3, 6), (5, 4), (7, 2)}
n(E) = 4

$P(E) = \frac{1}{5}$.

Q.43 $f(x) = x^3 - x^2 + 4x + 2\sin^{-1}x$

$f'(x) = 3x^2 - 2x + 4 + \frac{2}{\sqrt{1-x^2}} > 0$

∀ x ∈ (0, 1)
⇒ Range is [0, 4 + π].

Q.44 Put sin x = 1 + t ⇒ if $x \to \frac{\pi}{2} \Rightarrow t \to 0$

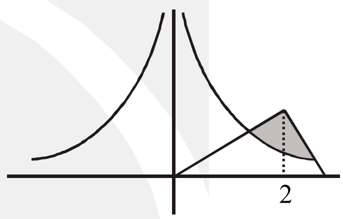
$$\lim_{t\to 0} \frac{(1+t)-(1+t)^{(1+t)}}{-t+\ln(1+t)} = \lim_{t\to 0} \frac{(1+t)^t - 1}{t - \ln(1+t)}$$

$$= \lim_{t\to 0} \frac{1+t^2+\frac{t(t+1)t^2}{2!}+\ldots\ldots-1}{t-\left(t-\frac{t^2}{2}+\frac{t^3}{3}+\ldots\ldots\right)} = 2.$$

Q.45 f '(x) = 3x² + 2ax + b – 5sin 2x > 0 ∀ x ∈ R
⇒ a² – 3 (b – 5) < 0 ⇒ a² – 3b + 15 < 0.

Q.46 Let the point be (2t², 4t)
Equation of normal is tx + y = 4t + 2t³
⇒ 2t³ + 4t + 6 = 0 ⇒ t³ + 2t + 3 = 0
⇒ (t + 1) (t² – t + 3) = 0 ⇒ t = – 1
point be (2, – 4).

Q.47 $y = \begin{cases} x, & x < 2 \\ 4-x, & x \geq 2 \end{cases}; \; y = \begin{cases} \frac{3}{x}, & x > 0 \\ \frac{-3}{x}, & x < 0 \end{cases}$



Hence, required area

$$= \left|\int_{\sqrt{3}}^{2}\left(x-\frac{3}{x}\right)dx\right| + \left|\int_{2}^{3}\left((4-x)-\frac{3}{x}\right)dx\right|$$

$$= \frac{5-3\ln 3}{2}.$$

Q.48 Put x + y = t ⇒ $1 + \frac{dy}{dx} = \frac{dt}{dx}$

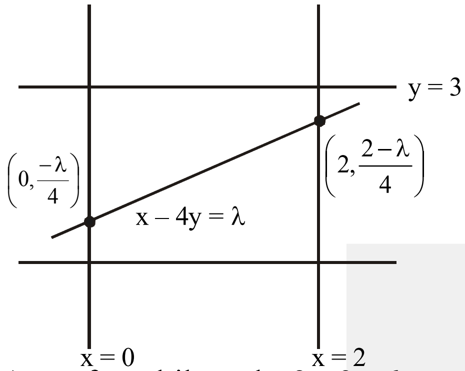
$$\left(\frac{t-1}{t-2}\right)\left(\frac{dt}{dx}-1\right) = \frac{t+1}{t+2}$$

$$\Rightarrow \frac{(t^2+t-2)\,dt}{(t^2+2)} = 2dx$$

$$\Rightarrow t + \frac{\ln|t^2-2|}{2} = 2x + C$$

$$\Rightarrow (y-x) + \frac{\ln|(x+y)^2-2|}{2} = C.$$

Q.49



Area of quadrilateral = 2 × 3 = 6

$\Rightarrow 3 = \frac{1}{2}\left(\frac{2-2\lambda}{4}\right) \times 2 \Rightarrow \lambda = -5.$

Q.50 $\frac{\sin^4\theta}{5} + \frac{\cos^4\theta}{1} = \frac{(\sin^2\theta + \cos^2\theta)^2}{5+1}$

$\Rightarrow \frac{\sin^4\theta}{5} = \frac{\cos^4\theta}{1} \Rightarrow \tan^2\theta = 5.$

Q.51 Let the circle be
$x^2 + y^2 - a^2 + \lambda\,(x\cos\alpha + y\sin\alpha - P) = 0$

Centre is $\left(\frac{-\lambda\cos\alpha}{2}, \frac{-\lambda\sin\alpha}{2}\right)$.

