FINAL JEE(Advanced) EXAMINATION - 2020

(Held On Sunday 27th SEPTEMBER, 2020)

PAPER-1 | **TEST PAPER WITH ANSWER**

PART-3 : MATHEMATICS

SECTION-1 : (Maximum Marks : 18)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : +3 If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : –1 In all other cases.

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1. Suppose a, b denote the distinct real roots of the quadratic polynomial $x^2 + 20x - 2020$ and suppose c,d denote the distinct complex roots of the quadratic polynomial $x^2 - 20x + 2020$. Then the value of

$$ac(a-c) + ad(a-d) + bc(b-c) + bd(b-d)$$

is

(A) 0 (B) 8000 (C) 8080 (D) 16000

Ans. (D)

2. If the function $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = |x|(x - \sin x)$, then which of the following statements is **TRUE ?**

(A) f is one-one, but **NOT** onto
(B) f is onto, but **NOT** one-one
(C) f is **BOTH** one-one and onto
(D) f is **NEITHER** one-one **NOR** onto

Ans. (C)

3. Let the functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be defined by

$f(x) = e^{x-1} - e^{-|x-1|}$ and $g(x) = \frac{1}{2}\left(e^{x-1} + e^{1-x}\right)$.

Then the area of the region in the first quadrant bounded by the curves $y = f(x)$, $y = g(x)$ and $x = 0$ is

(A) $\left(2-\sqrt{3}\right)+\frac{1}{2}(e-e^{-1})$
(B) $\left(2+\sqrt{3}\right)+\frac{1}{2}(e-e^{-1})$
(C) $\left(2-\sqrt{3}\right)+\frac{1}{2}(e+e^{-1})$
(D) $\left(2+\sqrt{3}\right)+\frac{1}{2}(e+e^{-1})$

Ans. (A)

4. Let a, b and λ be positive real numbers. Suppose P is an end point of the latus rectum of the parabola $y^2 = 4\lambda x$, and suppose the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through the point P. If the tangents to the parabola and the ellipse at the point P are perpendicular to each other, then the eccentricity of the ellipse is

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{2}{5}$

Ans. (A)

5. Let C_1 and C_2 be two biased coins such that the probabilities of getting head in a single toss are $\frac{2}{3}$ and $\frac{1}{3}$, respectively. Suppose α is the number of heads that appear when C_1 is tossed twice, independently, and suppose β is the number of heads that appear when C_2 is tossed twice, independently, Then probability that the roots of the quadratic polynomial $x^2 - \alpha x + \beta$ are real and equal, is

(A) $\frac{40}{81}$ (B) $\frac{20}{81}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Ans. (B)

6. Consider all rectangles lying in the region

$$\left\{(x,y)\in\mathbb{R}\times\mathbb{R}: 0\le x\le\frac{\pi}{2} \text{ and } 0\le y\le 2\sin(2x)\right\}$$

and having one side on the x-axis. The area of the rectangle which has the maximum perimeter among all such rectangles, is

(A) $\frac{3\pi}{2}$ (B) π (C) $\frac{\pi}{2\sqrt{3}}$ (D) $\frac{\pi\sqrt{3}}{2}$

Ans. (C)

SECTION-2 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks	: +4	If only (all) the correct option(s) is(are) chosen;
Partial Marks	: +3	If all the four options are correct but ONLY three options are chosen;
Partial Marks	: +2	If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks	: +1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	: 0	If none of the options is chose (i.e. the question is unanswered);
Negative Marks	: –2	In all other cases

7. Let the function $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^3 - x^2 + (x - 1)\sin x$ and let $g : \mathbb{R} \to \mathbb{R}$ be an arbitrary function. Let $fg : \mathbb{R} \to \mathbb{R}$ be the product function defined by $(f g)(x) = f(x)\, g(x)$. Then which of the following statements is/are TRUE ?

