Exercise 3.1

Question 1:

In the matrix $A=\begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$, write:

(i) The order of the matrix (ii) The number of elements,

(iii) Write the elements a_{13}, a_{21}, a_{33}, a_{24}, a_{23}

Answer

(i) In the given matrix, the number of rows is 3 and the number of columns is 4. Therefore, the order of the matrix is 3 × 4.

(ii) Since the order of the matrix is 3 × 4, there are 3 × 4 = 12 elements in it.

(iii) $a_{13} = 19$, $a_{21} = 35$, $a_{33} = -5$, $a_{24} = 12$, $a_{23} = \frac{5}{2}$

Question 2:

If a matrix has 24 elements, what are the possible order it can have? What, if it has 13 elements?

Answer

We know that if a matrix is of the order $m \times n$, it has mn elements. Thus, to find all the possible orders of a matrix having 24 elements, we have to find all the ordered pairs of natural numbers whose product is 24.

The ordered pairs are: (1, 24), (24, 1), (2, 12), (12, 2), (3, 8), (8, 3), (4, 6), and (6, 4)

Hence, the possible orders of a matrix having 24 elements are: 1 × 24, 24 × 1, 2 × 12, 12 × 2, 3 × 8, 8 × 3, 4 × 6, and 6 × 4

(1, 13) and (13, 1) are the ordered pairs of natural numbers whose product is 13. Hence, the possible orders of a matrix having 13 elements are 1 × 13 and 13 × 1.

Question 3:

If a matrix has 18 elements, what are the possible orders it can have? What, if it has 5 elements?

Answer

We know that if a matrix is of the order *m* × *n*, it has *mn* elements. Thus, to find all the possible orders of a matrix having 18 elements, we have to find all the ordered pairs of natural numbers whose product is 18.

The ordered pairs are: (1, 18), (18, 1), (2, 9), (9, 2), (3, 6,), and (6, 3)
Hence, the possible orders of a matrix having 18 elements are:

1 × 18, 18 × 1, 2 × 9, 9 × 2, 3 × 6, and 6 × 3

(1, 5) and (5, 1) are the ordered pairs of natural numbers whose product is 5.
Hence, the possible orders of a matrix having 5 elements are 1 × 5 and 5 × 1.

Question 5:

Construct a 3 × 4 matrix, whose elements are given by

(i) $a_{ij} = \frac{1}{2}\left|-3i + j\right|$ (ii) $a_{ij} = 2i - j$

Answer

In general, a 3 × 4 matrix is given by $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$

(i) $a_{ij} = \frac{1}{2}\left|-3i + j\right|, \ i = 1,2,3 \text{ and } j = 1,2,3,4$

$$\therefore a_{11} = \frac{1}{2}|-3\times 1+1| = \frac{1}{2}|-3+1| = \frac{1}{2}|-2| = \frac{2}{2} = 1$$
$$a_{21} = \frac{1}{2}|-3\times 2+1| = \frac{1}{2}|-6+1| = \frac{1}{2}|-5| = \frac{5}{2}$$
$$a_{31} = \frac{1}{2}|-3\times 3+1| = \frac{1}{2}|-9+1| = \frac{1}{2}|-8| = \frac{8}{2} = 4$$

$$a_{12} = \frac{1}{2}|-3\times 1+2| = \frac{1}{2}|-3+2| = \frac{1}{2}|-1| = \frac{1}{2}$$
$$a_{22} = \frac{1}{2}|-3\times 2+2| = \frac{1}{2}|-6+2| = \frac{1}{2}|-4| = \frac{4}{2} = 2$$
$$a_{32} = \frac{1}{2}|-3\times 3+2| = \frac{1}{2}|-9+2| = \frac{1}{2}|-7| = \frac{7}{2}$$

$$a_{13} = \frac{1}{2}|-3\times 1+3| = \frac{1}{2}|-3+3| = 0$$
$$a_{23} = \frac{1}{2}|-3\times 2+3| = \frac{1}{2}|-6+3| = \frac{1}{2}|-3| = \frac{3}{2}$$
$$a_{33} = \frac{1}{2}|-3\times 3+3| = \frac{1}{2}|-9+3| = \frac{1}{2}|-6| = \frac{6}{2} = 3$$

$$a_{14} = \frac{1}{2}|-3\times 1+4| = \frac{1}{2}|-3+4| = \frac{1}{2}|1| = \frac{1}{2}$$
$$a_{24} = \frac{1}{2}|-3\times 2+4| = \frac{1}{2}|-6+4| = \frac{1}{2}|-2| = \frac{2}{2} = 1$$
$$a_{34} = \frac{1}{2}|-3\times 3+4| = \frac{1}{2}|-9+4| = \frac{1}{2}|-5| = \frac{5}{2}$$

Therefore, the required matrix is $A = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$

(ii) $a_{ij} = 2i - j,\ i = 1,2,3$ and $j = 1,2,3,4$

$$\therefore a_{11} = 2\times 1-1 = 2-1 = 1$$
$$a_{21} = 2\times 2-1 = 4-1 = 3$$
$$a_{31} = 2\times 3-1 = 6-1 = 5$$

$$a_{12} = 2\times 1-2 = 2-2 = 0$$
$$a_{22} = 2\times 2-2 = 4-2 = 2$$
$$a_{32} = 2\times 3-2 = 6-2 = 4$$

$$a_{13} = 2\times 1-3 = 2-3 = -1$$
$$a_{23} = 2\times 2-3 = 4-3 = 1$$
$$a_{33} = 2\times 3-3 = 6-3 = 3$$

$$a_{14} = 2\times 1-4 = 2-4 = -2$$
$$a_{24} = 2\times 2-4 = 4-4 = 0$$
$$a_{34} = 2\times 3-4 = 6-4 = 2$$

Therefore, the required matrix is $A = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$

Question 6:

Find the value of *x*, *y*, and *z* from the following equation:

(i) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$ (ii) $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

(iii) $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Answer

(i) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

As the given matrices are equal, their corresponding elements are also equal.

Comparing the corresponding elements, we get:

$x = 1$, $y = 4$, and $z = 3$

(ii) $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

As the given matrices are equal, their corresponding elements are also equal.
Comparing the corresponding elements, we get:

$x + y = 6$, $xy = 8$, $5 + z = 5$ Now, $5 + z = 5 \Rightarrow z = 0$

We know that:

$(x - y)^2 = (x + y)^2 - 4xy$

$\Rightarrow (x - y)2 = 36 - 32 = 4$

$\Rightarrow x - y = \pm 2$

Now, when $x - y = 2$ and $x + y = 6$, we get $x = 4$ and $y = 2$

When $x - y = -2$ and $x + y = 6$, we get $x = 2$ and $y = 4$

$\therefore x = 4$, $y = 2$, and $z = 0$ or $x = 2$, $y = 4$, and $z = 0$

(iii) $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

As the two matrices are equal, their corresponding elements are also equal.
Comparing the corresponding elements, we get:

$x + y + z = 9$ … (1)

$x + z = 5$ … (2)

$y + z = 7$ … (3)

From (1) and (2), we have: $y + 5 = 9$

$\Rightarrow y = 4$

Then, from (3), we have: $4 + z = 7$

$\Rightarrow z = 3$

$\therefore x + z = 5$

$\Rightarrow x = 2$

$\therefore x = 2$, $y = 4$, and $z = 3$

Question 7:

Find the value of a, b, c, and d from the equation:

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

Answer

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

As the two matrices are equal, their corresponding elements are also equal.
Comparing the corresponding elements, we get:

$a - b = -1$ … (1)

$2a - b = 0$ … (2)

$2a + c = 5$ … (3)

$3c + d = 13$ … (4)

From (2), we have: $b = 2a$

Then, from (1), we have: $a - 2a = -1$
$\Rightarrow a = 1$
$\Rightarrow b = 2$
Now, from (3), we have:

$2 \times 1 + c = 5$
$\Rightarrow c = 3$
From (4) we have:

$3 \times 3 + d = 13$
$\Rightarrow 9 + d = 13 \Rightarrow d = 4$

$\therefore a = 1, b = 2, c = 3, \text{ and } d = 4$

Question 8:

$A = \left[a_{ij}\right]_{m\times n}$ is a square matrix, if

(A) $m < n$

(B) $m > n$

(C) *m* = *n*

(D) None of these Answer

The correct answer is C.

It is known that a given matrix is said to be a square matrix if the number of rows is equal to the number of columns.

Therefore, $A=\left[a_{ij}\right]_{m\times n}$ is a square matrix, if *m* = *n*.

Question 9:

Which of the given values of *x* and *y* make the following pair of matrices equal

$$\begin{bmatrix}3x+7 & 5\\ y+1 & 2-3x\end{bmatrix}=\begin{bmatrix}0 & y-2\\ 8 & 4\end{bmatrix}$$

(A) $x=\frac{-1}{3}, y=7$

(B) Not possible to find

(C) $y=7, x=\frac{-2}{3}$

(D) $x=\frac{-1}{3}, y=\frac{-2}{3}$

Answer

The correct answer is B.

It is given that $\begin{bmatrix}3x+7 & 5\\ y+1 & 2-3x\end{bmatrix}=\begin{bmatrix}0 & y-2\\ 8 & 4\end{bmatrix}$

Equating the corresponding elements, we get:

$$3x+7=0 \Rightarrow x=-\frac{7}{3}$$
$$5=y-2 \Rightarrow y=7$$
$$y+1=8 \Rightarrow y=7$$
$$2-3x=4 \Rightarrow x=-\frac{2}{3}$$

We find that on comparing the corresponding elements of the two matrices, we get two different values of x, which is not possible.

Hence, it is not possible to find the values of x and y for which the given matrices are equal.

Question 10:

The number of all possible matrices of order 3 × 3 with each entry 0 or 1 is:

(A) 27

(B) 18

(C) 81

(D) 512

Answer

The correct answer is D.

The given matrix of the order 3 × 3 has 9 elements and each of these elements can be either 0 or 1.

Now, each of the 9 elements can be filled in two possible ways.