$\Rightarrow \lambda = -2P.$

Q.52 Let P(h, k), then

$ky = 4b\left(\frac{2ax}{k} + \frac{2ah}{k}\right) = 0 \Rightarrow D = 0$

$\left(\frac{8ab}{k}\right)^2 = \frac{8ah}{k} \Rightarrow xy = \text{constant}.$

Q.53 Centre is point of intersection of
$2x - y + 1 = 0$ and $x + 2y - 3 = 0$.

Q.54 Let the tangent be $y = mx + \frac{2}{m}$

$x\left(mx + \frac{2}{m}\right) = -1 \Rightarrow mx^2 + \frac{2x}{m} + 1 = 0$

$D = \frac{4}{m^2} - 4m = 0 \Rightarrow m = 1.$

Q.55 $R = \sqrt{3}\,a$

$|\vec{a}| = |\vec{b}| = |\vec{c}| = \frac{a}{\sqrt{3}} = |\vec{p}|$

where $\overrightarrow{OA} = \vec{a}$; $\overrightarrow{OB} = \vec{b}$; $\overrightarrow{OC} = \vec{c}$; $\overrightarrow{OP} = \vec{p}$

$\left|\overrightarrow{PA}\right|^2 + \left|\overrightarrow{PB}\right|^2 + \left|\overrightarrow{PC}\right|^2$

$= |\vec{a} - \vec{p}|^2 + |\vec{b} - \vec{p}|^2 + |\vec{c} - \vec{p}|^2 = 6\left(\frac{a}{\sqrt{3}}\right)^2 = 2a^2$

Q.56 From theory.

Q.57 $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{\pi}{n}\sin\left(\frac{\pi r}{n}\right) = \int_0^1 \pi \sin \pi x \, dx$

$= -\cos \pi x \Big|_0^1 = 2.$

Q.58 $f(x) = [3 + 11\sin x] = 3 + [11\sin x]$
Number of points at which y = f(x) is not differentiable is 21.

Q.59 $f(x) = (\sin^{-1}x)^4 + (\cos^{-1}x)^4$

$\Rightarrow f'(x) = \frac{4\left((\sin^{-1}x)^3 - (\cos^{-1}x)^3\right)}{\sqrt{1-x^2}}$

$\Rightarrow$ f(x) is decreasing in $\left(-1, \frac{1}{\sqrt{2}}\right)$

and increasing in $\left(\frac{1}{\sqrt{2}}, 1\right)$.

$f_{max.} = f(-1) = \frac{17\pi^4}{16}$; $f_{min} = f\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi^4}{128}$.

Q.60 $f(x) = \int \sqrt{\frac{\cos x - \cos^3 x}{1 - \cos^3 x}}\, dx = \frac{2}{3}\int \frac{dt}{\sqrt{1-t^2}}$

$f(x) = \frac{2}{3}\left(\sin^{-1}(\cos x)^{\frac{3}{2}}\right).$

PHYSICS

Q.61 For W to be maximum ; $\frac{dW}{dx} = 0$;
i.e. F(x) = 0 $\Rightarrow x = \ell$, x = 0
Clearly for d = 1, the work done is maximum.
Alternate Solution :
External force and displacement are in the same direction

$\therefore$ Work will be positive continuosly so it will be maximum when displacement is maximum.

Q.62 At equilibrium

$mg = 6\pi\eta rv$ or $\rho\frac{4\pi}{3}r^3 g = 6\pi\eta rv$

$\therefore \frac{v_r}{v_{2r}} = \frac{(r)^2}{(2r)^2}$ or $v_{2r} = (v_r) \times 4$

= 4 cm/s.

