

MATHEMATICS

PART II

Textbook for Class XII



12080



राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

ISBN 81-7450-629-2 (Part-I)
81-7450-653-5 (Part-II)

First Edition
January 2007 *Magha 1928*

Reprinted
October 2007 *Kartika 1929*
January 2009 *Pausa 1930*
December 2009 *Agrahayana 1931*
January 2012 *Magha 1933*
November 2012 *Kartika 1934*
November 2013 *Kartika 1935*
December 2014 *Pausa 1936*
December 2015 *Pausa 1937*
December 2016 *Pausa 1938*
December 2017 *Pausa 1939*
December 2018 *Agrahayana 1940*
August 2019 *Shravana 1941*

PD 450T BS

© **National Council of Educational
Research and Training, 2007**

₹ **150.00**

*Printed on 80 GSM paper with
NCERT watermark*

Published at the Publication Division
by the Secretary, National Council of
Educational Research and Training, Sri
Aurobindo Marg, New Delhi 110 016
and printed at Prabhat Printing Press,
D-23, Industrial Area, Site-A, Mathura-
281 001 (U.P.)

ALL RIGHTS RESERVED

- No part of this publication may be reproduced, stored in a retrieval system or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise without the prior permission of the publisher.
- This book is sold subject to the condition that it shall not, by way of trade, be lent, re-sold, hired out or otherwise disposed of without the publisher's consent, in any form of binding or cover other than that in which it is published.
- The correct price of this publication is the price printed on this page. Any revised price indicated by a rubber stamp or by a sticker or by any other means is incorrect and should be unacceptable.

**OFFICES OF THE PUBLICATION
DIVISION, NCERT**

NCERT Campus
Sri Aurobindo Marg
New Delhi 110 016 **Phone : 011-26562708**

108, 100 Feet Road
Hosdakere Halli Extension
Banashankari III Stage
Bangaluru 560 085 **Phone : 080-26725740**

Navjivan Trust Building
P.O. Navjivan
Ahmedabad 380 014 **Phone : 079-27541446**

CWC Campus
Opp. Dhankal Bus Stop
Panihati
Kolkata 700 114 **Phone : 033-25530454**

CWC Complex
Maligaon
Guwahati 781 021 **Phone : 0361-2674869**

Publication Team

Head, Publication Division : *M. Siraj Anwar*

Chief Editor : *Shweta Uppal*

Chief Production Officer : *Arun Chitkara*

Chief Business Manager : *Bibash Kumar Das*

Editor : *Bijnan Sutar*

Production Assistant : *Mukesh Gaur*

Cover and Illustration

Arvinder Chawla

Foreword

The National Curriculum Framework, 2005, recommends that children's life at school must be linked to their life outside the school. This principle marks a departure from the legacy of bookish learning which continues to shape our system and causes a gap between the school, home and community. The syllabi and textbooks developed on the basis of NCF signify an attempt to implement this basic idea. They also attempt to discourage rote learning and the maintenance of sharp boundaries between different subject areas. We hope these measures will take us significantly further in the direction of a child-centred system of education outlined in the National Policy on Education (1986).

The success of this effort depends on the steps that school principals and teachers will take to encourage children to reflect on their own learning and to pursue imaginative activities and questions. We must recognise that, given space, time and freedom, children generate new knowledge by engaging with the information passed on to them by adults. Treating the prescribed textbook as the sole basis of examination is one of the key reasons why other resources and sites of learning are ignored. Inculcating creativity and initiative is possible if we perceive and treat children as participants in learning, not as receivers of a fixed body of knowledge.

These aims imply considerable change in school routines and mode of functioning. Flexibility in the daily time-table is as necessary as rigour in implementing the annual calendar so that the required number of teaching days are actually devoted to teaching. The methods used for teaching and evaluation will also determine how effective this textbook proves for making children's life at school a happy experience, rather than a source of stress or boredom. Syllabus designers have tried to address the problem of curricular burden by restructuring and reorienting knowledge at different stages with greater consideration for child psychology and the time available for teaching. The textbook attempts to enhance this endeavour by giving higher priority and space to opportunities for contemplation and wondering, discussion in small groups, and activities requiring hands-on experience.