Therefore, by the multiplication principle, the required number of possible matrices is $2^9 = 512$

Exercise 3.2

Question 1:

Let $A=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}, B=\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}, C=\begin{bmatrix}-2 & 5\\ 3 & 4\end{bmatrix}$

Find each of the following

(i) $A+B$ (ii) $A-B$ (iii) $3A-C$

(iv) AB (v) BA

Answer

(i)

$$A+B=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}+\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}=\begin{bmatrix}2+1 & 4+3\\ 3-2 & 2+5\end{bmatrix}=\begin{bmatrix}3 & 7\\ 1 & 7\end{bmatrix}$$

(ii)

$$A-B=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}-\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}=\begin{bmatrix}2-1 & 4-3\\ 3-(-2) & 2-5\end{bmatrix}=\begin{bmatrix}1 & 1\\ 5 & -3\end{bmatrix}$$

(iii)

$$3A-C=3\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}-\begin{bmatrix}-2 & 5\\ 3 & 4\end{bmatrix}$$

$$=\begin{bmatrix}3\times2 & 3\times4\\ 3\times3 & 3\times2\end{bmatrix}-\begin{bmatrix}-2 & 5\\ 3 & 4\end{bmatrix}$$

$$=\begin{bmatrix}6 & 12\\ 9 & 6\end{bmatrix}-\begin{bmatrix}-2 & 5\\ 3 & 4\end{bmatrix}$$

$$=\begin{bmatrix}6+2 & 12-5\\ 9-3 & 6-4\end{bmatrix}$$

$$=\begin{bmatrix}8 & 7\\ 6 & 2\end{bmatrix}$$

(iv) Matrix *A* has 2 columns. This number is equal to the number of rows in matrix *B*. Therefore, *AB* is defined as:

$$AB=\begin{bmatrix}2 & 4\\3 & 2\end{bmatrix}\begin{bmatrix}1 & 3\\-2 & 5\end{bmatrix}=\begin{bmatrix}2(1)+4(-2) & 2(3)+4(5)\\3(1)+2(-2) & 3(3)+2(5)\end{bmatrix}$$
$$=\begin{bmatrix}2-8 & 6+20\\3-4 & 9+10\end{bmatrix}=\begin{bmatrix}-6 & 26\\-1 & 19\end{bmatrix}$$

(v) Matrix *B* has 2 columns. This number is equal to the number of rows in matrix *A*. Therefore, *BA* is defined as:

$$BA=\begin{bmatrix}1 & 3\\-2 & 5\end{bmatrix}\begin{bmatrix}2 & 4\\3 & 2\end{bmatrix}=\begin{bmatrix}1(2)+3(3) & 1(4)+3(2)\\-2(2)+5(3) & -2(4)+5(2)\end{bmatrix}$$
$$=\begin{bmatrix}2+9 & 4+6\\-4+15 & -8+10\end{bmatrix}=\begin{bmatrix}11 & 10\\11 & 2\end{bmatrix}$$

Question 2:

Compute the following:

(i) $\begin{bmatrix}a & b\\-b & a\end{bmatrix}+\begin{bmatrix}a & b\\b & a\end{bmatrix}$ (ii) $\begin{bmatrix}a^2+b^2 & b^2+c^2\\a^2+c^2 & a^2+b^2\end{bmatrix}+\begin{bmatrix}2ab & 2bc\\-2ac & -2ab\end{bmatrix}$

(iii) $\begin{bmatrix}-1 & 4 & -6\\8 & 5 & 16\\2 & 8 & 5\end{bmatrix}+\begin{bmatrix}12 & 7 & 6\\8 & 0 & 5\\3 & 2 & 4\end{bmatrix}$

(v) $\begin{bmatrix}\cos^2 x & \sin^2 x\\\sin^2 x & \cos^2 x\end{bmatrix}+\begin{bmatrix}\sin^2 x & \cos^2 x\\\cos^2 x & \sin^2 x\end{bmatrix}$

Answer

(i) $\begin{bmatrix}a & b\\-b & a\end{bmatrix}+\begin{bmatrix}a & b\\b & a\end{bmatrix}=\begin{bmatrix}a+a & b+b\\-b+b & a+a\end{bmatrix}=\begin{bmatrix}2a & 2b\\0 & 2a\end{bmatrix}$

(ii) $\begin{bmatrix}a^2+b^2 & b^2+c^2\\a^2+c^2 & a^2+b^2\end{bmatrix}+\begin{bmatrix}2ab & 2bc\\-2ac & -2ab\end{bmatrix}$

$$=\begin{bmatrix} a^2+b^2+2ab & b^2+c^2+2bc \\ a^2+c^2-2ac & a^2+b^2-2ab \end{bmatrix}$$

$$=\begin{bmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{bmatrix}$$

(iii) $\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix}+\begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$

$$=\begin{bmatrix} -1+12 & 4+7 & -6+6 \\ 8+8 & 5+0 & 16+5 \\ 2+3 & 8+2 & 5+4 \end{bmatrix}$$

$$=\begin{bmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{bmatrix}$$

(iv) $\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix}+\begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$

$$=\begin{bmatrix} \cos^2 x+\sin^2 x & \sin^2 x+\cos^2 x \\ \sin^2 x+\cos^2 x & \cos^2 x+\sin^2 x \end{bmatrix}$$

$$=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad \left(\because \sin^2 x+\cos^2 x=1\right)$$

Question 3:

Compute the indicated products

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

(ii) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$

(iii) $\begin{bmatrix}1 & -2\\2 & 3\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\end{bmatrix}$

(iv) $\begin{bmatrix}2 & 3 & 4\\3 & 4 & 5\\4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & -3 & 5\\0 & 2 & 4\\3 & 0 & 5\end{bmatrix}$

(v) $\begin{bmatrix}2 & 1\\3 & 2\\-1 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 1\\-1 & 2 & 1\end{bmatrix}$

(vi) $\begin{bmatrix}3 & -1 & 3\\-1 & 0 & 2\end{bmatrix}\begin{bmatrix}2 & -3\\1 & 0\\3 & 1\end{bmatrix}$

Answer

(i) $\begin{bmatrix}a & b\\-b & a\end{bmatrix}\begin{bmatrix}a & -b\\b & a\end{bmatrix}$

$$\begin{bmatrix}a & b\\-b & a\end{bmatrix}\begin{bmatrix}a & -b\\b & a\end{bmatrix}$$

$$=\begin{bmatrix}a(a)+b(b) & a(-b)+b(a)\\-b(a)+a(b) & -b(-b)+a(a)\end{bmatrix}$$

$$=\begin{bmatrix}a^2+b^2 & -ab+ab\\-ab+ab & b^2+a^2\end{bmatrix}=\begin{bmatrix}a^2+b^2 & 0\\0 & a^2+b^2\end{bmatrix}$$

(ii) $\begin{bmatrix}1\\2\\3\end{bmatrix}\begin{bmatrix}2 & 3 & 4\end{bmatrix}=\begin{bmatrix}1(2) & 1(3) & 1(4)\\2(2) & 2(3) & 2(4)\\3(2) & 3(3) & 3(4)\end{bmatrix}=\begin{bmatrix}2 & 3 & 4\\4 & 6 & 8\\6 & 9 & 12\end{bmatrix}$

(iii) $\begin{bmatrix}1 & -2\\2 & 3\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\end{bmatrix}$

$$=\begin{bmatrix} 1(1)-2(2) & 1(2)-2(3) & 1(3)-2(1) \\ 2(1)+3(2) & 2(2)+3(3) & 2(3)+3(1) \end{bmatrix}$$

$$=\begin{bmatrix} 1-4 & 2-6 & 3-2 \\ 2+6 & 4+9 & 6+3 \end{bmatrix}=\begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

(iv) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$

$$=\begin{bmatrix} 2(1)+3(0)+4(3) & 2(-3)+3(2)+4(0) & 2(5)+3(4)+4(5) \\ 3(1)+4(0)+5(3) & 3(-3)+4(2)+5(0) & 3(5)+4(4)+5(5) \\ 4(1)+5(0)+6(3) & 4(-3)+5(2)+6(0) & 4(5)+5(4)+6(5) \end{bmatrix}$$

$$=\begin{bmatrix} 2+0+12 & -6+6+0 & 10+12+20 \\ 3+0+15 & -9+8+0 & 15+16+25 \\ 4+0+18 & -12+10+0 & 20+20+30 \end{bmatrix}=\begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$$

(v) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

$$=\begin{bmatrix} 2(1)+1(-1) & 2(0)+1(2) & 2(1)+1(1) \\ 3(1)+2(-1) & 3(0)+2(2) & 3(1)+2(1) \\ -1(1)+1(-1) & -1(0)+1(2) & -1(1)+1(1) \end{bmatrix}$$

$$=\begin{bmatrix} 2-1 & 0+2 & 2+1 \\ 3-2 & 0+4 & 3+2 \\ -1-1 & 0+2 & -1+1 \end{bmatrix}=\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$$

(vi) $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$

$$=\begin{bmatrix} 3(2)-1(1)+3(3) & 3(-3)-1(0)+3(1) \\ -1(2)+0(1)+2(3) & -1(-3)+0(0)+2(1) \end{bmatrix}$$

$$=\begin{bmatrix} 6-1+9 & -9-0+3 \\ -2+0+6 & 3+0+2 \end{bmatrix}=\begin{bmatrix} 14 & -6 \\ 4 & 5 \end{bmatrix}$$

Question 4:

If $A=\begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$, and $C=\begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$, then compute $(A+B)$ and $(B-C)$. Also, verify that $A+(B-C)=(A+B)-C$

Answer

$$A+B=\begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}+\begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$$

$$=\begin{bmatrix} 1+3 & 2-1 & -3+2 \\ 5+4 & 0+2 & 2+5 \\ 1+2 & -1+0 & 1+3 \end{bmatrix}=\begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix}$$

$$B-C=\begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}-\begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$$

$$=\begin{bmatrix} 3-4 & -1-1 & 2-2 \\ 4-0 & 2-3 & 5-2 \\ 2-1 & 0-(-2) & 3-3 \end{bmatrix}=\begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$$

$$A+(B-C)=\begin{bmatrix}1 & 2 & -3\\5 & 0 & 2\\1 & -1 & 1\end{bmatrix}+\begin{bmatrix}-1 & -2 & 0\\4 & -1 & 3\\1 & 2 & 0\end{bmatrix}$$
$$=\begin{bmatrix}1+(-1) & 2+(-2) & -3+0\\5+4 & 0+(-1) & 2+3\\1+1 & -1+2 & 1+0\end{bmatrix}=\begin{bmatrix}0 & 0 & -3\\9 & -1 & 5\\2 & 1 & 1\end{bmatrix}$$

$$(A+B)-C=\begin{bmatrix}4 & 1 & -1\\9 & 2 & 7\\3 & -1 & 4\end{bmatrix}-\begin{bmatrix}4 & 1 & 2\\0 & 3 & 2\\1 & -2 & 3\end{bmatrix}$$
$$=\begin{bmatrix}4-4 & 1-1 & -1-2\\9-0 & 2-3 & 7-2\\3-1 & -1-(-2) & 4-3\end{bmatrix}=\begin{bmatrix}0 & 0 & -3\\9 & -1 & 5\\2 & 1 & 1\end{bmatrix}$$

Hence, we have verified that $A+(B-C)=(A+B)-C$.

Question 5:

If $A=\begin{bmatrix}\frac{2}{3} & 1 & \frac{5}{3}\\\frac{1}{3} & \frac{2}{3} & \frac{4}{3}\\\frac{7}{3} & 2 & \frac{2}{3}\end{bmatrix}$ and $B=\begin{bmatrix}\frac{2}{5} & \frac{3}{5} & 1\\\frac{1}{5} & \frac{2}{5} & \frac{4}{5}\\\frac{7}{5} & \frac{6}{5} & \frac{2}{5}\end{bmatrix}$ then compute $3A-5B$.