Q.63 At maximum depth the ray graze the surface (i.e. the angle made by the ray with normal will become 90°)

Applying Snell's law

$$1 \times \sin 45^0 = \left(\sqrt{2} - \frac{1}{\sqrt{2}}x\right)\sin 90^0$$

$$\Rightarrow \quad \sqrt{2} - \frac{1}{\sqrt{2}}x = \frac{1}{\sqrt{2}} \text{ or } x = 1 \text{ m}$$

Q.64 (2) $dB = 10\log\left(\frac{I}{I_0}\right) = 10\log\left(\frac{K/r^2}{I_0}\right) = 10$ $[\log(K^1) - 2\log r]$

$dB_1 = 10(\log K' - 2\log r_1)$

$dB_2 = 10(\log K' - 2\log r_2)$

$$3 = dB_1 - dB_2 = 20\log\left(\frac{r_2}{r_1}\right)$$

$$(0.3) = \log\left(\frac{r_2}{r_1}\right)^2 \quad \Rightarrow \quad \left(\frac{r_1}{r_2}\right) = \frac{1}{\sqrt{2}}$$

Q.65 Induced emf in the rod $\varepsilon = Blv$

Current in the circuit

$$I = \frac{\varepsilon}{R}e^{-t/RC} = \frac{Blv}{R}e^{-t/RC}$$

Since the net force on the rod should be zero, the external force will be equal in magnitude but opposite to the magnetic force.

$$\Rightarrow \quad F = I\,l\,B = \frac{B^2 l^2 v}{R}e^{-t/RC}$$

Q.67 $n = \lambda N = \lambda = \frac{n}{N}$

$$\therefore t_{1/2} = \frac{0.69}{\lambda} = \frac{0.69\ N}{n}$$

Q.68 Energy released = (80 × 7 + 120 × 8 – 200 × 6.5) = 220 MeV.

Q.69 Angular momentum $= \frac{nh}{2\pi} = \frac{h}{2\pi}$

($\because n = 1$)

Q.70 $C = \frac{\varepsilon_0 A}{L}$

$$\therefore \quad \log C = \log\varepsilon_0 + \log A - \log L$$

$$\frac{dC}{C} = \frac{dA}{A} - \frac{dL}{L}$$

$$\frac{dC}{C} = 2\alpha_1\,dT - \alpha_2 dT = 0$$

$$\therefore \quad 2\alpha_1 = \alpha_2$$

Q.71 Since elasticity of balloon is negligible, pressure inside ballon ≃ pressure outside baloon = P_{atm}.

$\therefore W = P_{atm}\Delta V$

$V_{in} = 10$ litre.

$$\frac{V_{in}}{T_{in}} = \frac{V_{fin}}{T_{fin}} \Rightarrow V_{final} = \left(\frac{V_{in}T_{final}}{T_{in}}\right) \text{litre.}$$

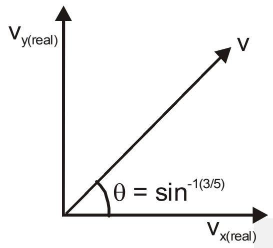
$$\Rightarrow W = P_{atm}V_{in}\left(\frac{T_{final}}{T_{in}} - 1\right)$$

$$\Rightarrow 10^5 \times 10^{-2}\left(\frac{58}{290}\right) = 200 \text{ J}$$

Q.72 Let y-axis be vertically upwards and x-axis be horizontal.

$$V_y(\text{app.}) = \frac{V_y(\text{real})}{\left(\frac{1}{\mu}\right)}$$

$$V_x(\text{app.}) = V_x(\text{real})$$



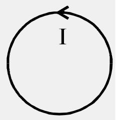
$$\tan\phi = \frac{V_y(app)}{V_x(app)} = \frac{4}{3}\tan\theta = \frac{4}{3}\times\frac{3}{4} = 1$$

Q.73 $E = \pi \times 10^{-9}\ \omega\sin\omega t$

Also $E = i \times 2$.

