## CBSE

## TERM-1 SAMPLE PAPER

## SOIVFD

## MATHEMATICS <br> (STANDARD)

## General Instructions:

(i) The question paper contains THREE parts $\mathrm{A}, \mathrm{B}$ and C .
(ii) Section $A$ consists of 20 questions of 1 mark each. Any 16 questions are to be attempted.
(iii) Section B consists of 20 questions of 1 mark each. Any 16 questions are to be attempted.
(iv) Section C consists of 10 questions based on two Case Studies. Attempt any 8 questions.
(v) There is no negative marking.

## SECTION - A

16 marks
(Section A consists of 20 questions of 1 mark each. Any 16 questions are to be attempted)

1. The ratio of LCM and HCF of the least composite and the least prime numbers is:
(a) 1: 2
(b) 2: 1
(c) 1: 1
(d) 1: 3

1
Ans. (b) Least composite number is 4 and the least prime number is 2
$\operatorname{LCM}(4,2): \operatorname{HCF}(4,2)=4: 2=2: 1$
Explanation: Least composite number

$$
=4=2^{2}
$$

and, $\quad$ Least prime number $=2$
$\therefore \operatorname{HCF}(4,2)=2$ and $\operatorname{LCM}(4,2)=4$

$$
\therefore \quad \frac{\operatorname{LCM}(4,2)}{\operatorname{HCF}(4,2)}=\frac{4}{2}=\frac{2}{1}
$$

2. The value of $k$ for which the lines $5 x+7 y=$ 3 and $15 x+21 y=k$ coincide is:
(a) 9
(b) 5
(c) 7
(d) 18

1
Ans.
(a) For lines to coincide: $\frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}}=\frac{c_{1}}{c_{2}}$

$$
\text { So, } \frac{5}{15}=\frac{7}{21}=\frac{-3}{=k} \text { i.e., } k=9
$$

Explanation: The lines $5 x+7 y=3$ and $15 x+$ $21 y=k$ will coincide,

$$
\begin{array}{rlrl}
\text { If } & & \frac{a_{1}}{a_{2}} & =\frac{b_{1}}{b_{2}}=\frac{c_{1}}{c_{2}} \\
\text { or, } & \frac{5}{15} & =\frac{7}{21}=\frac{(-3)}{-k} \\
\Rightarrow & \frac{1}{3} & =\frac{3}{k} \\
\Rightarrow & k & =9
\end{array}
$$

3. A girl walks 200 m towards East and then 150 m towards North. The distance of the girl from the starting point is:
(a) 350 m
(b) 250 m
(c) 300 m
(d) 225 m

1
Ans. (b) By Pythagoras theorem
The required distance $=\sqrt{\left(200^{2}+150^{2}\right)}$
$=\sqrt{(40000+22500)}=\sqrt{(62500)}$
$=250 \mathrm{~m}$.
So, the distance of the girl from the starting point is 250 m .

Explanation: Let $A$ be the starting point of the girl, $A B$ be the distance of 200 m covered by her towards East and BC be the distance of 150 m covered by her towards North. So, C is her end point.

$\therefore$ Distance of the girl from the starting point

$$
\begin{aligned}
& =C A \\
& =\sqrt{(\mathrm{AB})^{2}+(\mathrm{BC})^{2}}
\end{aligned}
$$

[Using Pythagoras theorem in $\triangle A B C]$
$=\sqrt{(200)^{2}+(150)^{2}}$
$=\sqrt{40000+22500}$
$=\sqrt{62500}$

$$
=250
$$

4. The lengths of the diagonals of a rhombus are 24 cm and 32 cm , then the length of the altitude of the rhombus is:
(a) 12 cm
(b) 12.8 cm
(c) 19 cm
(d) 19.2 cm 1

Ans. (d) Area of the Rhombus

$$
=\frac{1}{2} d_{1} d_{2}=\frac{1}{2} \times 24 \times 32=384 \mathrm{~cm}^{2}
$$

Using Pythagoras theorem,
Side ${ }^{2}=\left(\frac{1}{2} d_{1}\right)^{2}+\left(\frac{1}{2} d_{2}\right)^{2}$
$=12^{2}+16^{2}=144+256=400$
$\therefore$ Side $=20 \mathrm{~cm}$
Area of the Rhombus $=$ base $\times$ altitude
$384=20 \times$ altitude
So, altitude $=\frac{384}{20}=19.2 \mathrm{~cm}$
Explanation: Let $A B C D$ be the rhombus and DM be its altitude.


Let $A C$ and $B D$ be the diagonals of the rhombus ABCD, bisecting each other perpendicularly at point O .
$\therefore \quad \mathrm{OA}=\frac{1}{2} \mathrm{AC}=\frac{1}{2} \times 24=12 \mathrm{~cm}$
and, $\quad \mathrm{OB}=\frac{1}{2} \mathrm{BD}=\frac{1}{2} \times 32=16 \mathrm{~cm}$
So, in $\triangle A O B$,

$$
\begin{aligned}
\mathrm{AB}^{2} & =\mathrm{OA}^{2}+\mathrm{OB}^{2} \\
{[ } & {[\text { Using Pythagoras theorem }] } \\
& =(12)^{2}+(16)^{2} \\
& =144+256 \\
& =400=(20)^{2} \\
\Rightarrow \quad \mathrm{AB} & =20 \mathrm{~cm}
\end{aligned}
$$

Here, $D M$ is the altitude of rhombus $A B C D$. Then,
area of rhombus ABCD

$$
\begin{aligned}
& =\frac{1}{2} \times \mathrm{AC} \times \mathrm{BD}=\mathrm{AB} \times \mathrm{DM} \\
\Rightarrow \frac{1}{2} \times 24 \times 32 & =20 \times \mathrm{DM} \\
\Rightarrow \quad \mathrm{DM} & =19.2
\end{aligned}
$$

5. Two fair coins are tossed. What is the probability of getting at the most one head?
(a) $\frac{3}{4}$
(b) $\frac{1}{4}$
(c) $\frac{1}{2}$
(d) $\frac{3}{8}$

1

Ans. (a) Possible outcomes are (HH), (HT), (TH), (TT)
Favourable outcomes (at the most one head) are (HT), (TH), (TT).
So, probability of getting at the most one head $=\frac{3}{4}$

Explanation: On tossing two fair coins,
Total possible outcomes

$$
=\{H H, H T, T H, T T\} \text { i.e., } 4
$$

Favourable outcomes (at most one head)

$$
=\{H T, T H, T T\} \text { i.e., } 3
$$

$\therefore \mathrm{P}($ at most one head $)=\frac{3}{4}$
6. $\triangle A B C \sim \triangle P Q R$. If $A M$ and $P N$ are altitudes of $\triangle A B C$ and $\triangle P Q R$ respectively and $A B^{2}: P Q^{2}$ = 4: 9, then AM: $\mathrm{PN}=$
(a) 16: 81
(b) 4: 9
(c) 3: 2
(d) 2: 3

1
Ans. (d) Ratio of altitudes = Ratio of sides for similar triangles
So, $A M$ : $P N=A B: P Q=2: 3$
Explanation: $\because \Delta \mathrm{ABC} \sim \Delta \mathrm{PQR}$


$$
\therefore \quad \frac{A B}{P Q}=\frac{B C}{Q R}=\frac{A C}{P R}=\frac{A M}{P N}
$$

$$
\begin{aligned}
& \Rightarrow \quad \frac{A B}{P Q}= \\
& \Rightarrow \quad \frac{A M}{P N} \\
& \Rightarrow \frac{A M}{P N}=\frac{2}{3} \\
& \\
& {\left[\because \frac{A B^{2}}{P Q^{2}}=\frac{4}{9} \Rightarrow \frac{A B}{P Q}=\frac{2}{3}\right] }
\end{aligned}
$$