NCERT appreciates the hard work done by the textbook development committee responsible for this book. We wish to thank the Chairperson of the advisory group in Science and Mathematics, Professor J.V. Narlikar and the Chief Advisor for this book, Professor P.K. Jain for guiding the work of this committee. Several teachers contributed to the development of this textbook; we are grateful to their principals for making this possible. We are indebted to the institutions and organisations which have generously permitted us to draw upon their resources, material and personnel. As an organisation committed to systemic reform and continuous improvement in the quality of its products, NCERT welcomes comments and suggestions which will enable us to undertake further revision and refinement.

New Delhi
20 November 2006

Director
National Council of Educational
Research and Training

© NCERT
not to be republished

Preface

The National Council of Educational Research and Training (NCERT) had constituted 21 Focus Groups on Teaching of various subjects related to School Education, to review the National Curriculum Framework for School Education - 2000 (NCFSE - 2000) in face of new emerging challenges and transformations occurring in the fields of content and pedagogy under the contexts of National and International spectrum of school education. These Focus Groups made general and specific comments in their respective areas. Consequently, based on these reports of Focus Groups, National Curriculum Framework (NCF)-2005 was developed.

NCERT designed the new syllabi and constituted Textbook Development Teams for Classes XI and XII to prepare textbooks in Mathematics under the new guidelines and new syllabi. The textbook for Class XI is already in use, which was brought in 2005.

The first draft of the present book (Class XII) was prepared by the team consisting of NCERT faculty, experts and practicing teachers. The draft was refined by the development team in different meetings. This draft of the book was exposed to a group of practicing teachers teaching Mathematics at higher secondary stage in different parts of the country, in a review workshop organised by the NCERT at Delhi. The teachers made useful comments and suggestions which were incorporated in the draft textbook. The draft textbook was finalised by an editorial board constituted out of the development team. Finally, the Advisory Group in Science and Mathematics and the Monitoring Committee constituted by the HRD Ministry, Government of India have approved the draft of the textbook.

In the fitness of things, let us cite some of the essential features dominating the textbook. These characteristics have reflections in almost all the chapters. The existing textbook contains thirteen main chapters and two appendices. Each chapter contains the followings :

- Introduction: Highlighting the importance of the topic; connection with earlier studied topics; brief mention about the new concepts to be discussed in the chapter.
- Organisation of chapter into sections comprising one or more concepts/subconcepts.
- Motivating and introducing the concepts/subconcepts. Illustrations have been provided wherever possible.

- Proofs/problem solving involving deductive or inductive reasoning, multiplicity of approaches wherever possible have been inducted.
- Geometric viewing / visualisation of concepts have been emphasized whenever needed.
- Applications of mathematical concepts have also been integrated with allied subjects like Science and Social Sciences.
- Adequate and variety of examples/exercises have been given in each section.
- For refocusing and strengthening the understanding and skill of problem solving and applicabilities, miscellaneous types of examples/exercises have been provided involving two or more subconcepts at a time at the end of the chapter. The scope of challenging problems to talented minority have been reflected conducive to the recommendation as reflected in NCF-2005.
- For more motivational purpose, brief historical background of topics have been provided at the end of the chapter and at the beginning of each chapter, relevant quotation and photograph of eminent mathematician who have contributed significantly in the development of the topic undertaken, are also provided.
- Lastly, for direct recapitulation of main concepts, formulas and results, brief summary of the chapter has also been provided.

I am thankful to Professor Krishan Kumar, Director, NCERT who constituted the team and invited me to join this national endeavour for the improvement of Mathematics education. He has provided us with an enlightened perspective and a very conducive environment. This made the task of preparing the book much more enjoyable and rewarding. I express my gratitude to Professor J. V. Narlikar, Chairperson of the Advisory Group in Science and Mathematics, for his specific suggestions and advice towards the improvement of the book from time to time. I, also, thank Professor G. Ravindra, Joint Director, NCERT for his help from time to time.