Answer

$$3A-5B=3\begin{bmatrix}\frac{2}{3} & 1 & \frac{5}{3}\\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3}\\ \frac{7}{3} & 2 & \frac{2}{3}\end{bmatrix}-5\begin{bmatrix}\frac{2}{5} & \frac{3}{5} & 1\\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5}\\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5}\end{bmatrix}$$

$$=\begin{bmatrix}2 & 3 & 5\\ 1 & 2 & 4\\ 7 & 6 & 2\end{bmatrix}-\begin{bmatrix}2 & 3 & 5\\ 1 & 2 & 4\\ 7 & 6 & 2\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

Question 6:

Simplify $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$

Answer

$$\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ -\sin\theta\cos\theta & \cos^2\theta\end{bmatrix}+\begin{bmatrix}\sin^2\theta & -\sin\theta\cos\theta\\ \sin\theta\cos\theta & \sin^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\theta+\sin^2\theta & \cos\theta\sin\theta-\sin\theta\cos\theta\\ -\sin\theta\cos\theta+\sin\theta\cos\theta & \cos^2\theta+\sin^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\qquad\left(\because \cos^2\theta+\sin^2\theta=1\right)$$

Question 7:

Find *X* and *Y*, if

(i) $X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$ and $X-Y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$

(ii) $2X+3Y=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$ and $3X+2Y=\begin{bmatrix}2 & -2\\ 1 & 5\end{bmatrix}$

Answer

(i)

$$X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix} \quad ...(1)$$

$$X-Y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix} \quad ...(2)$$

Adding equations (1) and (2), we get:

$$2X=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}+\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}=\begin{bmatrix}7+3 & 0+0\\ 2+0 & 5+3\end{bmatrix}=\begin{bmatrix}10 & 0\\ 2 & 8\end{bmatrix}$$

$$\therefore X=\frac{1}{2}\begin{bmatrix}10 & 0\\ 2 & 8\end{bmatrix}=\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}$$

$$\text{Now, } X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$$

$$\Rightarrow Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}-\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}$$

$$\Rightarrow Y=\begin{bmatrix}7-5 & 0-0\\ 2-1 & 5-4\end{bmatrix}$$

$$\therefore Y=\begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}$$

(ii)

$$2X+3Y=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix} \quad ...(3)$$

$$3X+2Y=\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix} \quad ...(4)$$

Multiplying equation (3) with (2), we get:

$$2(2X+3Y)=2\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$$

$$\Rightarrow 4X+6Y=\begin{bmatrix}4 & 6\\ 8 & 0\end{bmatrix} \qquad ...(5)$$

Multiplying equation (4) with (3), we get:

$$3(3X+2Y)=3\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}$$

$$\Rightarrow 9X+6Y=\begin{bmatrix}6 & -6\\ -3 & 15\end{bmatrix} \qquad ...(6)$$

From (5) and (6), we have:

$$(4X+6Y)-(9X+6Y)=\begin{bmatrix}4 & 6\\ 8 & 0\end{bmatrix}-\begin{bmatrix}6 & -6\\ -3 & 15\end{bmatrix}$$

$$\Rightarrow -5X=\begin{bmatrix}4-6 & 6-(-6)\\ 8-(-3) & 0-15\end{bmatrix}=\begin{bmatrix}-2 & 12\\ 11 & -15\end{bmatrix}$$

$$\therefore X=-\frac{1}{5}\begin{bmatrix}-2 & 12\\ 11 & -15\end{bmatrix}=\begin{bmatrix}\frac{2}{5} & -\frac{12}{5}\\ -\frac{11}{5} & 3\end{bmatrix}$$

Now, $2X+3Y=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$

$$\Rightarrow 2\begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix} + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{4}{5} & -\frac{24}{5} \\ -\frac{22}{5} & 6 \end{bmatrix} + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} \frac{4}{5} & -\frac{24}{5} \\ -\frac{22}{5} & 6 \end{bmatrix}$$

$$\Rightarrow 3Y = \begin{bmatrix} 2-\frac{4}{5} & 3+\frac{24}{5} \\ 4+\frac{22}{5} & 0-6 \end{bmatrix} = \begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix}$$

$$\therefore Y = \frac{1}{3}\begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$

Question 8:

Find *X*, if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

Answer

$$2X+Y=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}$$

$$\Rightarrow 2X+\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}$$

$$\Rightarrow 2X=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}-\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}=\begin{bmatrix}1-3 & 0-2\\ -3-1 & 2-4\end{bmatrix}$$

$$\Rightarrow 2X=\begin{bmatrix}-2 & -2\\ -4 & -2\end{bmatrix}$$

$$\therefore X=\frac{1}{2}\begin{bmatrix}-2 & -2\\ -4 & -2\end{bmatrix}=\begin{bmatrix}-1 & -1\\ -2 & -1\end{bmatrix}$$

Question 9:

Find *x* and *y*, if $2\begin{bmatrix}1 & 3\\ 0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$

Answer

$$2\begin{bmatrix}1 & 3\\ 0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}2 & 6\\ 0 & 2x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}2+y & 6\\ 1 & 2x+2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we have:

$$2+y=5$$
$$\Rightarrow y=3$$

$\therefore x = 3$ and $y = 3$

$$2x+2=8$$
$$\Rightarrow x=3$$

Question 10:

Solve the equation for *x*, *y*, *z* and *t* if

$$2\begin{bmatrix} x & z \\ y & t \end{bmatrix}+3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}=3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

Answer

$$2\begin{bmatrix} x & z \\ y & t \end{bmatrix}+3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}=3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix}+\begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix}=\begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+3 & 2z-3 \\ 2y & 2t+6 \end{bmatrix}=\begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we get:

$$2x+3=9$$
$$\Rightarrow 2x=6$$
$$\Rightarrow x=3$$

$$2y=12$$
$$\Rightarrow y=6$$

$$2z-3=15$$
$$\Rightarrow 2z=18$$
$$\Rightarrow z=9$$

$$2t+6=18$$
$$\Rightarrow 2t=12$$
$$\Rightarrow t=6$$

$$\therefore x=3,\ y=6,\ z=9,\ \text{and } t=6$$

Question 11:

If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix}+y\begin{bmatrix} -1 \\ 1 \end{bmatrix}=\begin{bmatrix} 10 \\ 5 \end{bmatrix}$, find the value of x and y.

Answer

$$x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$$
$$\Rightarrow\begin{bmatrix}2x\\3x\end{bmatrix}+\begin{bmatrix}-y\\y\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$$
$$\Rightarrow\begin{bmatrix}2x-y\\3x+y\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we get: $2x - y = 10$ and $3x + y = 5$

Adding these two equations, we have: $5x = 15$
$\Rightarrow x = 3$

Now, $3x + y = 5$
$\Rightarrow y = 5 - 3x$
$\Rightarrow y = 5 - 9 = -4$
$\therefore x = 3$ and $y = -4$

Question 12:

Given $3\begin{bmatrix}x & y\\z & w\end{bmatrix}=\begin{bmatrix}x & 6\\-1 & 2w\end{bmatrix}+\begin{bmatrix}4 & x+y\\z+w & 3\end{bmatrix}$, find the values of x, y, z and w.

Answer

$$3\begin{bmatrix}x & y\\z & w\end{bmatrix}=\begin{bmatrix}x & 6\\-1 & 2w\end{bmatrix}+\begin{bmatrix}4 & x+y\\z+w & 3\end{bmatrix}$$
$$\Rightarrow\begin{bmatrix}3x & 3y\\3z & 3w\end{bmatrix}=\begin{bmatrix}x+4 & 6+x+y\\-1+z+w & 2w+3\end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we get:

$3x = x + 4$
$\Rightarrow 2x = 4$
$\Rightarrow x = 2$

$3y = 6 + x + y$
$\Rightarrow 2y = 6 + x = 6 + 2 = 8$
$\Rightarrow y = 4$

$3w = 2w + 3$
$\Rightarrow w = 3$

$3z = -1 + z + w$
$\Rightarrow 2z = -1 + w = -1 + 3 = 2$
$\Rightarrow z = 1$

$\therefore x = 2,\ y = 4,\ z = 1,\ \text{and } w = 3$

Question 13:

If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that $F(x)F(y) = F(x+y)$.

Answer

$$F(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix},\ F(y)=\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$F(x+y)=\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$F(x)F(y)$$
$$=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$$
$$=\begin{bmatrix}\cos x\cos y-\sin x\sin y+0 & -\cos x\sin y-\sin x\cos y+0 & 0\\ \sin x\cos y+\cos x\sin y+0 & -\sin x\sin y+\cos x\cos y+0 & 0\\ 0 & 0 & 0\end{bmatrix}$$
$$=\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}$$
$$=F(x+y)$$

$$\therefore F(x)F(y)=F(x+y)$$

Question 14:

Show that

(i) $\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}\neq\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}$

(ii) $\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 0\\ 1 & 1 & 0\end{bmatrix}\begin{bmatrix}-1 & 1 & 0\\ 0 & -1 & 1\\ 2 & 3 & 4\end{bmatrix}\neq\begin{bmatrix}-1 & 1 & 0\\ 0 & -1 & 1\\ 2 & 3 & 4\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 0\\ 1 & 1 & 0\end{bmatrix}$

Answer

(i)

$$\begin{bmatrix}5 & -1\\6 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\3 & 4\end{bmatrix}$$
$$=\begin{bmatrix}5(2)-1(3) & 5(1)-1(4)\\6(2)+7(3) & 6(1)+7(4)\end{bmatrix}$$
$$=\begin{bmatrix}10-3 & 5-4\\12+21 & 6+28\end{bmatrix}=\begin{bmatrix}7 & 1\\33 & 34\end{bmatrix}$$

$$\begin{bmatrix}2 & 1\\3 & 4\end{bmatrix}\begin{bmatrix}5 & -1\\6 & 7\end{bmatrix}$$
$$=\begin{bmatrix}2(5)+1(6) & 2(-1)+1(7)\\3(5)+4(6) & 3(-1)+4(7)\end{bmatrix}$$
$$=\begin{bmatrix}10+6 & -2+7\\15+24 & -3+28\end{bmatrix}=\begin{bmatrix}16 & 5\\39 & 25\end{bmatrix}$$

$$\therefore\begin{bmatrix}5 & -1\\6 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\3 & 4\end{bmatrix}\neq\begin{bmatrix}2 & 1\\3 & 4\end{bmatrix}\begin{bmatrix}5 & -1\\6 & 7\end{bmatrix}$$

(ii)

$$\begin{bmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{bmatrix}\begin{bmatrix}-1 & 1 & 0\\0 & -1 & 1\\2 & 3 & 4\end{bmatrix}$$
$$=\begin{bmatrix}1(-1)+2(0)+3(2) & 1(1)+2(-1)+3(3) & 1(0)+2(1)+3(4)\\0(-1)+1(0)+0(2) & 0(1)+1(-1)+0(3) & 0(0)+1(1)+0(4)\\1(-1)+1(0)+0(2) & 1(1)+1(-1)+0(3) & 1(0)+1(1)+0(4)\end{bmatrix}$$
$$=\begin{bmatrix}5 & 8 & 14\\0 & -1 & 1\\-1 & 0 & 1\end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1(1)+1(0)+0(1) & -1(2)+1(1)+0(1) & -1(3)+1(0)+0(0) \\ 0(1)+(-1)(0)+1(1) & 0(2)+(-1)(1)+1(1) & 0(3)+(-1)(0)+1(0) \\ 2(1)+3(0)+4(1) & 2(2)+3(1)+4(1) & 2(3)+3(0)+4(0) \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & -3 \\ 1 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Question 15:

