Time Allowed : 3 Hours **Maximum Marks : 80**

General Instructions:

1. All questions are compulsory.
2. This question paper contains **36** questions.
3. Question **1-20** in **Section A** are (objective) Very Short-Answer Type questions carrying **1** mark each.
4 Question **21-26** in **Section B** are Short-Answer Type questions carrying **2** marks each.
5. Question **27-32** in **Section C** are Long-Answer-I Type questions carrying **4** marks each.
6. Question **33-36** in **Section D** are Long-Answer-II type questions carrying **6** marks each.
7. There is no overall choice. However, internal choice has been provided in **3** questions of **Section A**, **2** questions of **Section B, 2** questions of **Section C** and **2** questions of **Section D**. You have to attempt only one of the alternatives in all such questions.
8. Use of calculator is not permitted. You may ask for logarithmic tables, if required.

SECTION - A

(Q. 1 – Q. 10) Multiple choice type questions. Select the correct option.

1. Let X = {– 1, 0, 1}, Y = {0, 2} and a function $f : X \to Y$ defined by $y = 2x^4$, is
   (a) one-one onto (b) one-one into (c) many-one onto (d) many-one into

2. If $f(x) = \begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix}$, then
   (a) $f(a) = 0$ (b) $f(b) = 0$ (c) $f(0) = 0$ (d) $f(1) = 0$

3. Consider the following statements
   I. To every rectangular matrix $A = [a_{ij}]$ of order n, we can associate a number (real or complex) called determinant of A.
   II. Determinant is a function which associates each square matrix with a unique number (real or complex).
   (a) Only I is true (b) Only II is true
   (c) Both I and II are true (d) Neither I nor II is true

4. The order and degree of the differential equation whose solution is $y = cx + c^2 - 3c^{3/2} + 2$, where c is a parameter, is
   (a) order = 4, degree = 4 (b) order = 4, degree = 1
   (c) order = 1, degree = 4 (d) None of these

5. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are linearly dependent vectors and $|\vec{c}| = \sqrt{3}$, then
   (a) $\alpha = 1, \beta = -1$ (b) $\alpha = 1, \beta = \pm 1$ (c) $\alpha = -1, \beta = \pm 1$ (d) $\alpha = \pm 1, \beta = 1$

6. If $|\vec{a}| = 3, |\vec{b}| = 4$, then a value of λ for which $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{a} - \lambda\vec{b}$ is :
   (a) $\frac{9}{16}$ (b) $\frac{3}{4}$ (c) $\frac{3}{2}$ (d) $\frac{4}{3}$

7. If the directions cosines of a line are k, k, k, then

(a) $k>0$ (b) $0<k<1$ (c) $k=1$ (d) $k=\frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$

8. The projection of the line segment joining the points (–1, 0, 3) and (2, 5, 1) on the line whose direction ratios are (6, 2, 3) is

(a) 6 (b) 7 (c) $\frac{22}{7}$ (d) 3

9. The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5) (15, 15), (0, 20). Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the maximum of Z occurs at both the points (15, 15) and (0, 20) is

(a) $p = q$ (b) $p = 2q$ (c) $q = 2p$ (d) $q = 3p$

10. If two events A and B are such that $P(A') = 0.3$, $P(B) = 0.4$ and $P(A \cap B') = 0.5$, then $P\left(\frac{B}{A \cup B'}\right) =$

(a) 1/4 (b) 1/5 (c) 3/5 (d) 2/5

(Q. 11 – Q. 15) Fill in the blanks.

11. The principal value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right) =$ ____________.

OR

The principal value of $\sin^{-1}\left(\sin\frac{5\pi}{3}\right) =$ ______.

12. If $A^2 - A + I = O$, then the inverse of A = ____________.

13. If $\sec\left(\frac{x-y}{x+y}\right) = a$, then $\frac{dy}{dx} =$ ______.

14. Value of $\int_0^{\pi/2} \frac{\cos x}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^3} dx =$ _______.

OR

Value of $\int \frac{dx}{\sqrt{x(a-x)}} =$ _______.

15. If $P(A \cap B) = 0.15$, $P(B') = 0.10$, then $P(A/B) =$ _______.

(Q. 16 – Q. 20) Answer the following questions.

16. If $A = \begin{bmatrix} \cos 20 & \sin 20° \\ \sin 70 & \cos 70° \end{bmatrix}$ find |A|.

17. Find the order and degree of the differential equation $y = x\frac{dy}{dx} + \sqrt{a^2\left(\frac{dy}{dx}\right)^2 + b^2}$.

OR

Find the order & degree of : $\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$

18. Integrate sin x sin (cos x) w.r.t x.
19. Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$
20. Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$.

SECTION - B

21. Evaluate : $\int \frac{1}{\cos(x-a)\cos(x-b)} dx.$

22. Find X if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X+Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$.

OR

If $A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$, show that $\left(A - A^T\right)$ is a skew-symmetric matrix, where A^T is the transpose of matrix A.

23. Show that cot x is a continuous function in its domain.

24. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, prove that $\vec{a} = \pm 2(\vec{b} \times \vec{c})$.

OR

Let $\vec{a}$ and $\vec{b}$ be two given vectors such that $|\vec{a}| = 2$, $|\vec{b}| = 1$ and $\vec{a}.\vec{b} = 1$. Find the angle between $\vec{a}$ and $\vec{b}$.

25. Find the value of the following $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$
26. A window is in the form of rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimension of the window to admit maximum sunlight through the whole opening.

SECTION - C

27. Let $A = R - \{3\}$ and $B = R - \{1\}$

Let $f : A \to B$ defined as $f(x) = \frac{x-2}{x-3} \forall x \in A$. Then show that f is bijective.

28. Solve the sysem of the following equations

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4\,;\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1$$

$$\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$$

OR

If x, y, z are non-zero real numbers, then find the inverse of matrix $A=\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ and show that $A^{-1}A=I$.

29. Solve the following differential equation: $ye^{x/y}\,dx=(xe^{x/y}+y)dy$.

OR

Solve the following differential equation: $(1+y+x^2y)dx+(x+x^3)\,dy=0$, where $y=0$ when $x=1$.

30. Differentiate : $(\sin x)^x+\sin^{-1}\sqrt{x}$

31. Minimise : $Z=-3x+4y$

Subject to : $x+2y\le 8,\ 3x+2y\le 12,\ x\ge 0,\ y\ge 0$

32. Find the area bounded by the curve $x^2=4y$ and the line $x=4y-2$.

SECTION - D

33. Evaluate : $\int x\cos^{-1}x\,dx$

OR

$$\int_0^{\pi}\log(1+\cos x)\,dx$$

34. Find the Cartesian as well as the vector equation of the plane through the intersection of the planes $\vec{r}.(2\hat{i}+6\hat{j})+12=0$ and $\vec{r}.(3\hat{i}-\hat{j}+4\hat{k})=0$, which is at a unit distance from the origin.

35. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a scooter, a car and a truck are 0.01, 0.03 and 0.15, respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

36. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

OR

Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with its vertex at one of the major axis.