I express my sincere thanks to Professor Hukum Singh, Chief Coordinator and Head, DESM, Dr. V. P. Singh, Coordinator and Professor, S. K. Singh Gautam who have been helping for the success of this project academically as well as administratively. Also, I would like to place on records my appreciation and thanks to all the members of the team and the teachers who have been associated with this noble cause in one or the other form.

PAWAN K. JAIN
Chief Advisor

Textbook Development Committee

CHAIRPERSON, ADVISORY GROUP IN SCIENCE AND MATHEMATICS

J.V. Narlikar, *Emeritus Professor*, Inter-University Centre for Astronomy and Astrophysics (IUCAA), Ganeshkhind, Pune University, Pune

CHIEF ADVISOR

P.K. Jain, *Professor*, Department of Mathematics, University of Delhi, Delhi

CHIEF COORDINATOR

Hukum Singh, *Professor and Head*, DESM, NCERT, New Delhi

MEMBERS

A.K. Rajput, *Reader*, RIE, Bhopal, M.P.

Arun Pal Singh, *Sr. Lecturer*, Department of Mathematics, Dayal Singh College, University of Delhi, Delhi

B.S.P. Raju, *Professor*, RIE Mysore, Karnataka

C.R. Pradeep, *Assistant Professor*, Department of Mathematics, Indian Institute of Science, Bangalore, Karnataka

D.R. Sharma, *P.G.T.*, Jawahar Navodaya Vidyalaya, Mungeshpur, Delhi

R.P. Maurya, *Reader*, DESM, NCERT, New Delhi

Ram Avtar, *Professor (Retd.) and Consultant*, DESM, NCERT, New Delhi

S.K. Kaushik, *Reader*, Department of Mathematics, Kirori Mal College, University of Delhi, Delhi

S.K.S. Gautam, *Professor*, DESM, NCERT, New Delhi

S.S. Khare, *Pro-Vice-Chancellor*, NEHU, Tura Campus, Meghalaya

Sangeeta Arora, *P.G.T.*, Apeejay School, Saket, New Delhi

Shailja Tewari, *P.G.T.*, Kendriya Vidyalaya, Barkakana, Hazaribagh, Jharkhand

Sunil Bajaj, *Sr. Specialist*, SCERT, Gurgaon, Haryana

Vinayak Bujade, *Lecturer*, Vidarbha Buniyadi Junior College, Sakkardara Chowk, Nagpur, Maharashtra

MEMBER-COORDINATOR

V.P. Singh, *Reader*, DESM, NCERT, New Delhi

THE CONSTITUTION OF INDIA

PREAMBLE

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a ¹**[SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC]** and to secure to all its citizens :

JUSTICE, social, economic and political;

LIBERTY of thought, expression, belief, faith and worship;

EQUALITY of status and of opportunity; and to promote among them all

FRATERNITY assuring the dignity of the individual and the ²[unity and integrity of the Nation];

IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949 do **HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.**

1. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Sovereign Democratic Republic" (w.e.f. 3.1.1977)
2. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Unity of the Nation" (w.e.f. 3.1.1977)