Find $A^2 - 5A + 6I$ if $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

Answer

We have $A^2 = A \times A$

$$A^2 = AA = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2(2)+0(2)+1(1) & 2(0)+0(1)+1(-1) & 2(1)+0(3)+1(0) \\ 2(2)+1(2)+3(1) & 2(0)+1(1)+3(-1) & 2(1)+1(3)+3(0) \\ 1(2)+(-1)(2)+0(1) & 1(0)+(-1)(1)+0(-1) & 1(1)+(-1)(3)+0(0) \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1+0 & 1-3+0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

$$\therefore A^2 - 5A + 6I$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 5-10 & -1-0 & 2-5 \\ 9-10 & -2-5 & 5-15 \\ 0-5 & -1+5 & -2-0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -5+6 & -1+0 & -3+0 \\ -1+0 & -7+6 & -10+0 \\ -5+0 & 4+0 & -2+6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$$

Question 16:

If $A=\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$, prove that $A^3-6A^2+7A+2I=O$

Answer

$$A^2=AA=\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1+0+4 & 0+0+0 & 2+0+6\\ 0+0+2 & 0+4+0 & 0+2+3\\ 2+0+6 & 0+0+0 & 4+0+9\end{bmatrix}=\begin{bmatrix}5 & 0 & 8\\ 2 & 4 & 5\\ 8 & 0 & 13\end{bmatrix}$$

Now $A^3=A^2\cdot A$

$$=\begin{bmatrix}5 & 0 & 8\\ 2 & 4 & 5\\ 8 & 0 & 13\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}5+0+16 & 0+0+0 & 10+0+24\\ 2+0+10 & 0+8+0 & 4+4+15\\ 8+0+26 & 0+0+0 & 16+0+39\end{bmatrix}$$

$$=\begin{bmatrix}21 & 0 & 34\\ 12 & 8 & 23\\ 34 & 0 & 55\end{bmatrix}$$

$\therefore A^3-6A^2+7A+2I$

$$=\begin{bmatrix}21 & 0 & 34\\ 12 & 8 & 23\\ 34 & 0 & 55\end{bmatrix}-6\begin{bmatrix}5 & 0 & 8\\ 2 & 4 & 5\\ 8 & 0 & 13\end{bmatrix}+7\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}+2\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}21 & 0 & 34\\12 & 8 & 23\\34 & 0 & 55\end{bmatrix}-\begin{bmatrix}30 & 0 & 48\\12 & 24 & 30\\48 & 0 & 78\end{bmatrix}+\begin{bmatrix}7 & 0 & 14\\0 & 14 & 7\\14 & 0 & 21\end{bmatrix}+\begin{bmatrix}2 & 0 & 0\\0 & 2 & 0\\0 & 0 & 2\end{bmatrix}$$

$$=\begin{bmatrix}21+7+2 & 0+0+0 & 34+14+0\\12+0+0 & 8+14+2 & 23+7+0\\34+14+0 & 0+0+0 & 55+21+2\end{bmatrix}-\begin{bmatrix}30 & 0 & 48\\12 & 24 & 30\\48 & 0 & 78\end{bmatrix}$$

$$=\begin{bmatrix}30 & 0 & 48\\12 & 24 & 30\\48 & 0 & 78\end{bmatrix}-\begin{bmatrix}30 & 0 & 48\\12 & 24 & 30\\48 & 0 & 78\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}=O$$

$$\therefore A^3-6A^2+7A+2I=O$$

Question 17:

If $A=\begin{bmatrix}3 & -2\\4 & -2\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$, find *k* so that $A^2=kA-2I$

Answer

$$A^2=A\cdot A=\begin{bmatrix}3 & -2\\4 & -2\end{bmatrix}\begin{bmatrix}3 & -2\\4 & -2\end{bmatrix}$$

$$=\begin{bmatrix}3(3)+(-2)(4) & 3(-2)+(-2)(-2)\\4(3)+(-2)(4) & 4(-2)+(-2)(-2)\end{bmatrix}=\begin{bmatrix}1 & -2\\4 & -4\end{bmatrix}$$

Now $A^2=kA-2I$

$$\Rightarrow\begin{bmatrix}1 & -2\\4 & -4\end{bmatrix}=k\begin{bmatrix}3 & -2\\4 & -2\end{bmatrix}-2\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}1 & -2\\4 & -4\end{bmatrix}=\begin{bmatrix}3k & -2k\\4k & -2k\end{bmatrix}-\begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}1 & -2\\4 & -4\end{bmatrix}=\begin{bmatrix}3k-2 & -2k\\4k & -2k-2\end{bmatrix}$$

Comparing the corresponding elements, we have:

$3k-2=1$

$\Rightarrow 3k=3$

$\Rightarrow k=1$

Thus, the value of *k* is 1.

Question 18:

If $A=\begin{bmatrix}0 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 0\end{bmatrix}$ and *I* is the identity matrix of order 2, show that

$I+A=(I-A)\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

Answer

On the L.H.S.

$$I+A$$

$$=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}+\begin{bmatrix}0 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 0\end{bmatrix}$$

$$=\begin{bmatrix}1 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 1\end{bmatrix} \qquad \ldots(1)$$

On the R.H.S.

$$(I-A)\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$$

$$=\left(\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}-\begin{bmatrix}0 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 0\end{bmatrix}\right)\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$$

$$=\begin{bmatrix}1 & \tan\frac{\alpha}{2}\\ -\tan\frac{\alpha}{2} & 1\end{bmatrix}\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$$

$$=\begin{bmatrix}\cos\alpha+\sin\alpha\tan\frac{\alpha}{2} & -\sin\alpha+\cos\alpha\tan\frac{\alpha}{2}\\ -\cos\alpha\tan\frac{\alpha}{2}+\sin\alpha & \sin\alpha\tan\frac{\alpha}{2}+\cos\alpha\end{bmatrix} \qquad \ldots(2)$$

$$=\begin{bmatrix} 1-2\sin^2\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\tan\frac{\alpha}{2} & -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}+\left(2\cos^2\frac{\alpha}{2}-1\right)\tan\frac{\alpha}{2} \\ -\left(2\cos^2\frac{\alpha}{2}-1\right)\tan\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} & 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\tan\frac{\alpha}{2}+1-2\sin^2\frac{\alpha}{2} \end{bmatrix}$$

$$=\begin{bmatrix} 1-2\sin^2\frac{\alpha}{2}+2\sin^2\frac{\alpha}{2} & -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}-\tan\frac{\alpha}{2} \\ -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}+\tan\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} & 2\sin^2\frac{\alpha}{2}+1-2\sin^2\frac{\alpha}{2} \end{bmatrix}$$

$$=\begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$$

Thus, from (1) and (2), we get L.H.S. = R.H.S.

Question 19:

A trust fund has Rs 30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide Rs 30,000 among the two types of bonds. If the trust fund must obtain an annual total interest of:

(a) Rs 1,800 (b) Rs 2,000

Answer

(a) Let Rs x be invested in the first bond. Then, the sum of money invested in the second bond will be Rs (30000 − x).

It is given that the first bond pays 5% interest per year and the second bond pays 7% interest per year.

Therefore, in order to obtain an annual total interest of Rs 1800, we have:

$$\begin{bmatrix} x & (30000-x) \end{bmatrix} \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 1800 \qquad \left[\text{S.I. for 1 year} = \frac{\text{Principal} \times \text{Rate}}{100} \right]$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000-x)}{100} = 1800$$
$$\Rightarrow 5x + 210000 - 7x = 180000$$
$$\Rightarrow 210000 - 2x = 180000$$
$$\Rightarrow 2x = 210000 - 180000$$
$$\Rightarrow 2x = 30000$$
$$\Rightarrow x = 15000$$

Thus, in order to obtain an annual total interest of Rs 1800, the trust fund should invest Rs 15000 in the first bond and the remaining Rs 15000 in the second bond.

(b) Let Rs *x* be invested in the first bond. Then, the sum of money invested in the second bond will be Rs (30000 − *x*).

Therefore, in order to obtain an annual total interest of Rs 2000, we have:

$$\begin{bmatrix} x & (30000-x) \end{bmatrix} \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 2000$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000-x)}{100} = 2000$$
$$\Rightarrow 5x + 210000 - 7x = 200000$$
$$\Rightarrow 210000 - 2x = 200000$$
$$\Rightarrow 2x = 210000 - 200000$$
$$\Rightarrow 2x = 10000$$
$$\Rightarrow x = 5000$$

Thus, in order to obtain an annual total interest of Rs 2000, the trust fund should invest Rs 5000 in the first bond and the remaining Rs 25000 in the second bond.

Question 20:

The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are Rs 80, Rs 60 and Rs 40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.

Answer

The bookshop has 10 dozen chemistry books, 8 dozen physics books, and 10 dozen economics books.

The selling prices of a chemistry book, a physics book, and an economics book are respectively given as Rs 80, Rs 60, and Rs 40.

The total amount of money that will be received from the sale of all these books can be represented in the form of a matrix as:

$$12\begin{bmatrix}10 & 8 & 10\end{bmatrix}\begin{bmatrix}80\\60\\40\end{bmatrix}$$

$$=12\left[10\times80+8\times60+10\times40\right]$$
$$=12\left(800+480+400\right)$$
$$=12\left(1680\right)$$
$$=20160$$

Thus, the bookshop will receive Rs 20160 from the sale of all these books.

Question 21:

Assume *X*, *Y*, *Z*, *W* and *P* are matrices of order $2\times n, 3\times k, 2\times p,\ n\times 3$, and $p\times k$ respectively. The restriction on *n*, *k* and *p* so that $PY+WY$ will be defined are:

A. *k* = 3, *p* = *n*

B. *k* is arbitrary, *p* = 2

C. *p* is arbitrary, *k* = 3

D. *k* = 2, *p* = 3

Answer

Matrices *P* and *Y* are of the orders *p* × *k* and 3 × *k* respectively.

Therefore, matrix *PY* will be defined if *k* = 3. Consequently, *PY* will be of the order *p* × *k*. Matrices *W* and *Y* are of the orders *n* × 3 and 3 × *k* respectively.

Since the number of columns in *W* is equal to the number of rows in *Y*, matrix *WY* is well-defined and is of the order $n \times k$.

Matrices *PY* and *WY* can be added only when their orders are the same.

However, *PY* is of the order $p \times k$ and *WY* is of the order $n \times k$. Therefore, we must have $p = n$.

Thus, $k = 3$ and $p = n$ are the restrictions on *n*, *k*, and *p* so that $PY + WY$ will be defined.

Question 22:

Assume *X*, *Y*, *Z*, *W* and *P* are matrices of order $2\times n, 3\times k, 2\times p, n\times 3$, and $p\times k$ respectively. If $n = p$, then the order of the matrix $7X - 5Z$ is

A $p \times 2$ **B** $2 \times n$ **C** $n \times 3$ **D** $p \times n$

Answer

The correct answer is B.

Matrix *X* is of the order $2 \times n$.

Therefore, matrix 7*X* is also of the same order.

Matrix *Z* is of the order $2 \times p$, i.e., $2 \times n$ [Since $n = p$]

Therefore, matrix 5*Z* is also of the same order.