$\Rightarrow i = \frac{\pi\omega}{2} \times 10^{-9}\sin\omega t$.

Q.74 As soon as the field changes, there will be an induced current (anticlockwise) in the ring. As there is always a electromagnetic force acting on a current carrying element. Hence, there will be a torque on the ring about its axis. Hence (2).

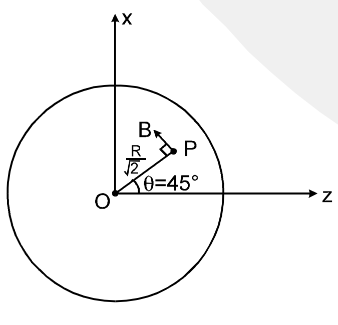


Q.75 The magnitude of magnetic field at $P\left(\frac{R}{2}, y, \frac{R}{2}\right)$ is

$$B = \frac{\mu_0 J r}{2} = \frac{\mu_0 i}{2\pi R^2} \times \frac{R}{\sqrt{2}} = \frac{\mu_0 i}{2\sqrt{2}\pi R}$$

unit vector in direction of magnetic field is

$$\hat{B} = \frac{\hat{i}-\hat{k}}{\sqrt{2}}$$



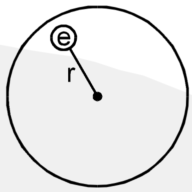
$$\therefore \quad \vec{B} = B\hat{B} = \frac{\mu_0 i}{4\pi R}(\hat{i}-\hat{k})$$

Alternate solution

$$\vec{B} = \frac{\mu_0}{2}\vec{J}\times\vec{r} = \frac{\mu_0}{2}\frac{i}{\pi R^2}\hat{j} \times \left(\frac{R}{2}\hat{i} + \frac{R}{2}\hat{k}\right) = \frac{\mu_0 i}{4\pi R}(\hat{i}-\hat{k})$$

Q.76 $eE = m_e\omega^2 r$

$$\Rightarrow \quad \int E\,dr = \frac{m_e\omega^2}{e}\int_0^R r\,dr$$



$$\Rightarrow \quad V = \frac{m_e\omega^2 R^2}{2e}$$

Q.77 As field is uniform

Acceleration 'a' $= \frac{qE}{m}$. $E = \frac{\sigma}{2\epsilon_0}$

Using $s = \frac{1}{2}at^2 \Rightarrow t = \frac{2s}{a}$

on putting values $t = 4\sqrt{2}\mu s$

Q.78 Strain $(\varepsilon) = \frac{\Delta\ell}{\ell} = \propto \Delta T = (10^{-5})(200)$

$= 2 \times 10^{-3}$

Stress = Y (strain)

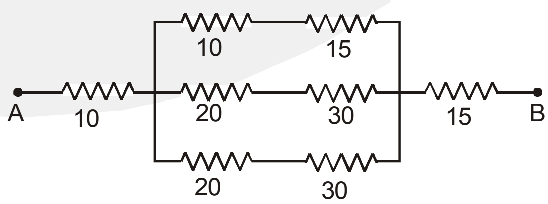
Stress $= 10^{11} \times 2 \times 10^{-3} = 2 \times 10^{8}\ N/m^2$

$\Rightarrow$ Required force = stress × Area = $(2 \times 10^8)(2 \times 10^{-6}) = 4 \times 10^2 = 400$ N

$\therefore$ Mass to be attached $= \frac{400}{g} = 40$ kg

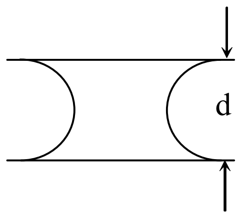
Ans.

Q.79 Equivalent circuit is



$= 37.5\ \Omega$

Q.80 The shape of water layer between the two plates is shown in the figure.
Thickness d of the film
= 0.12 mm = 0.012 cm.