Acknowledgements

The Council gratefully acknowledges the valuable contributions of the following participants of the Textbook Review Workshop: Jagdish Saran, *Professor*, Deptt. of Statistics, University of Delhi; Quddus Khan, *Lecturer*, Shibli National P.G. College, Azamgarh (U.P.); P.K. Tewari, *Assistant Commissioner* (Retd.), Kendriya Vidyalaya Sangathan; S.B. Tripathi, *Lecturer*, R.P.V.V., Surajmal Vihar, Delhi; O.N. Singh, *Reader*, RIE, Bhubaneswar, Orissa; Miss Saroj, *Lecturer*, Govt. Girls Senior Secondary School No.1, Roop Nagar, Delhi; P. Bhaskar Kumar, *P.G.T.*, Jawahar Navodaya Vidyalaya, Lepakshi, Anantapur, (A.P.); Mrs. S. Kalpagam, *P.G.T.*, K.V. NAL Campus, Bangalore; Rahul Sofat, *Lecturer*, Air Force Golden Jubilee Institute, Subroto Park, New Delhi; Vandita Kalra, *Lecturer*, Sarvodaya Kanya Vidyalaya, Vikaspuri, District Centre, New Delhi; Janardan Tripathi, *Lecturer*, Govt. R.H.S.S., Aizawl, Mizoram and Ms. Sushma Jaireth, *Reader*, DWS, NCERT, New Delhi.

The Council acknowledges the efforts of Deepak Kapoor, *Incharge*, Computer Station; Sajjad Haider Ansari, Rakesh Kumar and Nargis Islam, *D.T.P. Operators*; Monika Saxena, *Copy Editor*; and Abhimanu Mohanty, *Proof Reader*.

The contribution of APC-Office, administration of DESM and Publication Department is also duly acknowledged.

**Contents of
MATHEMATICS PART I
For Class XII**

Chapter 1	Relations and Functions	1 - 32
Chapter 2	Inverse Trigonometric Functions	33 - 55
Chapter 3	Matrices	56 - 102
Chapter 4	Determinants	103 - 146
Chapter 5	Continuity and Differentiability	147 - 193
Chapter 6	Application of Derivatives	194 - 246
	Appendix 1: Proofs in Mathematics	247 - 255
	Appendix 2: Mathematical Modelling	256 - 267
	<i>Answers</i>	268 - 286

Contents

PART II

<i>Foreword</i>	<i>iii</i>
Preface	v
7. Integrals	287
7.1 Introduction	288
7.2 Integration as an Inverse Process of Differentiation	288
7.3 Methods of Integration	300
7.4 Integrals of some Particular Functions	307
7.5 Integration by Partial Fractions	316
7.6 Integration by Parts	323
7.7 Definite Integral	331
7.8 Fundamental Theorem of Calculus	334
7.9 Evaluation of Definite Integrals by Substitution	338
7.10 Some Properties of Definite Integrals	341
8. Application of Integrals	359
8.1 Introduction	359
8.2 Area under Simple Curves	359
8.3 Area between Two Curves	366
9. Differential Equations	379
9.1 Introduction	379
9.2 Basic Concepts	379
9.3 General and Particular Solutions of a Differential Equation	383
9.4 Formation of a Differential Equation whose General Solution is given	385
9.5 Methods of Solving First order, First Degree Differential Equations	391
10. Vector Algebra	424
10.1 Introduction	424
10.2 Some Basic Concepts	424
10.3 Types of Vectors	427
10.4 Addition of Vectors	429

10.5	Multiplication of a Vector by a Scalar	432
10.6	Product of Two Vectors	441
11.	Three Dimensional Geometry	463
11.1	Introduction	463
11.2	Direction Cosines and Direction Ratios of a Line	463
11.3	Equation of a Line in Space	468
11.4	Angle between Two Lines	471
11.5	Shortest Distance between Two Lines	473
11.6	Plane	479
11.7	Coplanarity of Two Lines	487
11.8	Angle between Two Planes	488
11.9	Distance of a Point from a Plane	490
11.10	Angle between a Line and a Plane	492
12.	Linear Programming	504
12.1	Introduction	504
12.2	Linear Programming Problem and its Mathematical Formulation	505
12.3	Different Types of Linear Programming Problems	514
13.	Probability	531
13.1	Introduction	531
13.2	Conditional Probability	531
13.3	Multiplication Theorem on Probability	540
13.4	Independent Events	542
13.5	Bayes' Theorem	548
13.6	Random Variables and its Probability Distributions	557
13.7	Bernoulli Trials and Binomial Distribution	572
	Answers	588
	Supplementary Material	613