Now, both the matrices 7*X* and 5*Z* are of the order $2 \times n$.

Thus, matrix $7X - 5Z$ is well-defined and is of the order $2 \times n$.

Exercise 3.3

Question 1:

Find the transpose of each of the following matrices:

(i) $\begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ (iii) $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$

Answer

(i) $\text{Let } A = \begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}, \text{ then } A^{T} = \begin{bmatrix} 5 & \frac{1}{2} & -1 \end{bmatrix}$

(ii) $\text{Let } A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}, \text{ then } A^{T} = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

(iii) $\text{Let } A = \begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}, \text{ then } A^{T} = \begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}$

Question 2:

If $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$, then verify that

(i) $(A+B)' = A' + B'$

(ii) $(A-B)' = A' - B'$

Answer

We have:

$$A' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix}, B' = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$$

(i)

$$A+B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix}$$

$$\therefore (A+B)' = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$$

$$A'+B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$$

Hence, we have verified that $(A+B)' = A' + B'$

(ii)

$$A-B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{bmatrix}$$

$$\therefore (A-B)' = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$$

$$A'-B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$$

Hence, we have verified that $(A-B)' = A' - B'$.

Question 3:

If $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then verify that

(i) $(A+B)' = A' + B'$

(ii) $(A-B)' = A' - B'$

Answer

(i) It is known that $A = (A')'$

Therefore, we have:

$$A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$$

$$B' = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 5 & 4 & 4 \end{bmatrix}$$

$$\therefore (A+B)' = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$$

$$A' + B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$$

Thus, we have verified that $(A+B)' = A' + B'$.

(ii)

$$A-B=\begin{bmatrix}3 & -1 & 0\\ 4 & 2 & 1\end{bmatrix}-\begin{bmatrix}-1 & 2 & 1\\ 1 & 2 & 3\end{bmatrix}=\begin{bmatrix}4 & -3 & -1\\ 3 & 0 & -2\end{bmatrix}$$

$$\therefore (A-B)'=\begin{bmatrix}4 & 3\\ -3 & 0\\ -1 & -2\end{bmatrix}$$

$$A'-B'=\begin{bmatrix}3 & 4\\ -1 & 2\\ 0 & 1\end{bmatrix}-\begin{bmatrix}-1 & 1\\ 2 & 2\\ 1 & 3\end{bmatrix}=\begin{bmatrix}4 & 3\\ -3 & 0\\ -1 & -2\end{bmatrix}$$

Thus, we have verified that $(A-B)'=A'-B'$.

Question 4:

If $A'=\begin{bmatrix}-2 & 3\\ 1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 0\\ 1 & 2\end{bmatrix}$, then find $(A+2B)'$

Answer

We know that $A=(A')'$

$$\therefore A=\begin{bmatrix}-2 & 1\\ 3 & 2\end{bmatrix}$$

$$\therefore A+2B=\begin{bmatrix}-2 & 1\\ 3 & 2\end{bmatrix}+2\begin{bmatrix}-1 & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}-2 & 1\\ 3 & 2\end{bmatrix}+\begin{bmatrix}-2 & 0\\ 2 & 4\end{bmatrix}=\begin{bmatrix}-4 & 1\\ 5 & 6\end{bmatrix}$$

$$\therefore (A+2B)'=\begin{bmatrix}-4 & 5\\ 1 & 6\end{bmatrix}$$

Question 5:

For the matrices *A* and *B*, verify that $(AB)' = B'A'$ where

(i) $A=\begin{bmatrix}1\\-4\\3\end{bmatrix},\ B=\begin{bmatrix}-1 & 2 & 1\end{bmatrix}$

(ii) $A=\begin{bmatrix}0\\1\\2\end{bmatrix},\ B=\begin{bmatrix}1 & 5 & 7\end{bmatrix}$

Answer

(i)

$$AB=\begin{bmatrix}1\\-4\\3\end{bmatrix}\begin{bmatrix}-1 & 2 & 1\end{bmatrix}=\begin{bmatrix}-1 & 2 & 1\\4 & -8 & -4\\-3 & 6 & 3\end{bmatrix}$$

$$\therefore (AB)'=\begin{bmatrix}-1 & 4 & -3\\2 & -8 & 6\\1 & -4 & 3\end{bmatrix}$$

Now, $A'=\begin{bmatrix}1 & -4 & 3\end{bmatrix},\ B'=\begin{bmatrix}-1\\2\\1\end{bmatrix}$

$$\therefore B'A'=\begin{bmatrix}-1\\2\\1\end{bmatrix}\begin{bmatrix}1 & -4 & 3\end{bmatrix}=\begin{bmatrix}-1 & 4 & -3\\2 & -8 & 6\\1 & -4 & 3\end{bmatrix}$$

Hence, we have verified that $(AB)'=B'A'$.

(ii)

$$AB = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix}$$

$$\therefore (AB)' = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

Now, $A' = \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}, B' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$

$$\therefore B'A' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

Hence, we have verified that $(AB)' = B'A'$.

Question 6:

If (i) $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then verify that $A'A = I$

(ii) $A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$, then verify that $A'A = I$

Answer

(i)

$$A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$$

$$\therefore A'=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$$

$$A'A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$$
$$=\begin{bmatrix}(\cos\alpha)(\cos\alpha)+(-\sin\alpha)(-\sin\alpha) & (\cos\alpha)(\sin\alpha)+(-\sin\alpha)(\cos\alpha)\\ (\sin\alpha)(\cos\alpha)+(\cos\alpha)(-\sin\alpha) & (\sin\alpha)(\sin\alpha)+(\cos\alpha)(\cos\alpha)\end{bmatrix}$$
$$=\begin{bmatrix}\cos^2\alpha+\sin^2\alpha & \sin\alpha\cos\alpha-\sin\alpha\cos\alpha\\ \sin\alpha\cos\alpha-\sin\alpha\cos\alpha & \sin^2\alpha+\cos^2\alpha\end{bmatrix}$$
$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

Hence, we have verified that $A'A=I$.

(ii)

$$A=\begin{bmatrix}\sin\alpha & \cos\alpha\\ -\cos\alpha & \sin\alpha\end{bmatrix}$$

$$\therefore A'=\begin{bmatrix}\sin\alpha & -\cos\alpha\\ \cos\alpha & \sin\alpha\end{bmatrix}$$

$$A'A=\begin{bmatrix}\sin\alpha & -\cos\alpha\\ \cos\alpha & \sin\alpha\end{bmatrix}\begin{bmatrix}\sin\alpha & \cos\alpha\\ -\cos\alpha & \sin\alpha\end{bmatrix}$$

$$\begin{bmatrix}\sin\alpha & -\cos\alpha\\ \cos\alpha & \sin\alpha\end{bmatrix}\begin{bmatrix}\sin\alpha & \cos\alpha\\ -\cos\alpha & \sin\alpha\end{bmatrix}$$

$$=\begin{bmatrix}(\sin\alpha)(\sin\alpha)+(-\cos\alpha)(-\cos\alpha) & (\sin\alpha)(\cos\alpha)+(-\cos\alpha)(\sin\alpha)\\ (\cos\alpha)(\sin\alpha)+(\sin\alpha)(-\cos\alpha) & (\cos\alpha)(\cos\alpha)+(\sin\alpha)(\sin\alpha)\end{bmatrix}$$

$$=\begin{bmatrix}\sin^2\alpha+\cos^2\alpha & \sin\alpha\cos\alpha-\sin\alpha\cos\alpha\\ \sin\alpha\cos\alpha-\sin\alpha\cos\alpha & \cos^2\alpha+\sin^2\alpha\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

Hence, we have verified that $A'A=I$.

Question 7:

(i) Show that the matrix $A=\begin{bmatrix}1 & -1 & 5\\ -1 & 2 & 1\\ 5 & 1 & 3\end{bmatrix}$ is a symmetric matrix

(ii) Show that the matrix $A=\begin{bmatrix}0 & 1 & -1\\ -1 & 0 & 1\\ 1 & -1 & 0\end{bmatrix}$ is a skew symmetric matrix

Answer

(i) We have:

$$A'=\begin{bmatrix}1 & -1 & 5\\ -1 & 2 & 1\\ 5 & 1 & 3\end{bmatrix}=A$$

$\therefore A'=A$

Hence, *A* is a symmetric matrix.

(ii) We have:

$$A' = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = -A$$

$\therefore A' = -A$

Hence, *A* is a skew-symmetric matrix.

Question 8:

For the matrix $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify that

(i) $(A + A')$ is a symmetric matrix

(ii) $(A - A')$ is a skew symmetric matrix

Answer

$$A' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$$

(i) $A + A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$

$$\therefore (A + A')' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} = A + A'$$

Hence, $(A + A')$ is a symmetric matrix.

(ii) $A - A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$$(A - A')' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = -(A - A')$$

Hence, $(A - A')$ is a skew-symmetric matrix.

Question 9:

Find $\frac{1}{2}(A+A')$ and $\frac{1}{2}(A-A')$, when $A=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}$

Answer

The given matrix is $A=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}$

$$\therefore A'=\begin{bmatrix}0 & -a & -b\\ a & 0 & -c\\ b & c & 0\end{bmatrix}$$

$$A+A'=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}+\begin{bmatrix}0 & -a & -b\\ a & 0 & -c\\ b & c & 0\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$\therefore \frac{1}{2}(A+A')=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$\text{Now, } A-A'=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}-\begin{bmatrix}0 & -a & -b\\ a & 0 & -c\\ b & c & 0\end{bmatrix}=\begin{bmatrix}0 & 2a & 2b\\ -2a & 0 & 2c\\ -2b & -2c & 0\end{bmatrix}$$

$$\therefore \frac{1}{2}(A-A')=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}$$

Question 10:

Express the following matrices as the sum of a symmetric and a skew symmetric matrix:

(i) $\begin{bmatrix}3 & 5\\ 1 & -1\end{bmatrix}$

(ii) $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

(iv) $\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$

Answer

(i)

$$\text{Let } A=\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}, \text{then } A'=\begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$$

$$\text{Now, } A+A'=\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}+\begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}=\begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix}$$

$$\text{Let } P=\frac{1}{2}(A+A')=\frac{1}{2}\begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix}=\begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$$

$$\text{Now, } P'=\begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}=P$$

Thus, $P=\frac{1}{2}(A+A')$ is a symmetric matrix.

$$\text{Now, } A-A'=\begin{bmatrix}3 & 5\\1 & -1\end{bmatrix}-\begin{bmatrix}3 & 1\\5 & -1\end{bmatrix}=\begin{bmatrix}0 & 4\\-4 & 0\end{bmatrix}$$

$$\text{Let } Q=\frac{1}{2}(A-A')=\frac{1}{2}\begin{bmatrix}0 & 4\\-4 & 0\end{bmatrix}=\begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix}$$

$$\text{Now, } Q'=\begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix}=-Q$$

Thus, $Q=\frac{1}{2}(A-A')$ is a skew-symmetric matrix.