Radius R of cylindrical face $= \frac{d}{2}$.

Pressure difference across the surface

$= \frac{T}{R} = \frac{2T}{d}$.

Area of each plate wetted by water = A.
Force F required to separate the two plates is given by

F = pressure difference × area $= \frac{2T}{d}A$

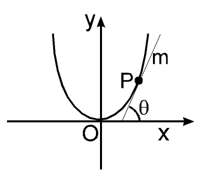
$= \frac{2 \times 75 \times 8}{0.012} = 10^5$ dynes

Q.81 Momentum of the system remains conserved as no external force is acting on the system in horizontal direction.
∴ (50 + 100) 10 = 50 × V + 100 × 0
⇒ V = 30 m/s towards right, as boat is at rest.
$V_{P_{boat}} = 30$ m/s

Q.82 $mg = m\omega^2 R$, $\omega = \sqrt{\frac{g}{R}}$

Q.83 $x^2 = 4ay$
Differentiating w.r.t. y, we get

$$\frac{dy}{dx} = \frac{x}{2a}$$



∴ At (2a, a), $\frac{dy}{dx} = 1$

⇒ hence θ = 45°

the component of weight along tangential direction is mg sin θ.

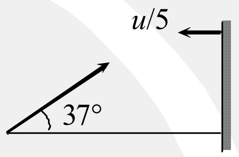
hence tangential acceleration is $g \sin\theta = \frac{g}{\sqrt{2}}$

Q.84 When connected in parallel
Potential difference across each capacitor = v
P.D. when connected in series = N.V.

Q.85 Let the ball collides with the wall after time t.
Let velocity of ball after collision is v.

$$\frac{-v - \left(-\frac{u}{5}\right)}{-\frac{u}{5} - u\cos 37} = \frac{1}{4} \;;\; -v + \frac{u}{5} = -\frac{u}{4} \;;$$

$$v = \frac{u}{5} + \frac{u}{4} = \frac{9u}{20}$$



Also, $(u\cos 37)\,t = \frac{9u}{20}(T - t)$

$$\frac{4ut}{5} = \frac{9u}{20}\left(\frac{2u}{g}\frac{3}{5} - t\right) \Rightarrow t = \frac{54u}{125g}$$

Q.86 The current lags the EMF by π/2, so the circuit should contain only an inductor.

Q.87 $x = A \sin\frac{2\pi}{T}t$; for $x = \frac{A}{2}$

$\Rightarrow \frac{A}{2} = A\sin\frac{2\pi}{T}t$

Solving $t = \frac{T}{6}$.

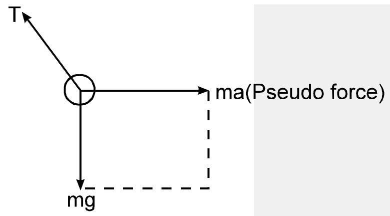
Q.88 $\omega_{rod} = \omega_{point} = \left(\frac{v_{rel.}}{r}\right)$; $v_{rel.}$ being the velocity of one point w.r.t. other.

$= \frac{3v - v}{r}$ and 'r' being the distance between them. $= \frac{2v}{r}$

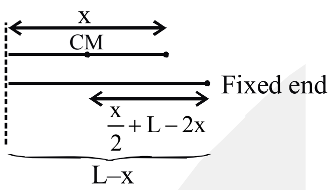
Q.89 Acceleration of box = 10 m/s^2

Inside the box forces acting on bob are shown in the figure

$T = \sqrt{(mg)^2 + (ma)^2} = 10\sqrt{2}$ N



Q.90



$$\frac{d}{dt}(L-2x) = 1 \text{ m/s}$$

$$\therefore -\frac{dx}{dt} = \frac{1}{2} \text{ m/s}$$

$$\therefore r_{CM} = \frac{2L-3x}{2}$$

$$\frac{dr_{CM}}{dt} = v_{CM} = \frac{-3}{2}\frac{dx}{dt} = \frac{3}{4} \text{ m/s}$$