7. If $2 \sin ^{2} \beta-\cos ^{2} \beta=2$, then $\beta$ is:
(a) $0^{\circ}$
(b) $90^{\circ}$
(c) $45^{\circ}$
(d) $30^{\circ}$

1
Ans.
(b) $2 \sin ^{2} \beta-\cos ^{2} \beta=2$

Then, $2 \sin ^{2} \beta-\left(1-\sin ^{2} \beta\right)=2$
$3 \sin ^{2} \beta=3$ or $\sin ^{2} \beta=1$ $\beta$ is $90^{\circ}$.
8. Prime factors of the denominator of a rational number with the decimal expansion 44.123 are:
(a) 2, 3
(b) 2, 3, 5
(c) 2,5
(d) 3, 5

1
Ans. (c) Since, it has a terminating decimal expansion, So, prime factors of the denominator will be 2, 5.

Explanation: We know, if the denominator of a rational number is of the form $2^{m} 5^{n}$, where $m, n$ are positive integers, then the rational number is a terminating decimal.
Since, the given rational number is terminating, so prime factors of the denominator must be 2 and 5.
9. The lines $x=a$ and $y=b$, are:
(a) Intersecting
(b) Parallel
(c) Overlapping
(d) None of these 1

Ans. (a) Lines $x=a$ is a line parallel to $y$-axis and $y=b$ is a line parallel to $x$-axis. So they will intersect.

Explanation: The lines $x=a$ and $y=b$ can be drawn on the graph paper as:


Therefore, lines $x=a$ and $y=b$ intersect at the point ( $a, b$ ).
10. The distance of point $A(-5,6)$ from the origin is:
(a) 11 units
(b) 61 units
(c) $\sqrt{11}$ units
(d) $\sqrt{61}$ units 1

Ans. (d) Distance of point $A(-5,6)$ from the origin $(0,0)$ is:

$$
\begin{aligned}
\sqrt{(0+5)^{2}+(0-6)^{2}} & =\sqrt{25+36} \\
& =\sqrt{61} \text { units }
\end{aligned}
$$

11. If $a^{2}=\frac{23}{25}$, then $a$ is:
(a) Rational
(b) Irrational
(c) whole number
(d) integer
1

Ans.
(b) $a^{2}=\frac{23}{25}$, then $a=\frac{\sqrt{23}}{5}$,
which is irrational.
12. If $\operatorname{LCM}(x, 18)=36$ and $\operatorname{HCF}(x, 18)=2$, then $x$ is:
(a) 2
(b) 3
(c) 4
(d) 5

1
Ans. (c) $\mathrm{LCM} \times \mathrm{HCF}=$ Product of two numbers

$$
\begin{aligned}
36 \times 2 & =18 \times x \\
x & =4
\end{aligned}
$$

13. In $\triangle A B C$ right angled at $B$, if tan $A=\sqrt{3}$, then then $\cos A \cos C-\sin A \sin$ C $=$
(a) -1
(b) 0
(c) 1
(d) $\frac{\sqrt{3}}{2}$

1

Ans. (b) $\tan A=\sqrt{3}=\tan 60^{\circ}$, so, $\angle A=60^{\circ}$, Hence, $\angle C=30^{\circ}$.
So, $\cos A \cos C-\sin A \sin C$

$$
=\frac{1}{2} \times \frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2} \times \frac{1}{2}=0
$$

Explanation: We have,

$$
\tan A=\sqrt{3}=\tan 60^{\circ}
$$

$\therefore \quad \angle A=60^{\circ}$
Also, $\quad \angle B=90^{\circ}$
$\therefore \quad \angle C=180^{\circ}-(\angle A+\angle B)$
(By angle sum property of triangle)
$\Rightarrow \quad \angle C=30^{\circ}$
Now, $\cos A \cos C-\sin A \sin C$
$=\cos 30^{\circ} \cos 60^{\circ}-\sin 30^{\circ} \sin 60^{\circ}$
$=\frac{\sqrt{3}}{2} \times \frac{1}{2}-\frac{1}{2} \times \frac{\sqrt{3}}{2}=0$
14. If the angles of $\triangle A B C$ are in ratio 1: $1: 2$, respectively (the largest angle being angle
C), then the value of $\frac{\sec A}{\operatorname{cosec} B}-\frac{\tan A}{\cot B}$ is:
(a) 0
(b) $\frac{1}{2}$
(c) 1
(d) $\frac{\sqrt{3}}{2}$

1

Ans. (a) $1 x+1 x+2 x=180^{\circ}, x=45^{\circ}$ $\angle A, \angle B$ and $\angle C$ are $45^{\circ}, 45^{\circ}$ and $90^{\circ}$ respectively.

$$
\frac{\sec A}{\operatorname{cosec} B}-\frac{\tan A}{\cot B}=\frac{\sec 45^{\circ}}{\operatorname{cosec} 45^{\circ}}-\frac{\tan 45^{\circ}}{\cot 45^{\circ}}
$$

$$
=\frac{\sqrt{2}}{\sqrt{2}}-\frac{1}{1}=1-1=0
$$

Explanation: Given, in $\triangle \mathrm{ABC}$

$$
\angle A: \angle B: \angle C=1: 1: 2
$$

Let, $\angle \mathrm{A}=x, \angle \mathrm{~B}=x$ and $\angle \mathrm{C}=2 x$

We know,

$$
\begin{array}{lc} 
& \angle A+\angle B+\angle C=180^{\circ} \\
& \text { (Angle sum property of triangle) } \\
\therefore & x+x+2 x=180^{\circ} \\
\Rightarrow & 4 x=180^{\circ} \\
\Rightarrow & x=45^{\circ} \\
\therefore \angle A=\angle B=45^{\circ} \text { and } \angle C=2 \times 45^{\circ}=90^{\circ} \\
\text { Now, } \frac{\sec A}{\operatorname{cosec} B}-\frac{\tan A}{\cot B}
\end{array}
$$

$$
\begin{aligned}
& =\frac{\sec 45^{\circ}}{\operatorname{cosec} 45^{\circ}}-\frac{\tan 45^{\circ}}{\cot 45^{\circ}} \\
& =\frac{\sqrt{2}}{\sqrt{2}}-\frac{1}{1} \\
& =1-1=0
\end{aligned}
$$

15. The number of revolutions made by a circular wheel of radius 0.7 m in rolling a distance of 176 m is:
(a) 22
(b) 24
(c) 75
(d) 40

Ans. (d) Number of revolutions

$$
=\frac{\text { total distance }}{\text { circumference }}=\frac{176}{2 \times \frac{22}{7} \times 0.7}=40
$$

Explanation: Total distance covered by wheel $=176 \mathrm{~cm}$

Distance covered in one revolution

$$
\begin{aligned}
& =\text { Circumference of wheel } \\
& =2 \pi r \\
& =2 \times \frac{22}{7} \times 0.1 \\
& =4.4 \mathrm{~m}
\end{aligned}
$$

So, number of revolutions made by the wheel
$=\frac{\text { Total distance covered by the wheel }}{\text { Circumference of wheel }}$
$=\frac{176}{4.4}=40$
16. $\triangle A B C$ is such that $A B=3 \mathrm{~cm}, B C=2 \mathrm{~cm}$, $C A=2.5 \mathrm{~cm}$. If $\triangle A B C \sim \triangle D E F$ and $E F=4 \mathrm{~cm}$, then perimeter of $\triangle D E F$ is:
(a) 7.5 cm
(b) 15 cm
(c) 22.5 cm
(d) 30 cm

1
Ans.
(b) $\frac{\text { perimeter of } \triangle A B C}{\text { perimeter of } \triangle D E F}=\frac{B C}{E F}$

$$
\frac{7.5}{\text { perimeter of } \triangle D E F}=\frac{2}{4} \text {. }
$$

So, perimeter of $\triangle D E F=15 \mathrm{~cm}$
Explanation: $\because \triangle \mathrm{ABC} \sim \triangle \mathrm{DEF}$

$$
\begin{array}{ll}
\therefore & \frac{A B}{D E}=\frac{B C}{E F}=\frac{A C}{D F} \\
\Rightarrow & \frac{3}{D E}=\frac{2}{4}=\frac{2.5}{D F} \\
\Rightarrow & \frac{3}{D E}=\frac{2}{4} \text { and } \frac{2}{4}=\frac{2.5}{D F} \\
\Rightarrow & D E=6 \text { and } D F=5
\end{array}
$$