Representing *A* as the sum of *P* and *Q*:

$$P+Q=\begin{bmatrix}3 & 3\\3 & -1\end{bmatrix}+\begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix}=\begin{bmatrix}3 & 5\\1 & -1\end{bmatrix}=A$$

(ii)

$$\text{Let } A=\begin{bmatrix}6 & -2 & 2\\-2 & 3 & -1\\2 & -1 & 3\end{bmatrix}, \text{ then } A'=\begin{bmatrix}6 & -2 & 2\\-2 & 3 & -1\\2 & -1 & 3\end{bmatrix}$$

$$\text{Now, } A+A'=\begin{bmatrix}6 & -2 & 2\\-2 & 3 & -1\\2 & -1 & 3\end{bmatrix}+\begin{bmatrix}6 & -2 & 2\\-2 & 3 & -1\\2 & -1 & 3\end{bmatrix}=\begin{bmatrix}12 & -4 & 4\\-4 & 6 & -2\\4 & -2 & 6\end{bmatrix}$$

$$\text{Let } P=\frac{1}{2}(A+A')=\frac{1}{2}\begin{bmatrix}12 & -4 & 4\\-4 & 6 & -2\\4 & -2 & 6\end{bmatrix}=\begin{bmatrix}6 & -2 & 2\\-2 & 3 & -1\\2 & -1 & 3\end{bmatrix}$$

$$\text{Now, } P'=\begin{bmatrix}6 & -2 & 2\\-2 & 3 & -1\\2 & -1 & 3\end{bmatrix}=P$$

Thus, $P=\frac{1}{2}(A+A')$ is a symmetric matrix.

$$\text{Now, } A-A'=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}+\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$\text{Let } Q=\frac{1}{2}\left(A-A'\right)=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$\text{Now, } Q'=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}=-Q$$

Thus, $Q=\frac{1}{2}\left(A-A'\right)$ is a skew-symmetric matrix.
Representing *A* as the sum of *P* and *Q*:

$$P+Q=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}+\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}=A$$

(iii)

$$\text{Let } A=\begin{bmatrix}3 & 3 & -1\\ -2 & -2 & 1\\ -4 & -5 & 2\end{bmatrix},\ \text{then } A'=\begin{bmatrix}3 & -2 & -4\\ 3 & -2 & -5\\ -1 & 1 & 2\end{bmatrix}$$

$$\text{Now, } A+A'=\begin{bmatrix}3 & 3 & -1\\ -2 & -2 & 1\\ -4 & -5 & 2\end{bmatrix}+\begin{bmatrix}3 & -2 & -4\\ 3 & -2 & -5\\ -1 & 1 & 2\end{bmatrix}=\begin{bmatrix}6 & 1 & -5\\ 1 & -4 & -4\\ -5 & -4 & 4\end{bmatrix}$$

$$\text{Let } P=\frac{1}{2}(A+A')=\frac{1}{2}\begin{bmatrix}6 & 1 & -5\\ 1 & -4 & -4\\ -5 & -4 & 4\end{bmatrix}=\begin{bmatrix}3 & \frac{1}{2} & -\frac{5}{2}\\ \frac{1}{2} & -2 & -2\\ -\frac{5}{2} & -2 & 2\end{bmatrix}$$

$$\text{Now, } P'=\begin{bmatrix}3 & \frac{1}{2} & -\frac{5}{2}\\ \frac{1}{2} & -2 & -2\\ -\frac{5}{2} & -2 & 2\end{bmatrix}=P$$

Thus, $P=\frac{1}{2}(A+A')$ is a symmetric matrix.

$$\text{Now, } A-A'=\begin{bmatrix}3 & 3 & -1\\ -2 & -2 & 1\\ -4 & -5 & 2\end{bmatrix}-\begin{bmatrix}3 & -2 & -4\\ 3 & -2 & -5\\ -1 & 1 & 2\end{bmatrix}=\begin{bmatrix}0 & 5 & 3\\ -5 & 0 & 6\\ -3 & -6 & 0\end{bmatrix}$$

$$\text{Let } Q=\frac{1}{2}(A-A')=\frac{1}{2}\begin{bmatrix}0 & 5 & 3\\ -5 & 0 & 6\\ -3 & -6 & 0\end{bmatrix}=\begin{bmatrix}0 & \frac{5}{2} & \frac{3}{2}\\ -\frac{5}{2} & 0 & 3\\ -\frac{3}{2} & -3 & 0\end{bmatrix}$$

$$\text{Now, } Q'=\begin{bmatrix}0 & -\frac{5}{2} & -\frac{3}{2}\\ \frac{5}{2} & 0 & -3\\ \frac{3}{2} & 3 & 0\end{bmatrix}=-Q$$

Thus, $Q=\frac{1}{2}(A-A')$ is a skew-symmetric matrix.

Representing *A* as the sum of *P* and *Q*:

$$P+Q=\begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix}+\begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix}=\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}=A$$

(iv)

$$\text{Let } A=\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}, \text{ then } A'=\begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$$

$$\text{Now } A+A'=\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}+\begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}=\begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix}$$

$$\text{Let } P=\frac{1}{2}(A+A')=\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$$

$$\text{Now, } P'=\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}=P$$

Thus, $P=\frac{1}{2}(A+A')$ is a symmetric matrix.

$$\text{Now, } A-A'=\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}-\begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}=\begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix}$$

$$\text{Let } Q=\frac{1}{2}(A-A')=\begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$$

$$\text{Now, } Q'=\begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}=-Q$$

Thus, $Q=\frac{1}{2}(A-A')$ is a skew-symmetric matrix.

Representing *A* as the sum of *P* and *Q*:

$$P+Q=\begin{bmatrix}1 & 2\\2 & 2\end{bmatrix}+\begin{bmatrix}0 & 3\\-3 & 0\end{bmatrix}=\begin{bmatrix}1 & 5\\-1 & 2\end{bmatrix}=A$$

Question 11:

If *A*, *B* are symmetric matrices of same order, then *AB* − *BA* is a

A. Skew symmetric matrix **B.** Symmetric matrix

C. Zero matrix **D.** Identity matrix

Answer

The correct answer is A.

A and *B* are symmetric matrices, therefore, we have:

$$A' = A \text{ and } B' = B \qquad ...(1)$$

$$\begin{aligned}\text{Consider } (AB-BA)' &= (AB)'-(BA)' && [(A-B)' = A'-B']\\ &= B'A'-A'B' && [(AB)' = B'A']\\ &= BA-AB && [\text{by } (1)]\\ &= -(AB-BA)\end{aligned}$$

$$\therefore (AB-BA)' = -(AB-BA)$$

Thus, (*AB* − *BA*) is a skew-symmetric matrix.

Question 12:

If $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$, then $A+A'=I$, if the value of α is

A. $\frac{\pi}{6}$ **B.** $\frac{\pi}{3}$

C. π **D.** $\frac{3\pi}{2}$

Answer

The correct answer is B.

$$A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$$
$$\Rightarrow A'=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$$

Now, $A+A'=I$

$$\therefore \begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}+\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$
$$\Rightarrow \begin{bmatrix}2\cos\alpha & 0\\ 0 & 2\cos\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Comparing the corresponding elements of the two matrices, we have:

$$2\cos\alpha=1$$
$$\Rightarrow \cos\alpha=\frac{1\pi}{2}=\cos\frac{}{3}$$
$$\therefore \alpha=\frac{\pi}{3}$$

Exercise 3.4

Question 1:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix}1 & -1\\ 2 & 3\end{bmatrix}$$

Answer

Let $A=\begin{bmatrix}1 & -1\\ 2 & 3\end{bmatrix}$

We know that $A = IA$

$$\therefore \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A \qquad \left(R_2 \to R_2 - 2R_1\right)$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A \qquad \left(R_2 \to \frac{1}{5} R_2\right)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A \qquad \left(R_1 \to R_1 + R_2\right)$$

$$\therefore A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

Question 2:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

Answer

$$\text{Let } A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A \qquad \left(R_1 \to R_1 - R_2\right)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A \qquad \left(R_2 \to R_2 - R_1\right)$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

Question 3:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$$

Answer

$$\text{Let } A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A \qquad (R_2 \to R_2 - 2R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} A \qquad (R_1 \to R_1 - 3R_2)$$

$$\therefore A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$$

Question 4:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$$

Answer

$$\text{Let } A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{3}{2} \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A \qquad \left(R_1 \to \frac{1}{2} R_1\right)$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{3}{2} \\ 0 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{5}{2} & 1 \end{bmatrix} A \qquad \left(R_2 \to R_2 - 5R_1\right)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ -\frac{5}{2} & 1 \end{bmatrix} A \qquad \left(R_1 \to R_1 + 3R_2\right)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix} A \qquad \left(R_2 \to -2R_1\right)$$

$$\therefore A^{-1} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$$

Question 5:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$$

Answer

$$\text{Let } A = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{1}{2} \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A \qquad \left(R_1 \to \frac{1}{2} R_1\right)$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{7}{2} & 1 \end{bmatrix} A \qquad \left(R_2 \to R_2 - 7R_1\right)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -\frac{7}{2} & 1 \end{bmatrix} A \qquad \left(R_1 \to R_1 - R_2\right)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} A \qquad \left(R_2 \to 2R_2\right)$$

$$\therefore A^{-1} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$$

Question 6:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

Answer

$$\text{Let } A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{5}{2} \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A \qquad \left(R_1 \to \frac{1}{2}R_1\right)$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{5}{2} \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & 1 \end{bmatrix} A \qquad \left(R_2 \to R_2 - R_1\right)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -\frac{1}{2} & 1 \end{bmatrix} A \qquad \left(R_1 \to R_2 - 5R_2\right)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} A \qquad \left(R_2 \to 2R_2\right)$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

Question 7:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$$

Answer

$$\text{Let } A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$$

We know that $A = AI$

$$\therefore \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} = A\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = A\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \qquad (C_1 \to C_1 - 2C_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = A\begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} \qquad (C_2 \to C_2 - C_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} \qquad (C_1 \to C_1 - C_2)$$

$$\therefore A^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$$

Question 8:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$$

Answer

$$\text{Let } A = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}A$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}A \qquad (R_1 \to R_1 - R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix}A \qquad (R_2 \to R_2 - 3R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}A \qquad (R_1 \to R_1 - R_2)$$

$$\therefore A^{-1} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$$

Question 9:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$$

Answer

$$\text{Let } A = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A \qquad (R_1 \to R_1 - R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} A \qquad (R_2 \to R_2 - 2R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix} A \qquad (R_1 \to R_1 - 3R_2)$$

$$\therefore A^{-1} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$$

Question 10:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$$

Answer

$$\text{Let } A = \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$$

We know that $A = AI$

$$\therefore \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \qquad (C_1 \to C_1 + 2C_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \qquad (C_2 \to C_2 + C_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix} \qquad \left(C_2 \to \frac{1}{2} C_2\right)$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix}$$

Question 11:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$$

Answer

$$\text{Let } A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$$

We know that $A = AI$

$$\therefore \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \qquad (C_2 \to C_2 + 3C_1)$$

$$\Rightarrow \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} -2 & 3 \\ -1 & 1 \end{bmatrix} \qquad (C_1 \to C_1 - C_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix} \qquad \left(C_1 \to \frac{1}{2} C_1\right)$$

$$\therefore A^{-1} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix}$$

Question 12:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$$

Answer

$$\text{Let } A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ 0 & 1 \end{bmatrix} A \qquad \left(R_1 \to \frac{1}{6} R_1\right)$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ \frac{1}{3} & 1 \end{bmatrix} A \qquad \left(R_2 \to R_2 + 2R_1\right)$$

Now, in the above equation, we can see all the zeros in the second row of the matrix on the L.H.S.