(A) If g is continuous at $x = 1$, then fg is differentiable at $x = 1$

(B) If fg is differentiable at $x = 1$, then g is continuous at $x = 1$

(C) If g is differentiable at $x = 1$, then fg is differentiable at $x = 1$

(D) If fg is differentiable at $x = 1$, then g is differentiable at $x = 1$

Ans. (A,C)

8. Let M be a 3×3 invertible matrix with real entries and let I denote the 3×3 identity matrix. If $M^{-1} = \text{adj}(\text{adj } M)$, then which of the following statement is/are ALWAYS TRUE ?

(A) $M = I$ (B) $\det M = 1$ (C) $M^2 = I$ (D) $(\text{adj } M)^2 = I$

Ans. (B,C,D)

9. Let S be the set of all complex numbers z satisfying $|z^2 + z + 1| = 1$. Then which of the following statements is/are TRUE ?

(A) $\left|z+\frac{1}{2}\right| \le \frac{1}{2}$ for all $z \in S$

(B) $|z| \le 2$ for all $z \in S$

(C) $\left|z+\frac{1}{2}\right| \ge \frac{1}{2}$ for all $z \in S$

(D) The set S has exactly four elements

Ans. (B,C)

10. Let x, y and z be positive real numbers. Suppose x, y and z are lengths of the sides of a triangle opposite to its angles X, Y and Z, respectively. If

$$\tan\frac{X}{2} + \tan\frac{Z}{2} = \frac{2y}{x+y+z},$$

then which of the following statements is/are TRUE?

(A) $2Y = X + Z$

(B) $Y = X + Z$

(C) $\tan\frac{X}{2} = \frac{x}{y+z}$

(D) $x^2 + z^2 - y^2 = xz$

Ans. (B,C)

11. Let L_1 and L_2 be the following straight line.

$$\mathbf{L_1}: \frac{x-1}{1}=\frac{y}{-1}=\frac{z-1}{3} \text{ and } L_2: \frac{x-1}{-3}=\frac{y}{-1}=\frac{z-1}{1}$$

Suppose the straight line

$$L: \frac{x-\alpha}{l}=\frac{y-1}{m}=\frac{z-\gamma}{-2}$$

lies in the plane containing L_1 and L_2, and passes through the point of intersection of L_1 and L_2. If the line L bisects the acute angle between the lines L_1 and L_2, then which of the following statements is/are TRUE?

(A) $\alpha-\gamma=3$ (B) $l+m=2$ (C) $\alpha-\gamma=1$ (D) $l+m=0$

Ans. (A,B)

12. Which of the following inequalities is/are TRUE?

(A) $\int_0^1 x\cos x\,dx \ge \frac{3}{8}$ (B) $\int_0^1 x\sin x\,dx \ge \frac{3}{10}$ (C) $\int_0^1 x^2\cos x\,dx \ge \frac{1}{2}$ (D) $\int_0^1 x^2\sin x\,dx \ge \frac{2}{9}$

Ans. (A,B,D)

SECTION-3 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme :
  Full Marks : +4 If ONLY the correct numerical value is entered;
  Zero Marks : 0 In all other cases.

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13. Let m be the minimum possible value of $\log_3\left(3^{y_1}+3^{y_2}+3^{y_3}\right)$, where y_1, y_2, y_3 are real numbers for which $y_1 + y_2 + y_3 = 9$. Let M be the maximum possible value of $(\log_3 x_1 + \log_3 x_2 + \log_3 x_3)$, where x_1, x_2, x_3 are positive real numbers for which $x_1 + x_2 + x_3 = 9$. Then the value of $\log_2(m^3) + \log_3(M^2)$ is ______.