Now, perimeter of $\triangle \mathrm{DEF}=\mathrm{DE}+\mathrm{EF}+\mathrm{DF}$

$$
\begin{aligned}
& =6+4+5 \\
& =15 \mathrm{~cm}
\end{aligned}
$$

17. In the figure, if $D E \| B C, A D=3 \mathrm{~cm}$, $B D=4 \mathrm{~cm}$ and $B C=14 \mathrm{~cm}$, then $D E$ equals:

(a) 7 cm
(b) 6 cm
(c) 4 cm
(d) 3 cm

1

Ans. (b) Since, $D E \| B C, \triangle A B C \sim \triangle A D E$
(By AA rule of similarity)
So, $\frac{A D}{A B}=\frac{D E}{B C}$ i.e., $\frac{3}{7}=\frac{D E}{14}$
So, $D E=6 \mathrm{~cm}$
Explanation: $\because \mathrm{DE}|\mid \mathrm{BC}$
$\therefore \quad \angle A D E=\angle A B C$ [determinate pair of angles] ....(i)
Now, in $\triangle A D E$ and $\triangle A B C$,

$$
\angle A D E=\angle A B C \quad[\text { Proved in (i) }]
$$

$$
\angle A=\angle A \quad \text { [Common angle] }
$$

$$
\begin{array}{ll}
\therefore & \triangle \mathrm{ADE} \sim \triangle \mathrm{ABC} \\
& {[\mathrm{By} \text { AA similarity axiom] }}
\end{array}
$$

$$
\therefore \quad \frac{A D}{A B}=\frac{D E}{B C}
$$

$[\because$ Corresponding sides of similar triangles are proportional]

$$
\begin{array}{cc}
\Rightarrow & \frac{\mathrm{AD}}{\mathrm{AD}+\mathrm{BD}}=\frac{\mathrm{DE}}{\mathrm{BC}} \\
\Rightarrow & \frac{3}{3+4}=\frac{\mathrm{DE}}{14} \\
\Rightarrow & \mathrm{DE}=6
\end{array}
$$

18. If $4 \tan \beta=3$, then $\frac{4 \sin \beta-3 \cos \beta}{4 \sin \beta+3 \cos \beta}=$
(a) 0
(b) $\frac{1}{3}$
(c) $\frac{2}{3}$
(d) $\frac{3}{4}$

Ans. (a) Dividing both numerator and denominator by $\cos \beta$,

$$
\begin{aligned}
& \Rightarrow \frac{4 \sin \beta-3 \cos \beta}{4 \sin \beta+3 \cos \beta}=\frac{4 \tan \beta-3}{4 \tan \beta+3} \\
& =\frac{3-3}{3+3}=0
\end{aligned}
$$

19. One equation of a pair of dependent linear equations is $-5 x+7 y=2$. The second equation can be:
(a) $10 x+14 y+4=0$
(b) $-10 x-14 y+4=0$
(c) $-10 x+14 y+4=0$
(d) $10 x-14 y=-4$

Ans. (d) $-2(-5 x+7 y=2)$ gives $10 x-14 y=-4$

$$
\text { Now } \frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}}=\frac{c_{1}}{c_{2}}=-2
$$

Explanation: Dependent pair of linear equations has an infinite numbers of solutions
$\because \quad 2 \times(-5 x+7 y=2)$
$\Rightarrow \quad-10 x+14 y=4$
$\Rightarrow \quad 10 x-14 y=-4$
$\therefore$ Option (d) is correct.
20. A letter of English alphabets is chosen at random. What is the probability that it is a letter of the word 'MATHEMATICS'?
(a) $\frac{4}{13}$
(b) $\frac{9}{26}$
(c) $\frac{5}{13}$
(d) $\frac{11}{26}$

Ans. (a) Number of possible outcomes are 26 Favourable outcomes are

$$
\begin{array}{r}
\text { M, A, T, H, E, I, C, S. } \\
\text { Probability }=\frac{8}{26}=\frac{4}{13} .
\end{array}
$$

Explanation: Unique alphabets in the word MATHEMATICS

$$
=\{M, A, T, H, E, I, C, S\}
$$

$\therefore$ Total number of unique alphabets in the word MATHEMATICS $=8$
and total number of English alphabets = 26
$\therefore$ Required probability $=\frac{8}{26}=\frac{4}{13}$

## SECTION - B

## 16 marks

(Section B consists of 20 questions of 1 mark each. Any 16 questions are to be attempted.)
21. If sum of two numbers is 1215 and their HCF is 81 , then the possible number of pairs of such numbers are:
(a) 2
(b) 3
(c) 4
(d) 5

1
Ans. (c) Since, $H C F=81$, two numbers can be taken as $81 x$ and $81 y$,
ATQ, $81 x+81 y=1215$
Or, $\quad x+y=15$
Which gives four co-prime pairs:
1, 14
2, 13
4, 11
7, 8
22. Given below is the graph representing two linear equations by lines $A B$ and $C D$ respectively. What is the area of the triangle formed by these two lines and the line $x=0$ ?

(a) 3 sq. units
(b) 4 sq. units
(c) 6 sq. units
(d) 8 sq. units
1

Ans. (c) Required area is area of triangle $A C D=$ $\frac{1}{2} \times 6 \times 2=6$ sq. units

## Explanation:

$\therefore$ Required area $=$ Area of $\triangle A C D$

$$
\begin{aligned}
& =\frac{1}{2} \times A D \times \text { Distance of } \\
& \quad \text { point C from } y \text {-axis } \\
& =\frac{1}{2} \times(4-(-2)) \times 2 \\
& =\frac{1}{2} \times 6 \times 2=6
\end{aligned}
$$

23. If $\tan \alpha+\cot \alpha=2$, then $\tan ^{20} \alpha+\cot ^{20} \alpha$ $=$
(a) 0
(b) 2
(c) 20
(d) $2^{20}$

1
Ans. (b) $\tan \alpha+\cot \alpha=2$ gives $\alpha=45^{\circ}$.

$$
\begin{aligned}
& \text { So } \tan \alpha=\cot \alpha=1 \\
& \tan ^{20} \alpha+\cot ^{20} \alpha=1^{20}+1^{20}=1+1=2
\end{aligned}
$$

Explanation: We have,

$$
\begin{aligned}
& \Rightarrow \quad \tan \alpha+\cot \alpha=2 \\
& \Rightarrow \quad \tan \alpha+\frac{1}{\tan \alpha}=2 \\
& \Rightarrow \quad \tan ^{2} \alpha-2 \tan \alpha+1=0 \\
& \Rightarrow \quad(\tan \alpha-1)^{2}=0 \\
& \Rightarrow \quad \tan \alpha-1=0 \\
& \Rightarrow \quad \tan \alpha=1 \\
& \text { and, } \quad \cot \alpha=\frac{1}{\tan \alpha}=\frac{1}{1}=1 \\
& \text { So, } \quad \tan ^{20} \alpha+\cot ^{20} \alpha=(\tan \alpha)^{20}+(\cot \alpha)^{20} \\
& =(1)^{20}+(1)^{20} \\
& =1+1=2
\end{aligned}
$$