Therefore, A^{-1} does not exist.

Question 13:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

Answer

$$\text{Let } A=\begin{bmatrix}2 & -3\\ -1 & 2\end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix}2 & -3\\ -1 & 2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}A$$

$$\Rightarrow \begin{bmatrix}1 & -1\\ -1 & 2\end{bmatrix}=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}A \qquad (R_1 \to R_1 + R_2)$$

$$\Rightarrow \begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 1\\ 1 & 2\end{bmatrix}A \qquad (R_2 \to R_2 + R_1)$$

$$\Rightarrow \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}A \qquad (R_1 \to R_1 + R_2)$$

$$\therefore A^{-1}=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$$

Question 14:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix}2 & 1\\ 4 & 2\end{bmatrix}$$

Answer

$$\text{Let } A=\begin{bmatrix}2 & 1\\ 4 & 2\end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix}2 & 1\\ 4 & 2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}A$$

Applying $R_1 \to R_1 - \frac{1}{2}R_2$, we have:

$$\begin{bmatrix}0 & 0\\ 4 & 2\end{bmatrix}=\begin{bmatrix}1 & -\frac{1}{2}\\ 0 & 1\end{bmatrix}A$$

Now, in the above equation, we can see all the zeros in the first row of the matrix on the L.H.S.

Therefore, A^{-1} does not exist.

Question 16:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$$

Answer

$$\text{Let } A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 + 3R_1$ and $R_3 \to R_3 - 2R_1$, we have:

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 + 3R_3$ and $R_2 \to R_2 + 8R_3$, we have:

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -2 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \to R_3 + R_2$, we have:

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & 0 & 25 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -15 & 1 & 9 \end{bmatrix} A$$

Applying $R_3 \to \frac{1}{25} R_3$, we have:

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

Applying $R_1 \to R_1 - 10R_3$, and $R_2 \to R_2 - 21R_3$, we have:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$$

Question 17:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

Answer

$$\text{Let } A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to \frac{1}{2} R_1$, we have:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 5R_1$, we have:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \to R_3 - R_2$, we have:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ \frac{5}{2} & -1 & 1 \end{bmatrix} A$$

Applying $R_3 \to 2R_3$, we have:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 5 & -2 & 2 \end{bmatrix} A$$

Applying $R_1 \to R_1 + \frac{1}{2}R_3$, and $R_2 \to R_2 - \frac{5}{2}R_3$, we have:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

Question 18:

Matrices *A* and *B* will be inverse of each other only if

A. $AB = BA$

C. $AB = 0$, $BA = I$

B. $AB = BA = 0$

D. $AB = BA = I$

Answer

Answer: D

We know that if A is a square matrix of order m, and if there exists another square matrix B of the same order m, such that $AB = BA = I$, then B is said to be the inverse of A. In this case, it is clear that A is the inverse of B.

Thus, matrices A and B will be inverses of each other only if $AB = BA = I$.

Miscellaneous Solutions

Question 1:

Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, show that $(aI + bA)^n = a^n I + na^{n-1} bA$, where I is the identity matrix of order 2 and $n \in \mathbf{N}$.

Answer

It is given that $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

To show: $\mathrm{P}(n): (aI + bA)^n = a^n I + na^{n-1} bA,\ n \in \mathbf{N}$

We shall prove the result by using the principle of mathematical induction.
For $n = 1$, we have:

$$\mathrm{P}(1): (aI + bA) = aI + ba^0 A = aI + bA$$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

That is,

$$\mathrm{P}(k): (aI + bA)^k = a^k I + ka^{k-1} bA$$

Now, we prove that the result is true for $n = k + 1$.

Consider

$$\begin{aligned} (aI + bA)^{k+1} &= (aI + bA)^k (aI + bA) \\ &= (a^k I + ka^{k-1} bA)(aI + bA) \\ &= a^{k+1} I + ka^k bAI + a^k bIA + ka^{k-1} b^2 A^2 \\ &= a^{k+1} I + (k+1) a^k bA + ka^{k-1} b^2 A^2 \qquad \ldots(1) \end{aligned}$$

$$\text{Now, } A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

From (1), we have:

$$\begin{aligned}(aI+bA)^{k+1} &= a^{k+1}I+(k+1)a^kbA+O\\ &= a^{k+1}I+(k+1)a^kbA\end{aligned}$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have:

$$(aI+bA)^n = a^nI+na^{n-1}bA \text{ where } A=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix},\ n\in\mathbf{N}$$

Question 2:

If $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$, prove that $A^n=\begin{bmatrix}3^{n-1} & 3^{n-1} & 3^{n-1}\\ 3^{n-1} & 3^{n-1} & 3^{n-1}\\ 3^{n-1} & 3^{n-1} & 3^{n-1}\end{bmatrix},\ n\in\mathbf{N}$

Answer

It is given that $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$

$$\text{To show:}\quad \mathrm{P}(n): A^n=\begin{bmatrix}3^{n-1} & 3^{n-1} & 3^{n-1}\\ 3^{n-1} & 3^{n-1} & 3^{n-1}\\ 3^{n-1} & 3^{n-1} & 3^{n-1}\end{bmatrix},\ n\in\mathbf{N}$$

We shall prove the result by using the principle of mathematical induction.

For $n = 1$, we have:

$$P(1):\begin{bmatrix}3^{1-1} & 3^{1-1} & 3^{1-1}\\ 3^{1-1} & 3^{1-1} & 3^{1-1}\\ 3^{1-1} & 3^{1-1} & 3^{1-1}\end{bmatrix}=\begin{bmatrix}3^0 & 3^0 & 3^0\\ 3^0 & 3^0 & 3^0\\ 3^0 & 3^0 & 3^0\end{bmatrix}=\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}=A$$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

That is $P(k):\ A^k=\begin{bmatrix}3^{k-1} & 3^{k-1} & 3^{k-1}\\ 3^{k-1} & 3^{k-1} & 3^{k-1}\\ 3^{k-1} & 3^{k-1} & 3^{k-1}\end{bmatrix}$

Now, we prove that the result is true for $n = k + 1$.

Now, $A^{k+1} = A \cdot A^k$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$$

$$= \begin{bmatrix} 3\cdot 3^{k-1} & 3\cdot 3^{k-1} & 3\cdot 3^{k-1} \\ 3\cdot 3^{k-1} & 3\cdot 3^{k-1} & 3\cdot 3^{k-1} \\ 3\cdot 3^{k-1} & 3\cdot 3^{k-1} & 3\cdot 3^{k-1} \end{bmatrix}$$

$$= \begin{bmatrix} 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \\ 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \\ 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \end{bmatrix}$$

Therefore, the result is true for $n = k + 1$.

Thus by the principle of mathematical induction, we have:

$$A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}, n \in \mathbf{N}$$

Question 3:

If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then prove $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ where n is any positive integer

Answer

It is given that $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

To prove: $\mathrm{P}(n): A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}, n \in \mathbf{N}$

We shall prove the result by using the principle of mathematical induction.

For $n = 1$, we have:

$$P(1):\ A^1=\begin{bmatrix}1+2 & -4\\ 1 & 1-2\end{bmatrix}=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}=A$$

Therefore, the result is true for n = 1.

Let the result be true for n = k.

That is,

$$P(k):\ A^k=\begin{bmatrix}1+2k & -4k\\ k & 1-2k\end{bmatrix},\ n\in\mathbf{N}$$

Now, we prove that the result is true for n = k + 1.

Consider

$$A^{k+1}=A^k\cdot A$$

$$=\begin{bmatrix}1+2k & -4k\\ k & 1-2k\end{bmatrix}\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$$

$$=\begin{bmatrix}3(1+2k)-4k & -4(1+2k)+4k\\ 3k+1-2k & -4k-1(1-2k)\end{bmatrix}$$

$$=\begin{bmatrix}3+6k-4k & -4-8k+4k\\ 3k+1-2k & -4k-1+2k\end{bmatrix}$$

$$=\begin{bmatrix}3+2k & -4-4k\\ 1+k & -1-2k\end{bmatrix}$$

$$=\begin{bmatrix}1+2(k+1) & -4(k+1)\\ 1+k & 1-2(k+1)\end{bmatrix}$$

Therefore, the result is true for n = k + 1.

Thus, by the principle of mathematical induction, we have:

$$A^n=\begin{bmatrix}1+2n & -4n\\ n & 1-2n\end{bmatrix},\ n\in\mathbf{N}$$

Question 4:

If A and B are symmetric matrices, prove that $AB - BA$ is a skew symmetric matrix. Answer

It is given that A and B are symmetric matrices. Therefore, we have:

$A' = A$ and $B' = B$ ...(1)

$$\begin{aligned}
\text{Now, } (AB - BA)' &= (AB)' - (BA)' && [(A-B)' = A' - B'] \\
&= B'A' - A'B' && [(AB)' = B'A'] \\
&= BA - AB && [\text{Using } (1)] \\
&= -(AB - BA)
\end{aligned}$$

$$\therefore (AB - BA)' = -(AB - BA)$$

Thus, (AB − BA) is a skew-symmetric matrix.

Question 5:

Show that the matrix $B'AB$ is symmetric or skew symmetric according as A is symmetric or skew symmetric.

Answer

We suppose that A is a symmetric matrix, then $A' = A$ … (1)

Consider

$$\begin{aligned}
(B'AB)' &= \{B'(AB)\}' \\
&= (AB)'(B')' && [(AB)' = B'A'] \\
&= B'A'(B) && [(B')' = B] \\
&= B'(A'B) \\
&= B'(AB) && [\text{Using } (1)]
\end{aligned}$$

$$\therefore (B'AB)' = B'AB$$

Thus, if A is a symmetric matrix, then $B'AB$ is a symmetric matrix.

Now, we suppose that A is a skew-symmetric matrix.

Then, $A' = -A$

Consider

$$\begin{aligned}(B'AB)' &= [B'(AB)]' = (AB)'(B')' \\ &= (B'A')B = B'(-A)B \\ &= -B'AB\end{aligned}$$

$$\therefore (B'AB)' = -B'AB$$

Thus, if *A* is a skew-symmetric matrix, then $B'AB$ is a skew-symmetric matrix.

Hence, if A is a symmetric or skew-symmetric matrix, then $B'AB$ is a symmetric or skew-symmetric matrix accordingly.

Question 6:

Solve system of linear equations, using matrix method.

$2x - y = -2$

$3x + 4y = 3$

Answer

The given system of equations can be written in the form of *AX* = *B*, where

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 \\ 3 \end{bmatrix}.$$

Now,

$$|A| = 8 + 3 = 11 \neq 0$$

Thus, *A* is non-singular. Therefore, its inverse exists.