Ans. (8.00)

14. Let a_1, a_2, a_3, ….. be a sequence of positive integers in arithmetic progression with common difference 2. Also, let b_1, b_2, b_3, ….. be a sequence of positive integers in geometric progression with common ratio 2. If $a_1 = b_1 = c$, then the number of all possible values of c, for which the equality

$$2(a_1 + a_2 + \ldots + a_n) = b_1 + b_2 + \ldots + b_n$$

holds for some positive integer n, is ______

Ans. (1.00)

15. Let $f : [0, 2] \to \mathbb{R}$ be the function defined by

$$f(x) = (3 - \sin(2\pi x))\sin\left(\pi x - \frac{\pi}{4}\right) - \sin\left(3\pi x + \frac{\pi}{4}\right)$$

If $\alpha, \beta \in [0, 2]$ are such that $\{x \in [0, 2] : f(x) \geq 0\} = [\alpha, \beta]$, then the value of $\beta - \alpha$ is _____

Ans. (1.00)

16. In a triangle PQR, let $\vec{a} = \overline{QR}, \vec{b} = \overline{RP}$ and $\vec{c} = \overline{PQ}$. If

$|\vec{a}| = 3$, $|\vec{b}| = 4$ and $\dfrac{\vec{a}.(\vec{c}-\vec{b})}{\vec{c}.(\vec{a}-\vec{b})} = \dfrac{|\vec{a}|}{|\vec{a}|+|\vec{b}|}$, then the value of $|\vec{a}\times\vec{b}|^2$ is ______

Ans. (108.00)

17. For a polynomial g(x) with real coefficient, let m_g denote the number of distinct real roots of g(x). Suppose S is the set of polynomials with real coefficient defined by

$S = \{(x^2 - 1)^2 (a_0 + a_1x + a_2x^2 + a_3x^3) : a_0, a_1, a_2, a_3 \in \mathbb{R}\}$.

For a polynomial f, let f' and f'' denote its first and second order derivatives, respectively. Then the minimum possible value of $(m_{f'} + m_{f''})$, where $f \in S$, is _______

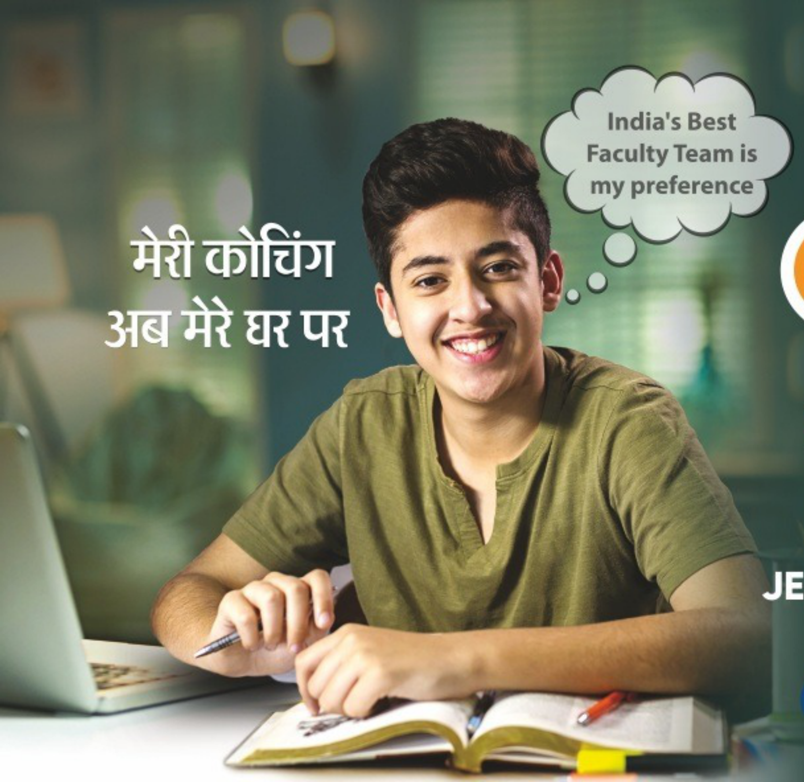
Ans. (5.00)

18. Let e denote the base of the natural logarithm. The value of the real number a for which the right hand limit

$$\lim_{x\to 0^+} \frac{(1-x)^{\frac{1}{x}} - e^{-1}}{x^a}$$

is equal to a nonzero real number, is ______.

Ans. (1.00)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