24. If $217 x+131 y=913,131 x+217 y=827$, then $x+y$ is:
(a) 5
(b) 6
(c) 7
(d) 8

1
Ans. (a) Adding the two given equations we get: $348 x+348 y=1740$.
So $x+y=5$

## Explanation: Given,

$$
\begin{equation*}
217 x+131 y=913 \tag{i}
\end{equation*}
$$

$$
\text { and } \quad 131 x+217 y=827
$$

Adding equations (i) and (ii), we get,

$$
\begin{array}{rlrl} 
& 217 x+131 y & =913 \\
131 x+217 y & =827 \\
+\quad \\
& \Rightarrow \quad 348 x+348 y & =1740 \\
\Rightarrow \quad 348(x+y) & =1740 \\
\Rightarrow \quad x+y & =\frac{1740}{348}=5
\end{array}
$$

25. The LCM of two prime numbers $p$ and $q(p>q)$ is 221 . Find the value of $3 p-q$.
(a) 4
(b) 28
(c) 38
(d) 48

Ans. (c) LCM of two prime numbers = product of the numbers

$$
\begin{aligned}
221 & =13 \times 17 \\
\text { So } p & =17 \text { and } q=13 \\
3 p-q & =51-13=38
\end{aligned}
$$

Explanation: The numbers $p$ and $q$ are prime numbers,
$\therefore \operatorname{HCF}(p, q)=1$
$\operatorname{Here}, \operatorname{LCM}(p, q)=221$
As, $p>q$
$\therefore p=17, q=13($ As $p \times q=221)$
Now, $3 p-q=3 \times 17-13$

$$
\begin{gathered}
=51-13 \\
=38
\end{gathered}
$$

26. A card is drawn from a well shuffled deck of cards. What is the probability that the card drawn is neither a king nor a queen?
(a) $\frac{11}{13}$
(b) $\frac{12}{13}$
(c) $\frac{11}{26}$
(d) $\frac{11}{52}$

Ans. (a) Probability that the card drawn is neither a king nor a queen

$$
=\frac{52-8}{52}=\frac{44}{52}=\frac{11}{13}
$$

Explanation: Total number of cords in a deck $=52$
Number of kings and queens $=4+4=8$
$\therefore \quad P($ neither king nor queen $)=\frac{52-8}{52}$

$$
=\frac{44}{52}=\frac{11}{13}
$$

27. Two fair dice are rolled simultaneously. The probability that 5 will come up at least once is:
(a) $\frac{5}{36}$
(b) $\frac{11}{36}$
(c) $\frac{12}{36}$
(d) $\frac{23}{36}$
1

Ans. (b) Outcomes when 5 will come up at least once are:
$(1,5),(2,5),(3,5),(4,5),(5,5),(6,5),(5,1)$, $(5,2),(5,3),(5,4)$ and $(5,6)$
Probability that 5 will come up at leastonce $=\frac{11}{36}$.

Explanation: On rolling two dice simultaneously, total number of outcomes $=36$
Outcomes in which 5 come up at least once $=\{(1,5)(2,5),(3,5),(4,5),(5,1)$, $(5,2),(5,3),(5,4),(5,5),(5,6),(6,5)\}$

$$
\Rightarrow \quad \text { Favourable outcomes }=11
$$

$\therefore \mathrm{P}(5$ will come up atleast once $)=\frac{11}{36}$
28. If $1+\sin ^{2} \alpha=3 \sin \alpha \cos \alpha$, then values of cot $\alpha$ are:
(a) $-1,1$
(b) 0,1
(c) 1,2
(d) $-1,-1$

1
Ans.
(c) $1+\sin ^{2} \alpha=3 \sin \alpha \cos \alpha$
$\sin ^{2} \alpha+\cos ^{2} \alpha+\sin ^{2} \alpha=3 \sin \alpha \cos \alpha$
$2 \sin ^{2} \alpha-3 \sin \alpha \cos \alpha+\cos ^{2} \alpha=0$
$(2 \sin \alpha-\cos \alpha)(\sin \alpha-\cos \alpha)=0$
$\therefore \cot \alpha=2$ or $\cot \alpha=1$
29. The vertices of a parallelogram in order are $A(1,2), B(4, y), C(x, 6)$ and $D(3,5)$. Then $(x, y)$ is:
(a) $(6,3)$
(b) $(3,6)$
(c) $(5,6)$
(d) $(1,4)$
1

Ans. (a) Since, $A B C D$ is a parallelogram, diagonals $A C$ and $B D$ bisect each other. $\therefore$ mid-point of $A C=$ mid point of $B D$.
$\left(\frac{x+1}{2}, \frac{6+2}{2}\right)=\left(\frac{3+4}{2}, \frac{5+y}{2}\right)$
Comparing the co-ordinates, we get
$\frac{x+1}{2}=\frac{3+4}{2}$. So, $x=6$
Similarly, $\frac{6+2}{2}=\frac{5+y}{2}$. So $y=3$
$\therefore(x, y)=(6,3)$

Explanation: We know, diagonals of a parallelogram bisect each other.


$$
\begin{array}{cc} 
& =\text { Mid-point of BD } \\
\Rightarrow & \left(\frac{1+x}{2}, \frac{2+6}{2}\right)=\left(\frac{4+3}{2}, \frac{y+5}{2}\right) \\
\Rightarrow & \frac{1+x}{2}=\frac{4+3}{2} ; \frac{2+6}{2}=\frac{y+5}{2} \\
\Rightarrow & x+1=7 ; 8=y+5 \\
\Rightarrow & x=6 ; y=3 \\
\therefore & (x, y)=(6,3)
\end{array}
$$

30. In the given figure, $\angle A C B=\angle C D A$, $A C=8 \mathrm{~cm}, A D=3 \mathrm{~cm}$, then $B D$ is:

(a) $\frac{22}{3} \mathrm{~cm}$
(b) $\frac{26}{3} \mathrm{~cm}$
(c) $\frac{55}{3} \mathrm{~cm}$
(d) $\frac{64}{3} \mathrm{~cm} 1$

Ans. (c) $\triangle A C D \sim \triangle A B C(A A)$

$$
\therefore \frac{A C}{A B}=\frac{A D}{A C}(C P S T)
$$

$$
\frac{8}{A B}=\frac{3}{8}
$$

This gives $A B=\frac{64}{3} \mathrm{~cm}$
So, $B D=A B-A D=\frac{64}{3}-3=\frac{55}{3} \mathrm{~cm}$
Explanation: In $\triangle A C D$ and $\triangle A B C$,

$$
\begin{aligned}
\angle \mathrm{ADC} & =\angle \mathrm{ACB} \quad \text { (Given) } \\
\angle \mathrm{A} & =\angle \mathrm{A} \quad \text { (Common angle) }
\end{aligned}
$$

$\therefore$ By AA similarity axiom

$$
\begin{array}{ll} 
& \Delta \mathrm{ACD} \sim \triangle \mathrm{ABC} \\
\therefore & \frac{\mathrm{AC}}{\mathrm{AB}}=\frac{A D}{\mathrm{AC}}=\frac{\mathrm{CD}}{\mathrm{BC}} \\
\Rightarrow & \frac{\mathrm{AC}}{\mathrm{AB}}=\frac{\mathrm{AD}}{\mathrm{AC}} \\
\Rightarrow & \frac{8}{\mathrm{AD}+\mathrm{BD}}=\frac{3}{8}
\end{array}
$$

$\Rightarrow \quad \frac{8}{3+B D}=\frac{3}{8}$
$\Rightarrow \quad 64=9+3 \mathrm{BD}$
$\Rightarrow \quad 3 B D=55$
$\Rightarrow \quad \mathrm{BD}=\frac{55}{3}$
31. The equation of the perpendicular bisector of line segment joining points $A(4,5)$ and $B(-2,3)$ is:
(a) $2 x-y+7=0$
(b) $3 x+2 y-7=0$
(c) $3 x-y-7=0$
(d) $3 x+y-7=0 \quad 1$

Ans. (d) Any point $(x, y)$ of perpendicular bisector will be equidistant from $A \& B$.

$$
\begin{aligned}
& \therefore \sqrt{(x-4)^{2}+(y-5)^{2}} \\
&=\sqrt{(x+2)^{2}+(y-3)^{2}}
\end{aligned}
$$