Now,

$$A^{-1}=\frac{1}{|A|}adjA=\frac{1}{11}\begin{bmatrix}4 & 1\\ -3 & 2\end{bmatrix}$$

$$\therefore X=A^{-1}B=\frac{1}{11}\begin{bmatrix}4 & 1\\ -3 & 2\end{bmatrix}\begin{bmatrix}-2\\ 3\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{11}\begin{bmatrix}-8+3\\ 6+6\end{bmatrix}=\frac{1}{11}\begin{bmatrix}-5\\ 12\end{bmatrix}=\begin{bmatrix}-\frac{5}{11}\\ \frac{12}{11}\end{bmatrix}$$

Hence, $x=\frac{-5}{11}$ and $y=\frac{12}{11}$.

Question 7:

For what values of x: $\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\ 2 & 0 & 1\\ 1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix}=O$?

Answer 7:

Given that: $\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\ 2 & 0 & 1\\ 1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix}=O$

$$\Rightarrow\begin{bmatrix}1+4+1 & 2+0+0 & 0+2+2\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix}=O$$

$$\Rightarrow\begin{bmatrix}6 & 2 & 4\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix}=O$$

$$\Rightarrow[0+4+4x]=[0]$$

$$\Rightarrow 4+4x=0$$

$$\Rightarrow x=-1$$

Question 8:

If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$, show that $A^2-5A+7I=O$

Answer

It is given that $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$

$$\therefore A^2=A\cdot A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$$
$$=\begin{bmatrix}3(3)+1(-1) & 3(1)+1(2)\\ -1(3)+2(-1) & -1(1)+2(2)\end{bmatrix}$$
$$=\begin{bmatrix}9-1 & 3+2\\ -3-2 & -1+4\end{bmatrix}=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}$$

$$\therefore \text{L.H.S.}=A^2-5A+7I$$
$$=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}-5\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}+7\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$
$$=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}-\begin{bmatrix}15 & 5\\ -5 & 10\end{bmatrix}+\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$$
$$=\begin{bmatrix}-7 & 0\\ 0 & -7\end{bmatrix}+\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$$
$$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$
$$=O=\text{R.H.S.}$$

$$\therefore A^2-5A+7I=O$$

Question 9:

Find x, if $\begin{bmatrix} x & -5 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$

Answer

We have:

$$\begin{bmatrix} x & -5 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} x+0-2 & 0-10+0 & 2x-5-3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} x-2 & -10 & 2x-8 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow \left[x(x-2)-40+2x-8\right] = O$$

$$\Rightarrow \left[x^2-2x-40+2x-8\right] = [0]$$

$$\Rightarrow \left[x^2-48\right] = [0]$$

$$\therefore x^2-48=0$$

$$\Rightarrow x^2 = 48$$

$$\Rightarrow x = \pm 4\sqrt{3}$$

Question 10:

A manufacturer produces three products x, y, z which he sells in two markets. Annual sales are indicated below:

Market	**Products**		
I	10000	2000	18000
II	6000	20000	8000

(a) If unit sale prices of x, y and z are Rs 2.50, Rs 1.50 and Rs 1.00, respectively, find the total revenue in each market with the help of matrix algebra.

(b) If the unit costs of the above three commodities are Rs 2.00, Rs 1.00 and 50 paise respectively. Find the gross profit.

Answer

(a) The unit sale prices of *x*, *y*, and *z* are respectively given as Rs 2.50, Rs 1.50, and Rs 1.00.

Consequently, the total revenue in market **I** can be represented in the form of a matrix as:

$$\begin{bmatrix}10000 & 2000 & 18000\end{bmatrix}\begin{bmatrix}2.50\\1.50\\1.00\end{bmatrix}$$
$$=10000\times 2.50+2000\times 1.50+18000\times 1.00$$
$$=25000+3000+18000$$
$$=46000$$

The total revenue in market **II** can be represented in the form of a matrix as:

$$\begin{bmatrix}6000 & 20000 & 8000\end{bmatrix}\begin{bmatrix}2.50\\1.50\\1.00\end{bmatrix}$$
$$=6000\times 2.50+20000\times 1.50+8000\times 1.00$$
$$=15000+30000+8000$$
$$=53000$$

Therefore, the total revenue in market **I** isRs 46000 and the same in market **II** isRs 53000.

(b) The unit cost prices of *x*, *y*, and *z* are respectively given as Rs 2.00, Rs 1.00, and 50 paise.

Consequently, the total cost prices of all the products in market **I** can be represented in the form of a matrix as:

$$\begin{bmatrix}10000 & 2000 & 18000\end{bmatrix}\begin{bmatrix}2.00\\1.00\\0.50\end{bmatrix}$$
$$=10000\times 2.00+2000\times 1.00+18000\times 0.50$$
$$=20000+2000+9000$$
$$=31000$$

Since the total revenue in market **I** isRs 46000, the gross profit in this marketis (Rs 46000 − Rs 31000) Rs 15000.

The total cost prices of all the products in market **II** can be represented in the form of a matrix as:

$$\begin{bmatrix}6000 & 20000 & 8000\end{bmatrix}\begin{bmatrix}2.00\\1.00\\0.50\end{bmatrix}$$
$$=6000\times 2.00+20000\times 1.00+8000\times 0.50$$
$$=12000+20000+4000$$
$$=\text{Rs } 36000$$

Since the total revenue in market **II** isRs 53000, the gross profit in this market is (Rs 53000 − Rs 36000) Rs 17000.

Question 11:

Find the matrix X so that $X\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$

Answer

It is given that:

$$X\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$$

The matrix given on the R.H.S. of the equation is a 2 × 3 matrix and the one given on the L.H.S. of the equation is a 2 × 3 matrix. Therefore, X has to be a 2 × 2 matrix.

Now, let $X=\begin{bmatrix}a & c\\b & d\end{bmatrix}$

Therefore, we have:

$$\begin{bmatrix}a & c\\b & d\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$$
$$\Rightarrow\begin{bmatrix}a+4c & 2a+5c & 3a+6c\\b+4d & 2b+5d & 3b+6d\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$$

Equating the corresponding elements of the two matrices, we have:

$a+4c=-7, \quad 2a+5c=-8, \quad 3a+6c=-9$
$b+4d=2, \quad 2b+5d=4, \quad 3b+6d=6$

Now, $a+4c=-7 \Rightarrow a=-7-4c$

$$\therefore 2a+5c=-8 \Rightarrow -14-8c+5c=-8$$
$$\Rightarrow -3c=6$$
$$\Rightarrow c=-2$$

$$\therefore a=-7-4(-2)=-7+8=1$$

Now, $b+4d=2 \Rightarrow b=2-4d$

$$\therefore 2b+5d=4 \Rightarrow 4-8d+5d=4$$
$$\Rightarrow -3d=0$$
$$\Rightarrow d=0$$

$$\therefore b=2-4(0)=2$$

Thus, $a = 1$, $b = 2$, $c = -2$, $d = 0$

Hence, the required matrix X is $\begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$.

Question 12:

If A and B are square matrices of the same order such that $AB = BA$, then prove by induction that $AB^n = B^n A$. Further, prove that $(AB)^n = A^n B^n$ for all $n \in \mathbf{N}$

Answer

A and B are square matrices of the same order such that $AB = BA$.

To prove: $\quad P(n): AB^n = B^n A, \ n \in \mathbf{N}$

For $n = 1$, we have:

$P(1): AB = BA \qquad$ [Given]

$\Rightarrow AB^1 = B^1 A$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

$$\mathrm{P}(k): AB^k = B^k A \qquad \text{...(1)}$$

Now, we prove that the result is true for $n = k + 1$.

$$\begin{aligned} AB^{k+1} &= AB^k \cdot B \\ &= \left(B^k A\right) B && \left[\text{By (1)}\right] \\ &= B^k (AB) && [\text{Associative law}] \\ &= B^k (BA) && \left[AB = BA \ (\text{Given})\right] \\ &= \left(B^k B\right) A && [\text{Associative law}] \\ &= B^{k+1} A \end{aligned}$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have $AB^n = B^n A,\ n \in \mathbf{N}.$

Now, we prove that $(AB)^n = A^n B^n$ for all $n \in \mathbf{N}$

For $n = 1$, we have:

$$(AB)^1 = A^1 B^1 = AB$$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

$$(AB)^k = A^k B^k \qquad \text{...(2)}$$

Now, we prove that the result is true for $n = k + 1$.

$$\begin{aligned} (AB)^{k+1} &= (AB)^k \cdot (AB) \\ &= \left(A^k B^k\right) \cdot (AB) && \left[\text{By (2)}\right] \\ &= A^k \left(B^k A\right) B && [\text{Associative law}] \\ &= A^k \left(AB^k\right) B && \left[AB^n = B^n A \text{ for all } n \in \mathbf{N}\right] \\ &= \left(A^k A\right) \cdot \left(B^k B\right) && [\text{Associative law}] \\ &= A^{k+1} B^{k+1} \end{aligned}$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have $(AB)^n = A^n B^n$, for all natural numbers.

Question 13:

Choose the correct answer in the following questions:

If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$ then

A. $1+\alpha^2+\beta\gamma=0$

B. $1-\alpha^2+\beta\gamma=0$

C. $1-\alpha^2-\beta\gamma=0$

D. $1+\alpha^2-\beta\gamma=0$

Answer

Answer: C

$$A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$$

$$\begin{aligned}\therefore A^2 = A \cdot A &= \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \\ &= \begin{bmatrix} \alpha^2+\beta\gamma & \alpha\beta-\alpha\beta \\ \alpha\gamma-\alpha\gamma & \beta\gamma+\alpha^2 \end{bmatrix} \\ &= \begin{bmatrix} \alpha^2+\beta\gamma & 0 \\ 0 & \beta\gamma+\alpha^2 \end{bmatrix}\end{aligned}$$

$$\text{Now, } A^2 = I \Rightarrow \begin{bmatrix} \alpha^2+\beta\gamma & 0 \\ 0 & \beta\gamma+\alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

On comparing the corresponding elements, we have:

$$\alpha^2 + \beta\gamma = 1$$
$$\Rightarrow \alpha^2 + \beta\gamma - 1 = 0$$
$$\Rightarrow 1 - \alpha^2 - \beta\gamma = 0$$

Question 14:

If the matrix *A* is both symmetric and skew symmetric, then

A. *A* is a diagonal matrix

B. *A* is a zero matrix

C. *A* is a square matrix

D. None of these Answer

Answer: B

If *A* is both symmetric and skew-symmetric matrix, then we should have

$$A' = A \text{ and } A' = -A$$
$$\Rightarrow A = -A$$
$$\Rightarrow A + A = O$$
$$\Rightarrow 2A = O$$
$$\Rightarrow A = O$$

Therefore, *A* is a zero matrix.

Question 15:

If *A* is square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to

A. *A* **B.** *I* − *A* **C.** *I* **D.** 3*A*

Answer: C

$$
\begin{aligned}
(I+A)^3-7A &= I^3+A^3+3I^2A+3A^2I-7A \\
&= I+A^3+3A+3A^2-7A \\
&= I+A^2\cdot A+3A+3A-7A \qquad \left[A^2=A\right] \\
&= I+A\cdot A-A \\
&= I+A^2-A \\
&= I+A-A \\
&= I
\end{aligned}
$$

$\therefore (I+A)^3-7A=I$