Solving we get,
$-12 x-4 y+28=0$ or $3 x+y-7=0$
Explanation: Let $P(x, y)$ be any point on the perpendicular bisector of $A B$. Then,

$$
P A=P B
$$

$\Rightarrow \sqrt{(x-4)^{2}+(y-5)^{2}}$

$$
=\sqrt{(x+2)^{2}+(y-3)^{2}}
$$

(Using distance formula)
$\Rightarrow(x-4)^{2}+(y-5)^{2}=(x+2)^{2}+(y-3)^{2}$
(Squaring both sides)

$$
\begin{array}{cc} 
& x^{2}-8 x+16 y+y^{2}-10 y+25 \\
& =x^{2}+4 x+4+y^{2}-6 y+9 \\
\Rightarrow & -8 x-10 y+41=4 x-6 y+13 \\
\Rightarrow & -12 x-4 y+28=0 \\
\Rightarrow &
\end{array} \quad 3 x+y-7=0
$$

[Dividing both sides by (-4)]
32. In the given figure, $D$ is the mid-point of $B C$, then the value of $\frac{\cot y^{\circ}}{\cot x^{\circ}}$ is:

(a) 2
(b) $\frac{1}{2}$
(c) $\frac{1}{3}$
(d) $\frac{1}{4}$

1
Ans.
(b) $\frac{\cot y^{\circ}}{\cot x^{\circ}}=\frac{A C / B C}{A C / C D}=\frac{C D}{B C}=\frac{C D}{2 C D}=\frac{1}{2}$

Explanation: In $\triangle A D C$,

$$
\begin{equation*}
\cot x^{\circ}=\frac{A C}{C D} \tag{i}
\end{equation*}
$$

and, in $A B C$,

$$
\begin{equation*}
\cot y^{\circ}=\frac{\mathrm{AC}}{\mathrm{BC}}=\frac{\mathrm{AC}}{2 \mathrm{CD}} \tag{ii}
\end{equation*}
$$

$[\because D$ is mid-point of $B C]$
So, $\quad \frac{\cot y^{\circ}}{\cot x^{\circ}}=\frac{\frac{A C}{2 C D}}{\frac{A C}{C D}}$

$$
=\frac{1}{2}
$$

33. The smallest number by which $\frac{1}{13}$ should be multiplied so that its decimal expansion terminates after two decimal places is:
(a) $\frac{13}{100}$
(b) $\frac{13}{10}$
(c) $\frac{10}{13}$
(d) $\frac{100}{13}$

1
Ans.
(a) The smallest number by which $\frac{1}{13}$ should be multiplied so that its decimal expansion terminates after two decimal points is

$$
\begin{aligned}
& \frac{13}{100} \text { as: } \\
& \frac{1}{13} \times \frac{13}{100}=\frac{1}{100}=0.01
\end{aligned}
$$

Explanation: $\because \frac{1}{13} \times \frac{13}{100}=\frac{1}{100}=0.01$ i.e., decimal expansion terminates after two decimal places
$\therefore \quad$ Smallest required number $=\frac{13}{100}$
34. Sides $A B$ and $B E$ of a right triangle, right angled at $B$ are of lengths 16 cm and 8 cm respectively. The length of the side of largest square FDGB that can be inscribed in the triangle ABE is:

(a) $\frac{32}{3} \mathrm{~cm}$
(b) $\frac{16}{3} \mathrm{~cm}$
(c) $\frac{8}{3} \mathrm{~cm}$
(d) $\frac{4}{3} \mathrm{~cm}$

Ans. (b) $\triangle A B E$ is a right triangle \& $F D G B$ is a square of side $\times \mathrm{cm}$


$$
\begin{aligned}
\triangle A F D \sim & \sim D G E(A A) \\
\therefore \quad \frac{A F}{D G} & =\frac{F D}{G E}(C P S T) \\
\frac{16-x}{x} & =\frac{x}{8-x}(C P S T) \\
128 & =24 x \text { or } x=\frac{16}{3} \mathrm{~cm} .
\end{aligned}
$$

Explanation: In $\triangle \mathrm{AFD}$ and $\triangle \mathrm{ABE}$

$$
\begin{aligned}
\angle \mathrm{AFD} & =\angle \mathrm{ABE}=90^{\circ} \\
\angle \mathrm{A} & =\angle \mathrm{A} \text { [Common angle] }
\end{aligned}
$$

$\therefore B y A A$ similarity axiom,

$$
\begin{aligned}
& \triangle \mathrm{AFD} \sim \triangle \mathrm{ABE} \\
& \therefore \quad \frac{A F}{A B}=\frac{F D}{B E} \\
& \Rightarrow \quad \frac{A B-B F}{A B}=\frac{B F}{B E} \\
& {[\because B F=F D=\text { Sides of square } F D G B]} \\
& \Rightarrow \quad \frac{16-B F}{16}=\frac{B F}{8} \\
& \Rightarrow \quad 128-8 B F=16 B F \\
& \Rightarrow \quad 24 \mathrm{BF}=128 \\
& \Rightarrow \quad \mathrm{BF}=\frac{128}{24}=\frac{16}{3}
\end{aligned}
$$

$\therefore$ Length of side of square FDGB

$$
=\frac{16}{3} \mathrm{~cm}
$$

35. Point $P$ divides the line segment joining $R(-1,3)$ and $S(9,8)$ in ratio $k$ : 1 . If $P$ lies on the line $x-y+2=0$, then value of $k$ is:
(a) $\frac{2}{3}$
(b) $\frac{1}{2}$
(c) $\frac{1}{3}$
(d) $\frac{1}{4}$

1

Ans. (a) Since $P$ divides the line segment joining $R(-1,3)$ and $S(9,8)$ in ratio k: 1
$\therefore$ Coordinates of $P$ are $\left(\frac{9 k-1}{k+1}, \frac{8 k+3}{k+1}\right)$
Since $P$ lies on the line $x-y+2$
$=0$,
then $\frac{9 k-1}{k+1}-\frac{8 k+3}{k+1}+2=0$
$9 k-1-8 k-3+2 k+2=0$
which gives $k=\frac{2}{3}$.

Explanation: Using section formula,

$$
\begin{aligned}
& \text { Coordinates of } \mathrm{P}=\left(\frac{k \times 9+1 \times(-1)}{k+1},\right. \\
&=\left(\frac{9 k-1}{k+1}, \frac{8 k+3+1 \times 3}{k+1}\right) \\
& \mathrm{R}(-1,3)
\end{aligned}
$$

Since, point $P$ lies on the line $x-y+2=0$, so, it must satisfy the equation,

$$
\begin{array}{rr}
\therefore & \frac{9 k-1}{k+1}-\frac{8 k+3}{k+1}+2=0 \\
\Rightarrow & \frac{9 k-1-(8 k+3)+2(k+1)}{k+1}=0 \\
\Rightarrow & \frac{3 k-2}{k+1}=0 \\
\Rightarrow & 3 k-2=0 \\
\Rightarrow & k=\frac{2}{3}
\end{array}
$$

36. In the figure given below, $A B C D$ is a square of side 14 cm with $\mathrm{E}, \mathrm{F}, \mathrm{G}$ and H as the mid points of sides $A B, B C, C D$ and DA respectively. The area of the shaded portion is:

(a) $44 \mathrm{~cm}^{2}$
(b) $49 \mathrm{~cm}^{2}$
(c) $98 \mathrm{~cm}^{2}$
(d) $\frac{49 \pi}{2} \mathrm{~cm}^{2}$

1
Ans. (c) Shaded area $=$ Area of semicircle + (Area of half square - Area of two quadrants)

$=$ Area of semicircle + (Area of half square - Area of semicircle)
= Area of half square
$=\frac{1}{2} \times 14 \times 14=98 \mathrm{~cm}^{2}$.

Explanation: Here, $\mathrm{HF}=\mathrm{DC}=$ side of square $=14 \mathrm{~cm}$ and $\mathrm{DH}=\mathrm{FC}=\mathrm{DG}=\mathrm{GC}=r=$ $\frac{1}{2}$ (side of square) $=7 \mathrm{~cm}$


Now, Area of shaded region

$$
=\text { Area of semi-circle with diameter }
$$

HF + (Area of rectangle DHFC - 2
$\times$ Area of quadrant of radius $D G$ )

$$
\begin{aligned}
& =\frac{1}{2} \pi r^{2}+D C \times D H-2 \times \frac{1}{4} \pi r^{2} \\
& =D C \times D H=14 \times 7=98 \mathrm{~cm}^{2}
\end{aligned}
$$

37. Given below is the picture of the Olympic rings made by taking five congruent circles of radius 1 cm each, intersecting in such a way that the chord formed by joining the point of intersection of two circles is also of length 1 cm . Total area of all the dotted regions assuming the thickness of the rings to be negligible is:

(a) $\frac{4 \pi}{12}-\sqrt{\frac{3}{4}} \mathrm{~cm}^{2}$
(b) $\frac{\pi}{6}-\sqrt{\frac{3}{4}} \mathrm{~cm}^{2}$
(c) $\frac{4 \pi}{6}-\sqrt{\frac{3}{4}} \mathrm{~cm}^{2}$
(d) $\frac{8 \pi}{6}-\sqrt{\frac{3}{4}} \mathrm{~cm}^{2} 1$

Ans. (d)


Let $O$ the, centre of the circle.
$O A=O B=A B=1 \mathrm{~cm}$.
So, $\triangle O A B$ is a equilateral triangle and
$\therefore \angle A O B=60^{\circ}$

Required area $=8 \times$ Area of one segment with $r=1 \mathrm{~cm}, \theta=60^{\circ}$

$$
\begin{aligned}
& =8 \times\left(\frac{60}{360} \times \pi \times 1^{2}-\frac{\sqrt{3}}{4} \times 1^{2}\right) \\
& =8\left(\frac{\pi}{6}-\frac{\sqrt{3}}{4}\right) \mathrm{cm}^{2}
\end{aligned}
$$

Explanation: Consider the first ring with centre $O$ and chord $A B$.

$\therefore \quad \mathrm{OA}=\mathrm{OB}=$ Radii $=1 \mathrm{~cm}$
and

$$
\mathrm{AB}=1 \mathrm{~cm}
$$

(Given),
$\Rightarrow \quad \mathrm{OA}=\mathrm{OB}=\mathrm{AB}=1 \mathrm{~cm}$
$\Rightarrow \triangle \mathrm{AOB}$ is an equilateral triangle
$\therefore \quad \angle A O B=60^{\circ}$
$\therefore$ Area of shaded region (A)
= Area of sector OAB - Area of $\triangle A O B$
So, area of region $\mathrm{AMBN}=2 \times \mathrm{A}$


So, area of dotted region

$$
\begin{aligned}
& =4 \times \text { Area of region AMBN } \\
& =4 \times(2 \mathrm{~A})=8 \mathrm{~A} \\
& =8\left(\frac{\theta}{360} \pi r^{2}-\frac{\sqrt{3}}{4} a^{2}\right) \\
= & 8\left[\frac{60^{\circ}}{360^{\circ}} \times \pi \times(1)^{2}-\frac{\sqrt{3}}{4} \times(1)^{2}\right] \\
& =8\left[\frac{\pi}{6}-\frac{\sqrt{3}}{4}\right] \mathrm{cm}^{2}
\end{aligned}
$$

38. If 2 and $\frac{1}{2}$ are the zeros of $p x^{2}+5 x+r$, then:
(a) $p=r=2$
(b) $p=r=-2$
(c) $p=2, r=-2$
(d) $p=-2, r=2 \quad 1$

Ans. (b) Sum of zeroes $=2+\frac{1}{2}=-\frac{5}{p}$
i.e., $\frac{5}{2}=-\frac{5}{p}$. So, $p=-2$

Product of zeroes $=2 \times \frac{1}{2}=\frac{r}{p}$
i.e., $\frac{r}{p}=1$ or $r=p=-2$

Explanation: Since 2 and $\frac{1}{2}$ are zeroes of polynomial $p x^{2}+5 x+r$
$\therefore \quad$ Sum of zeroes $=-\frac{\text { Coefficient of } x}{\text { Coefficient of } x^{2}}$

$$
\begin{align*}
\Rightarrow & 2+\frac{1}{2} & =-\frac{5}{p} \\
\Rightarrow & \frac{5}{2} & =-\frac{5}{p} \\
\Rightarrow & p & =-2 \tag{i}
\end{align*}
$$

Also,
Product of zeroes $=\frac{\text { Constant term }}{\text { Coefficient of } x^{2}}$

$$
\begin{aligned}
\Rightarrow & 2 \times \frac{1}{2} & =\frac{r}{p} \\
\Rightarrow & \frac{r}{p} & =1 \\
\Rightarrow & r & =p=-2
\end{aligned}
$$

[From (i)]
39. The circumference of a circle is 100 cm . The side of a square inscribed in the circle is:
(a) $50 \sqrt{2} \mathrm{~cm}$
(b) $100 / \pi \mathrm{cm}$
(c) $\frac{50 \sqrt{2}}{\pi} \mathrm{~cm}$
(d) $\frac{100 \sqrt{2}}{\pi} \mathrm{~cm} \quad 1$

Ans. (c) $2 \pi r=100$. So diameter $=2 r=\frac{100}{\pi}=$ diagonal of the square.
Side $\sqrt{2}=$ diagonal of square $=\frac{100}{\pi}$
$\therefore$ Side $=\frac{100}{\sqrt{2} \pi}=\frac{50 \sqrt{2}}{\pi} \mathrm{~cm}$
Explanation: Let $r$ be the radius of circle.
Then, circumference of circle $=2 \pi r$

$$
\Rightarrow \quad \begin{align*}
& \quad 100=2 \pi r \\
& \Rightarrow \quad r=\frac{100}{2 \pi}  \tag{i}\\
& \hline \text { OU }
\end{align*}
$$

Let $A B C D$ be the square inscribed in the circle of radius $r$. Then,

Diagonal of square $=$ Diameter of circle

$$
=2 r=\frac{100}{\pi}[\text { From (i) }]
$$

Now, in $\triangle A D C, \angle A D C=90^{\circ}$

$$
\therefore \quad A D^{2}+D C^{2}=A C^{2}
$$

(By Pythagoras theorem)

$$
\begin{array}{cc}
\Rightarrow & 2 A D^{2}=\left(\frac{100}{\pi}\right)^{2} \\
& {[\because A D=D C=\text { side of square }]} \\
\Rightarrow & A D^{2}=\frac{1}{2}\left(\frac{100}{\pi}\right)^{2}
\end{array}
$$

$$
\begin{aligned}
\Rightarrow \quad \mathrm{AD} & =\frac{1}{\sqrt{2}} \frac{100}{\pi} \\
& =\frac{1}{\sqrt{2}} \frac{100}{\pi} \times \frac{\sqrt{2}}{\sqrt{2}} \\
& =\frac{100 \sqrt{2}}{2 \pi}=\frac{50 \sqrt{2}}{\pi}
\end{aligned}
$$

40. The number of solutions of $3^{x+y}=243$ and $243^{x-y}=3$ is:
(a) 0
(b) 1
(c) 2
(d) infinite
1

Ans.

$$
\text { (b) } \begin{align*}
3^{x+y}=243 & =3^{5} \\
\text { So, } \quad x+y=5 & \cdots(1) \\
243^{x-y} & =3 \\
\left(3^{5}\right)^{x-y} & =31 \tag{2}
\end{align*}
$$

So, $5 x-5 y=1$

$$
\text { Since: } \frac{a_{1}}{a_{2}} \neq \frac{b_{1}}{b_{2}} \text {, so unique solution }
$$

Explanation: We have,

$$
\begin{array}{ll}
\Rightarrow & 3^{x+y}=243 ; 243^{x-y}=3 \\
\Rightarrow & 3^{x+y}=3^{5} ;\left(3^{5}\right)^{x-y}=3^{1} \\
\Rightarrow & 3^{x+y}=3^{5} ; 3^{5(x-y)}=3^{1} \\
\Rightarrow & x+y=5 ; 5(x-y)=1
\end{array}
$$

[ $\because$ Base is same, so powers will also be same]

Now, $\frac{a_{1}}{a_{2}}=\frac{1}{5} ; \frac{b_{1}}{b_{2}}=\frac{1}{-5} ; \frac{c_{1}}{c_{2}}=\frac{5}{1}$
i.e., $\quad \frac{a_{1}}{a_{2}} \neq \frac{b_{1}}{b_{2}} \neq \frac{c_{1}}{c_{2}}$
$\therefore$ Two equation of lines represent intersecting lines, which have a unique solution, i.e., number of solutions is one.

## SECTION - C

## Case Study Based Questions

(Section C consists of 10 questions of 1 mark each. Any 8 questions are to be attempted.)

## Q.41-45 are based on Case Study-1 Case Study-1 :



The figure given alongside shows the path of a diver, when she takes a jump from the diving board. Clearly it is a parabola.
Annie was standing on a diving board, 48 feet above the water level. She took a dive into the pool. Her height (in feet) above the water level at any time ' $t$ ' in seconds is given by the polynomial $h(t)$ such that

$$
h(t)=-16 t^{2}+8 t+k
$$

41. What is the value of $k$ ?
(a) 0
(b) -48
(c) 48
(d) 48/-16

Ans. (c) Initially, at $t=0$, Annie's height is 48 ft
So, at $t=0, h$ should be equal to 48

$$
h(0)=-16(0)^{2}+8(0)+k=48
$$

$$
\text { So, } \quad k=48
$$

Explanation: $\because$ Board is 48 feet above the water level and at $t=0$ seconds, Annie was standing on it

$$
\begin{array}{rlrl}
\therefore & & h(0) & =48 \mathrm{ft} \\
\Rightarrow & -16(0)^{2}+8(0)+k & =48 \\
\Rightarrow & & k & =48
\end{array}
$$

42. At what time will she touch the water in the pool?
(a) 30 seconds
(b) 2 seconds
(c) 1.5 seconds
(d) 0.5 seconds 1

Ans. (b) When Annie touches the pool, her height $=0$ feet.
i.e., $-16 t^{2}+8 t+48=0$ above water level

$$
\begin{aligned}
& 2 t^{2}-t-6=0 \\
& 2 t^{2}-4 t+3 t-6=0 \\
& 2 t(t-2)+3(t-2)=0 \\
&(2 t+3)(t-2)=0 \\
& \text { i.e.., } \quad t=2 \text { or } t= \\
&-\frac{3}{2} \quad \text { Since, time cannot be negative, so } \\
& t=2 \text { seconds. }
\end{aligned}
$$

Explanation: When Annie touch the pool, her height above the water level will be zero i.e., $h$ $(t)=0$

$$
\begin{array}{cc}
\therefore & -16 t^{2}+8 t+48=0 \\
\Rightarrow & -2 t^{2}+t+6=0
\end{array}
$$

$$
\begin{array}{lr}
\Rightarrow & -2 t^{2}+4 t-3 t+6=0 \\
\Rightarrow & -2 t(t-2)-3(t-2)=0 \\
\Rightarrow & (-2 t-3)(t-2)=0 \\
\Rightarrow & t=-\frac{3}{2}, 2
\end{array}
$$

Since, time cannot be negative

$$
\therefore \quad t=2
$$

So, Annie will touch the water in the pool after two seconds.
43. Rita's height (in feet) above the water level is given by another polynomial $p(t)$ with zeroes -1 and 2 . Then $p(t)$ is given by:
(a) $t^{2}+t-2$
(b) $t^{2}+2 t-1$
(c) $24 t^{2}-24 t+48$
(d) $-24 t^{2}+24 t+48$

Ans. (d) $t=-1 \& t=2$ are the two zeroes of the polynomial $p(t)$, then

$$
\begin{aligned}
p(t) & =k(t-(-1))(t-2) \\
& =k(t+1)(t-2)
\end{aligned}
$$

When $t=0$ (initially) $h=48 \mathrm{ft}$

$$
p(0)=k\left(0^{2}-0-2\right)=48
$$

i.e., $-2 k=48$

So the polynomial is $-24\left(t^{2}-t-2\right)$

$$
=-24 t^{2}+24 t+48
$$

Explanation: We know, a quadratic polynomial with zeroes, say $\alpha$ and $\beta$ is given as:

$$
p(t)=k\left(t^{2}-(\alpha+\beta) t+\alpha \beta\right),
$$

where $k$ is constant

$$
\begin{array}{ll}
\text { Here, } & \alpha \\
& \therefore
\end{array}
$$

Since, it is given that at $t=0$, height $=48 \mathrm{ft}$

$$
\begin{aligned}
\therefore & & p(0) & =48=k\left[(0)^{2}-0-2\right] \\
\Rightarrow & & -2 k & =48 \\
\Rightarrow & & k & =-24
\end{aligned}
$$

$\therefore$ The required polynomial is:

$$
\begin{aligned}
p(t) & =-24\left(t^{2}-t-2\right) \\
& =-24 t^{2}+24 t+48
\end{aligned}
$$

44. A polynomial $q(t)$ with sum of zeroes as 1 and the product as -6 is modelling Anu's height in feet above the water at any time $t$ (in seconds). Then $q(t)$ is given by:
(a) $t^{2}+t+6$
(b) $t^{2}+t-6$
(c) $-8 t^{2}+8 t+48$
(d) $8 t^{2}-8 t+48 \quad 1$

Ans. (c) A polynomial $q(t)$ with sum of zeroes as 1 and the product as -6 is given by $q(t)=$ $k$ ( $t^{2}$ - (sum of zeroes) $t$

+ product of zeroes)

$$
\begin{aligned}
& \quad=k\left(t^{2}-1 t+-6\right) \text {........(i) } \\
& \text { When } \quad t=0 \text { (initially) } q(0)=48 \mathrm{ft} \\
& q(0)=k\left(0^{2}-1(0)-6\right)=48 \\
& \text { i.e., } \quad-6 k=48 \text { or } k=-8 \\
& \text { Putting } k=-8 \text { in equation (i), required. } \\
& \text { polynomial is }-8\left(t^{2}-1 t-6\right)=-8 t^{2}+ \\
& 8 t+48
\end{aligned}
$$

45. The zeroes of the polynomial $r(t)=-12 t^{2}$ $+(k-3) t+48$ are negative of each other. Then $k$ is:
(a) 3
(b) 0
(c) -1.5
(d) -3

1
Ans. (a) When the zeroes are negative of each other,

$$
\text { Sum of zeroes }=0
$$

$$
\begin{aligned}
& \text { So, }-\frac{b}{a}=0,-\frac{(k-3)}{-12}=0, \frac{k-3}{12}=0 \\
& k-3=0, \text { i.e., } k=3
\end{aligned}
$$

Explanation: $r(t)=-12 t^{2}+(k-3) t+48$
Let one of the zeroes be $\alpha$.
Then, other zero $=-\alpha$
We know,

$$
\begin{array}{rlrl} 
& & \text { Sum of zeroes } & =-\frac{\text { Coefficient of } t}{\text { Coefficient of } t^{2}} \\
\Rightarrow & & \alpha+(-\alpha) & =-\frac{k-3}{-12} \\
\Rightarrow & 0 & =\frac{k-3}{12} \\
\Rightarrow & k-3 & =0 \Rightarrow k=3
\end{array}
$$

## Q.46-50 are based on Case Study -2

## Case Study-2 :

A hockey field is the playing surface for the game of hockey. Historically, the game was played on natural turf (grass) but nowadays it is predominantly played on an artificial turf. It is rectangular in shape - 100 yards by 60 yards. Goals consist of two upright posts placed equidistant from the centre of the backline, joined at the top by a horizontal crossbar. The inner edges of the posts must be 3.66 metres (4 yards) apart, and the lower edge of the crossbar must be 2.14 metres (7 feet) above the ground.
Each team plays with 11 players on the field during the game including the goalie. Positions you might play include-

- Forward: As shown by players A, B, C and D.
- Midfielders: As shown by players E, F and G.
- Fullbacks: As shown by players H, I and J.
- Goalie: As shown by player K

Using the picture of a hockey field below, answer the questions that follow:

46. The coordinates of the centroid of $\triangle E H J$ are:
(a) $\left(\frac{-2}{3}, 1\right)$
(b) $\left(1, \frac{-2}{3}\right)$
(c) $\left(\frac{2}{3}, 1\right)$
(d) $\left(\frac{-2}{3},-1\right)$
1

Ans.
(a) Centroid of $\triangle E H J$ with $E(2,1), H(-2,4)$ \& $J(-2,-2)$ is:
$\left(\frac{2+-2+-2}{3}, \frac{1+4+-2}{3}\right)=\left(-\frac{2}{3}, 1\right)$
Explanation: From the graph,
Coordinates of $\mathrm{E}=(2,1)$
Coordinates of $\mathrm{H}=(-2,4)$
Coordinates of $\mathrm{J}=(-2,-2)$
$\therefore \quad$ Centroid of $\triangle \mathrm{EHJ}$

$$
\begin{aligned}
& =\left(\frac{x_{1}+x_{2}+x_{3}}{3}, \frac{y_{1}+y_{2}+y_{3}}{3}\right) \\
& =\left(\frac{2+(-2)+(-2)}{3}, \frac{1+4+(-2)}{3}\right) \\
& =\left(-\frac{2}{3}, \frac{3}{3}\right)=\left(-\frac{2}{3}, 1\right)
\end{aligned}
$$

47. If a player $P$ needs to be at equal distances from $A$ and $G$, such that $A, P$ and $G$ are in straight line, then position of $P$ will be given by:
(a) $\left(\frac{-3}{2}, 2\right)$
(b) $\left(2, \frac{-3}{2}\right)$
(c) $\left(2, \frac{3}{2}\right)$
(d) $(-2,-3) 1$

Ans. (c) If $P$ needs to be at equal distance from $A(3,6)$ and $G(1,-3)$, such that $A, P$ and $G$ are collinear, then $P$ will be the mid point of $A G$.

So, coordinates of $P$ will be :

$$
\left(\frac{3+1}{2}, \frac{6+(-3)}{2}\right)=\left(2, \frac{3}{2}\right)
$$

48. The point on $x$ axis equidistant from $I$ and $E$ is:
(a) $\left(\frac{1}{2}, 0\right)$
(b) $\left(0, \frac{-1}{2}\right)$
(c) $\left(\frac{-1}{2}, 0\right)$
(d) $\left(0, \frac{1}{2}\right)$

Ans. (a) Let the point on x-axis equidistant from $I(-1,1)$ and $E(2,1)$ be $(x, 0)$ then

$$
\begin{aligned}
& \sqrt{(x+1)^{2}+(0-1)^{2}} \\
& \quad=\sqrt{(x-2)^{2}+(0-1)^{2}}
\end{aligned}
$$

$$
\begin{aligned}
& x^{2}+1+2 x+1=x^{2}+4-4 x+1 \\
& 6 x=3, \text { So } x=1 / 2 \\
& \text { The required point is }(1 / 2,0)
\end{aligned}
$$

Explanation: Let the required point on $x$-axis be $X(x, 0)$.

$$
\therefore \quad X I=E X
$$

Since, $I=(-1,1), E=(2,1)$
$\therefore$ Using distance formula,
$\Rightarrow \sqrt{(-1-x)^{2}+(1-0)^{2}}$

$$
\begin{array}{ccc} 
& =\sqrt{(x-2)^{2}+(0-1)^{2}} \\
\Rightarrow & & (-1-x)^{2}+1^{2}= \\
& (x-2)^{2}+(-1)^{2} \\
& & \text { (Squaring both sides) } \\
\Rightarrow & 1+2 x+x^{2}+1= & x^{2}-4 x+4+1 \\
\Rightarrow & & 2 x+2=-4 x+5 \\
\Rightarrow & & 6 x=3 \\
\Rightarrow & & x=\frac{3}{6}=\frac{1}{2} \\
& & \text { Required point on } x \text {-axis }=\left(\frac{1}{2}, 0\right)
\end{array}
$$

49. What are the coordinates of the position of a player Q such that his distance from K is twice his distance from $E$ and $K, Q$ and $E$ are collinear?
(a) $(1,0)$
(b) $(0,1)$
(c) $(-2,1)$
(d) $(-1,0)$

1
Ans. (b) Let the coordinates of the position of a player $Q$ such that his distance from $K(-4$,

1) is twice his distance from $E(2,1)$ be $Q(x$,
y) then $K Q: Q E=2: 1$

$$
\begin{aligned}
Q(x, y) & =\left(\frac{2 \times 2+1 \times(-4)}{3}, \frac{2 \times 1+1 \times 1}{3}\right) \\
& =(0,1)
\end{aligned}
$$

Explanation: Let the coordinates of Q be $(x, y)$.
From the graph, $K=(-4,1)$ and $E=(2,1)$
According to question,

$$
\begin{aligned}
& Q K=2 K E \\
& \Rightarrow \quad \frac{\mathrm{QK}}{\mathrm{KE}}=\frac{2}{1} \\
&
\end{aligned}
$$

$\because$ Points K, Q, E are collinear,
$\therefore$ Using section formula,

$$
\begin{aligned}
Q(x, y) & =\left(\frac{2 \times 2+1 \times(-4)}{2+1}, \frac{2 \times 1+1 \times 1}{2+1}\right) \\
& =\left(\frac{4-4}{3}, \frac{2+1}{3}\right) \\
& =\left(0, \frac{3}{3}\right)=(0,1)
\end{aligned}
$$

50. The point on $y$ axis equidistant from $B$ and C is:
(a) $(-1,0)$
(b) $(0,-1)$
(c) $(1,0)$
(d) $(0,1)$

1

Ans. (d) Let the point on $y$-axis equidistant from $B(4,3)$ and $C(4,-1)$ be $(0, y)$ then

$$
\begin{array}{r}
\sqrt{(4-0)^{2}+(3-y)^{2}} \\
=\sqrt{(4-0)^{2}+(y+1)^{2}} \\
16+y^{2}+9-6 y=16+y^{2}+1+2 y \\
-8 y=-8
\end{array}
$$

$$
\text { So, } y=1
$$

$\therefore$ the required point is $(0,1)$
Explanation: Let the required point on $y$-axis be $Y(0, y)$.

Then, according to question,

$$
\mathrm{BY}=\mathrm{CY}
$$

From the graph, $B=(4,3)$ and $C=(4,-1)$
$\therefore$ Using distance formula,

$$
\sqrt{(0-4)^{2}+(3-y)^{2}}=\sqrt{(0-4)^{2}+(y+1)^{2}}
$$

$$
\Rightarrow \quad(-4)^{2}+(3-y)^{2}=(-4)^{2}+(y+1)^{2}
$$

[Squaring both sides]
$\Rightarrow \quad 16+9-6 y+y^{2}=16+y^{2}+2 y+1$
$\Rightarrow \quad-6 y+25=2 y+17$
$\Rightarrow \quad-8 y=-8$
$\Rightarrow \quad y=1$
$\therefore$ Required point on $y$-axis $=(0,1)$

